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Graceful labeling of combination graph derived by coupling a graph with quadrilateral snake

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Abstract

We studied graphs which are quadrilateral with one chord, barycentric subdivision of quadrilateral with one chord, double quadrilateral snake, alternate quadrilateral snake and alternate double quadrilateral snake. It is observed that Graph derived by coupling barycentric subdivision of quadrilateral with one chord and alternate quadrilateral snake, graph derived by coupling barycentric subdivision of quadrilateral with one chord and double quadrilateral snake, graph derived by coupling barycentric subdivision of quadrilateral with one chord and alternate double quadrilateral snake and graph derived by coupling quadrilateral with one chord and alternate quadrilateral snake are graceful.

Keywords

Graceful labeling, Quadrilateral snake, barycentric subdivision, Double quadrilateral snake, Alternate quadrilateral snake, Alternate double quadrilateral snake.

AMS Subject Classification

05C78.

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Contents

1. Introduction

The concept of graceful labeling was introduced by Rosa [5] in 1967 and for numbering in graph was defined by S.W.Golomb [2]. Many researchers have studied gracefulness of graphs, refer Gallian survey [1]. A good number of papers are found with variety of applications in coding theory, radar communication, cryptography etc. A depth details about applications of graph labeling is found in Bloom and Golomb [2]. We accept all notations and terminology from Harary [3]. Let's look at towards some definitions which are used in this article.

A function f is called graceful labeling of a graph $G = (V,E)$ if $f : V \to \{0,1,...,q\}$ is injective and the induce function $f^* : E \to \{1,...,q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge $e = (u,v) \in E(G)$. A graph G is called graceful graph if it admits a graceful labeling [5].

Cycle is a closed trail in which the "first vertex = last vertex"

The quadrilateral snake Q_n is obtained from the path P_n by replacing every edge of a path by cycle C_4 [6].

A chord of a quadrilateral is an edge joining two non-adjacent vertices of quadrilateral [4].

An alternate quadrilateral snake $A(QS_n)$ is obtained from a path $u_1, u_2, ..., u_n$ by joined u_i, u_{i+1} (alternatively) to new vertices v_i and w_i respectively and adding edges $v_i\ w_i$ where

$(1 \leq i \leq n-1)$ for even n and $(1 \leq i \leq n-2)$ for odd n. That is every alternate edge of a path is replaced by a cycle C_4 [6] .

The double quadrilateral snake DQ_n consists of two quadrilateral snakes that have a common path [6].

An alternate double quadrilateral snake ADQ_n is consist of two alternate double quadrilateral snakes that have common path [6].

Let G = (V, E) be a graph. Let e = uv be an edge of G, and w is not a vertex of G. The edge e is subdivided when it is replaced by edges $e^{'} = uw$ and $e^{"} = wv$ [7].

Let G = (V, E) be a graph if every edge of graph G is subdivided, then the resulting graph is called barycentric subdivision of graph G. In other words barycentric subdivision is the graph obtained by inserting a vertex of degree 2 into every edge of original graph. The barycentric subdivision of any graph G is denoted by S(G). It is easy to observe that $|VS(G)| = |V(G)| + |E(G)|$ and $|ES(G)| = 2|E(G)|$ [7].

In this paper we introduced gracefulness of (i) graph derived by coupling barycentric subdivision of quadrilateral with one chord and alternate quadrilateral snake (ii) graph derived by coupling barycentric subdivision of quadrilateral with one chord and double quadrilateral snake (iii) graph derived by coupling barycentric subdivision of quadrilateral with one chord and alternate double quadrilateral snake (iv) Graph obtained by joining quadrilateral with one chord and alternate quadrilateral snake. For depth study of graph labeling, Gallian survey is refered [1].

2. Main Results:

2.1 Theorem

The graph derived by coupling barycentric subdivision of quadrilateral with one chord and alternate quadrilateral snake is graceful.

***Verification*:**

Here graph G = (V, E), derived by coupling two graphs. barycentric subdivision of quadrilateral with one chord G_1 and alternate quadrilateral snake G_2 by a path P_k of span k. If $\{u_1, u_2, u_3, u_4\}$ are vertices of quadrilateral with one chord and $\{x_1, x_2, x_3, x_4, x_5\}$ are inserted vertices due to barycentric subdivision that is $\{u_1x_1u_2x_2u_3x_3u_4x_4, u_2x_5u_4\}$ is of G_1 and $\{w_1, w_2, ..., w_k\}$ are vertices of alternate quadrilateral snake G_2 and $\{v_1, v_2, ..., v_k\}$ be the vertices of P_k with $v_1 = u_4$ and $v_k = w_1$

Below said Occurrences are considered.

***Occurrence-1*:** For odd value of k.

$f : v \to \{0, 1, ...q\}$ for q = 5j + (2l+8).

For vertices

$f(u_1)$ = 5j + (2l+8) $\quad$ $f(u_2)$ = 5j + (2l+7)

$f(u_3)$ = 5j + (2l+6) $\quad$ $f(u_4)$ = 5j + (2l+4)

$f(x_1) = 0$ $\quad$ $f(x_2) = 1$ $\quad$ $f(x_5) = 5$

$f(x_3) = 3$ $\quad$ $f(x_4) = 4$

$f(w_{4l-3})$ = 3l + (m+2) $\quad$ $f(w_{4l-2})$ = 5j + ((6+m)-2l)

$f(w_{4l-1})$ = 3l + (m+4) $\quad$ $f(w_{4l})$ = 5j + ((5+m)-2l)

$f(v_1)$ = 5j + (4+2l) $\quad$ $f(v_2) = 6$

$f(v_3)$ = 5j + (3+2l) $\quad$ $f(v_4) = 7$

.

$f(v_{2m-1})$ = 5j + ((5-m)+2l) $\quad$ $f(v_{2m})$ = 5 + m

.

$f(v_{2i})$ = 5 + i

2.2 Illustration

Graceful labeling of the graph derived by coupling barycentric subdivision of quadrilateral with one chord and alternate quadrilateral snake.

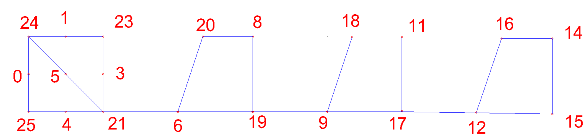

figure 1: The graph derived by coupling by 3 copies of alternate quadrilateral snake and barycentric subdivision of quadrilateral with one chord with p = 21 and q = 25 is graceful labeling.

***Occurrence-2*:** For even value of k.

$f : v \to \{0, 1, ...q\}$ for q = 5j + (2l+9).

For vertices

$f(u_1)$ = 5j + (2l+9) $\quad$ $f(u_2)$ = 5j + (2l+8)

$f(u_3)$ = 5j + (2l+7) $\quad$ $f(u_4)$ = 5j + (2l+5)

$f(x_1) = 0$ $\quad$ $f(x_2) = 1$ $\quad$ $f(x_5) = 5$

$f(x_3) = 3$ $\quad$ $f(x_4) = 4$

$f(w_{4l-3})$ = 5j + ((8+m)-3l) $\quad$ $f(w_{4l-2})$ = 2l + (4+m)

$f(w_{4l-1})$ = 5j + ((6+m)-3l) $\quad$ $f(w_{4l})$ = 2l + (5+m)

$f(v_1)$ = 5j + (5+2l) $\quad$ $f(v_2) = 6$

$f(v_3)$ = 5j + (4+2l) $\quad$ $f(v_4) = 7$

.

$f(v_{2m-1})$ = 5j + ((6-m)+2l) $\quad$ $f(v_{2m})$ = 5 + m

.

$f(v_{2i})$ = 5 + i

$(\forall\ j, l, m, i = N)$.

(j = no. of snakes in alternate quadrilateral, l = labeling in j^{th} graph, m = labeling in l^{th} graph, i = no. of vertices in a graph and N = Natural numbers)

From above results, conditions of graceful labeling is satisfied, hence graph G is graceful.

2.3 Theorem

The graph derived by coupling barycentric subdivision of quadrilateral with one chord and double quadrilateral snake is graceful.

***Verification*:**

Here graph G = (V, E), derived by coupling two graphs. barycentric subdivision of quadrilateral with one chord G_1 and double quadrilateral snake G_2 by a path P_k of span k.If $\{u_1,u_2,u_3,u_4\}$ are vertices of quadrilateral with one chord G_1 and $\{x_1,x_2,x_3,x_4,x_5\}$ are inserted vertices due to barycentric subdivision $\{u_1x_1u_2x_2u_3x_3u_4x_4,u_2x_5u_4\}$ of G_1 and vertex set $\{w_1,w_2,...,w_k,w'_1,...,w'_k,y_1,y_2,...,y_k\}$ of G_2. Here vertex set $\{v_1,v_2,...,v_k\}$ of P_k with $v_1 = u_4$ and $v_k = y_1$ and for G_2 join y_i to y_{i+1} (alternatively) to four new vertices w_i to w_{i+1}, w'_i and w'_{i+1} by the edges y_iw_i, w_iw_{i+1}, $w_{i+1}y_{i+1}$, $y_{i+1}w'_{i+1}$, $w'_iw'_{i+1}$
Below said Occurrences are considered.

***Occurrence-1*:** For odd value of k.
$f : v \rightarrow \{0,1,....q\}$ for q = 7j + (2l+9).
For vertices

$f(u_1)$ = 7j + (2l+9) $\quad$ $f(u_2)$ = 7j + (2l+8)
$f(u_3)$ = 7j + (2l+7) $\quad$ $f(u_4)$ = 7j + (2l+5)

$f(x_1) = 0$ $\quad$ $f(x_2) = 1$ $\quad$ $f(x_5) = 5$
$f(x_3) = 3$ $\quad$ $f(x_4) = 4$

$f(v_1)$ = 7j + (2l+5) $\quad$ $f(v_2) = 6$
$f(v_3)$ = 7j + (2l+4) $\quad$ $f(v_4) = 7$
.
$f(v_{2m-1})$ = 7j + (2l+(6-m)) $\quad$ $f(v_{2m})$ = 5 + m
.
$f(v_{2i})$ = 5 + i

$f(w_{4l-3})$ = 7j + ((12+m) -7l) $\quad$ $f(w_{4l-1})$ = 7l + (m + 2)
$f(w_{4l-2})$ = 7l + (m - 1) $\quad$ $f(w_{4l})$ = 7j + ((8+m) - 7l)

$f(w'_{4l-3})$ = 7j + ((10+m) -7l) $\quad$ $f(w'_{4l-1})$ = 7l + (m + 4)
$f(w'_{4l-2})$ = 7l + (m + 1) $\quad$ $f(w'_{4l})$ = 7j + ((6+m) - 7l)

$f(y_{2l-1})$ = 7l + (m-2) $\quad$ $f(y_{2l})$ = 7j + ((9+m) - 7l)

2.4 Illustration

Graceful labeling of the graph derived by coupling barycentric subdivision of quadrilateral with one chord and double quadrilateral snake.

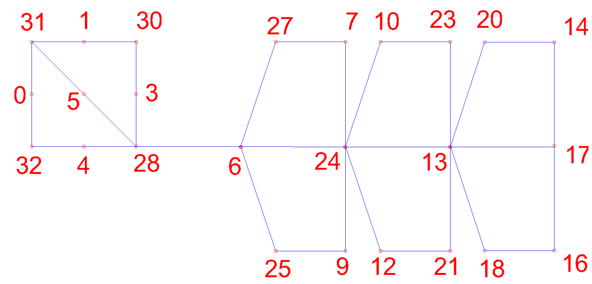

figure 2: The graph derived by coupling by 3 copies of double quadrilateral snake and barycentric subdivision of quadrilateral with one chord with p = 25 and q = 32 is graceful labeling.

***Occurrence-2*:** For even value of k.
$f : v \rightarrow \{0,1,....q\}$ for q = 7j + (2l+10).
For vertices

$f(u_1)$ = 7j + (2l+10) $\quad$ $f(u_3)$ = 7j + (2l+8)
$f(u_2)$ = 7j + (2l+9) $\quad$ $f(u_4)$ = 7j + (2l+6)

$f(x_1) = 0$ $\quad$ $f(x_2) = 1$ $\quad$ $f(x_5) = 5$
$f(x_3) = 3$ $\quad$ $f(x_4) = 4$

$f(v_1)$ = 7j + (2l+6) $\quad$ $f(v_2) = 6$
$f(v_3)$ = 7j + (2l+5) $\quad$ $f(v_4) = 7$
.
$f(v_{2m-1})$ = 7j + (2l+(7-m)) $\quad$ $f(v_{2m})$ = 5 + m
.
$f(v_{2i+1})$ = 7j + (2l+(6-i))

$f(w_{4l-3})$ = 7l + (m - 1) $\quad$ $f(w_{4l-1})$ = 7j + ((9+m) - 7l)
$f(w_{4l-2})$ = 7j + ((12+m) - 7l) $\quad$ $f(w_{4l})$ = 7l + (3+m)

$f(w'_{4l-3})$ = 7l + (m + 1) $\quad$ $f(w'_{4l-1})$ = 7j + ((7+m) -7l)
$f(w'_{4l-2})$ = 7j + ((10+m) -7l) $\quad$ $f(w'_{4l})$ = 7l + (5+m)

$f(y_{2l-1})$ = 7j + ((13+m) - 7l) $\quad$ $f(y_{2l})$ = 7l + (m+2)

$(\forall\ j = 1,2,....,\forall\ l = 1,2,...,\forall\ m = 1,2,...)$
(j = no. of snakes in double quadrilateral, l = labeling in j^{th} graph, m = labeling in l^{th} graph)

From above results, conditions of graceful labeling is satisfied, hence graph G is graceful.

2.5 Theorem

The graph derived by coupling barycentric subdivision of quadrilateral with one chord and alternate double quadrilateral snake is graceful.

***Verification*:**

Here graph G = (V, E), derived by coupling two graphs. barycentric subdivision of quadrilateral with one chord G_1

and alternate double quadrilateral snake G_2 by a path P_k of span k. If $\{u_1, u_2, u_3, u_4\}$ are vertices of quadrilateral with one chord G_1 and $\{x_1, x_2, x_3, x_4, x_5\}$ are inserted vertices due to barycentric subdivision $\{u_1x_1u_2x_2u_3x_3u_4x_4, u_2x_5u_4\}$ of G_1 and vertex set $\{w_1, w_2, ..., w_k, w'_1, ..., w'_k, y_1, y_2, ..., y_k\}$ of G_2. Here vertex set $\{v_1, v_2, ..., v_k\}$ of P_k with $v_1 = u_4$ and $v_k = y_1$ and for G_2 join y_i to y_{i+1} (alternatively) to four new vertices w_i to w_{i+1}, w'_i and w'_{i+1} by the edges y_iw_i, w_iw_{i+1}, $w_{i+1}y_{i+1}$, $y_iw'_{i+1}$, $w'_iw'_{i+1}$ and $y_iw'_i$. (i = 1,2,...j-1)
Below said Occurrences are considered.

***Occurrence-1*:** For odd value of k.
$f : v \to \{0, 1, ..., q\}$ for q = 7j + (11+2l).
For vertices
$f(u_1)$ = 7j + (2l+11) $f(u_3)$ = 7j + (2l+9)
$f(u_2)$ = 7j + (2l+10) $f(u_4)$ = 7j + (2l+7)

$f(x_1) = 0$ $f(x_2) = 1$ $f(x_5) = 5$
$f(x_3) = 3$ $f(x_4) = 4$

$f(v_1)$ = 7j + (2l+7) $f(v_2) = 6$
$f(v_3)$ = 7j + (2l+6) $f(v_4) = 7$
.
$f(v_{2m-1})$ = 7j + (2l+(8-m)) $f(v_{2m})$ = 5 + m
.
$f(v_{2i})$ = 5 + i

$f(w_{2l-1})$ = 8j + ((8+m) - 4l) $f(w_{2l})$ = 4l + (2+m)

$f(w'_{2l-1})$ = 8j + ((6+m) - 4l) $f(w'_{2l})$ = 4l + (4+m)

$f(y_{2l-1})$ = 4l + (m+1) $f(y_{2l})$ = 8j + ((5+m) - 4l)

2.6 Illustration

Graceful labeling of the graph derived by coupling barycentric subdivision of quadrilateral with one chord and alternate double quadrilateral snake.

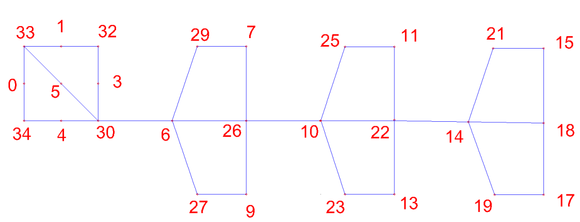

figure 3: The graph derived by coupling by 3 copies of alternate double quadrilateral snake and barycentric subdivision of quadrilateral with one chord with p = 27 and q = 34 is graceful labeling.

***Occurrence-2*:** For even value of k.
$f : v \to \{0, 1, ..., q\}$ for q = 7j + (12+2l).
For vertices
$f(u_1)$ = 7j + (2l+12) $f(u_3)$ = 7j + (2l+10)
$f(u_2)$ = 7j + (2l+11) $f(u_4)$ = 7j + (2l+8)

$f(x_1) = 0$ $f(x_2) = 1$ $f(x_5) = 5$
$f(x_3) = 3$ $f(x_4) = 4$

$f(v_1)$ = 7j + (2l+8) $f(v_2) = 6$
$f(v_3)$ = 7j + (2l+7) $f(v_4) = 7$
.
$f(v_{2m-1})$ = 7j + (2l + (9-m)) $f(v_{2m})$ = 5 + m
.
$f(v_{2i+1})$ = 7j + (2l +(8-i))

$f(w_{2l-1})$ = 4l + (2+m) $f(w_{2l})$ = 8j + ((8+m) - 4l)

$f(w'_{2l-1})$ = 4l + (4+m) $f(w'_{2l})$ = 8j + ((6+m) - 4l)

$f(y_{2l-1})$ = 8j + ((9+m) - 4l) $f(y_{2l})$ = 4l + (m+5)

$(\forall\ j, l, m, i = N)$.
(j = no. of snakes in alternate double quadrilateral, l = labeling in j^{th} graph, m = labeling in l^{th} graph, i = no. of vertices in a graph and N = Natural numbers)

From above results, conditions of graceful labeling is satisfied, hence graph G is graceful

2.7 Theorem

The graph derived by coupling quadrilateral with one chord and alternate quadrilateral snake is graceful.
***Verification*:**
Here graph G = (V, E), derived by coupling two graphs. Quadrilateral with one chord G_1 and alternate quadrilateral snake G_2 by a path P_k of span k. If $\{u_1, u_2, u_3, u_4\}$ are vertices of quadrilateral with one chord G_1 and $\{w_1, w_2, ..., w_k\}$ are vertices of alternate quadrilateral snake G_2. Here vertex set $\{v_1, v_2, ..., v_k\}$ of P_k with $v_1 = u_4$ and $v_k = w_1$
Below said Occurrences are considered.

***Occurrence-1*:** For odd value of k.
$f : v \to \{0, 1, ..., q\}$ for q = 5j + (3+2l).
For vertices
$f(u_1)$ = 5j + (2l+3) $f(u_3)$ = 5j + (2l+1)
$f(u_2) = 0$ $f(u_4) = 1$

$f(v_1) = 1$ $f(v_2)$ = 5j + (2l)
$f(v_3) = 2$ $f(v_4)$ = 5j + (2l - 1)
.
$f(v_{2m-1})$ = m $f(v_{2m})$ = 5j + (2l + (1-m))
.
$f(v_{2i})$ = 5j + (i+1)

$f(w_{4l-3})$ = 5j + ((4+m) - 3l) $f(w_{4l-2})$ = 2l + (m-1)
$f(w_{4l-1})$ = 5j + ((2+m) - 3l) $f(w_{4l})$ = 2l + m

2.8 Illustration

Graceful labeling of the graph derived by coupling quadrilateral with one chord and alternate quadrilateral snake.

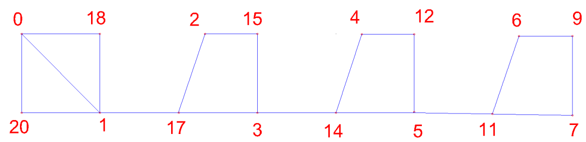

figure 4: The graph derived by coupling by 3 copies of alternate quadrilateral snake and quadrilateral with one chord with p = 16 and q = 20 is graceful labeling.

Occurrence-2**:** For even value of k.
$f : v \to \{0,1,....q\}$ for q = 5j + (4+2l).
For vertices
$f(u_1)$ = 5j + (2l+4) $f(u_3)$ = 5j + (2l+2)
$f(u_2) = 0$ $f(u_4) = 1$

$f(v_1) = 1$ $f(v_2)$ = 5j + (2l + 1)
$f(v_3) = 2$ $f(v_4)$ = 5j + (2l + 2)
.
$f(v_{2m-1})$ = m $f(v_{2m})$ = 5j + (2l + m)
.
$f(v_{2i+1})$ = i + 1

$f(w_{4l-3})$ = 3l + (m-2) $f(w_{4l-2})$ = 5j + ((3+m) - 2l)
$f(w_{4l-1})$ = 3l + m $f(w_{4l})$ = 5j + ((2+m) - 2l)

$(\forall\ j,l,m,i = N)$.
(j = no. of snakes in alternate quadrilateral, l = labeling in j^{th} graph, m = labeling in l^{th} graph, i = no. of vertices in a graph and N = Natural numbers)

From above results, conditions of graceful labeling is satisfied, hence graph G is graceful.

2.9 Concluding Remark

Current work come up with some new outcomes. We discussed gracefulness for (i) graph derived by coupling barycentric subdivision of quadrilateral with one chord and alternate quadrilateral snake (ii) graph derived by coupling barycentric subdivision of quadrilateral with one chord and double quadrilateral snake (iii) graph derived by coupling barycentric subdivision of quadrilateral with one chord and alternate double quadrilateral snake and (iv) Graph obtained by joining quadrilateral with one chord and alternate quadrilateral snake. The labeling example is exhibited by figures which gives easy explanations to determined outcomes.

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