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A note on multiplicative(generalized)-derivations and Lie ideals in prime and semiprime rings

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Abstract
Let R be a 2-torsion free semiprime ring and U be a square closed Lie ideal of R. A mapping $F: R \longrightarrow R$ is called a multiplicative(generalized)- derivation if there exists a map $d: R \longrightarrow R$ such that $F(xy) = F(x)y + xd(y)$, for all $x, y \in R$. Suppose that R admits a multiplicative(generalized)-derivation F associated with a map d such that $d(U) \subseteq U$. In the present paper, we shall prove that d is commuting on U if one of the following conditions holds: (i)$F([x,y]) = \pm[d(x),y], (ii) F(xoy) = \pm(d(x)oy), (iii) F([x,y]) = \pm(d(x)oy), (iv) F(xoy) = \pm[d(x)oy], (v) F([x,y]) = \pm[F(x),y], (vi) F(xoy) = \pm[F(x)oy], (vii) F([x,y]) = \pm[F(x)oy], (viii) F(xoy) = \pm[F(x),y]$ for all $x, y \in U$.

Keywords
Semiprime ring, Prime ring, Lie ideal, Multiplicative(generalized)-derivation.

AMS Subject Classification
16N60, 16W25.

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Contents

1. Introduction

Through out this paper R will denote an associative ring with centre $Z(R)$. For any $x, y \in R$, the symbol $[x,y]$ and xoy stands for the commutator $xy - yx$ and the anti-commutator $xy + yx$, respectively. A ring R is called 2-torsion free, if when ever $2x = 0$, with $x \in R$, then $x = 0$. Recall that a ring R is prime if for any $a, b \in R$, $aRb = 0$ implies $a = 0$ or $b = 0$ and is semiprime if for any $a \in R$, $aRa = 0$ implies $a = 0$. An additive subgroup U of R is said to be a Lie ideal of R if $[u,r] \in U$ for all $u \in U$ and $r \in R$. U is said to be a square closed Lie ideal of R if $u^2 \in U$ for all $u \in U$. Moreover if U is a square closed Lie ideal of R, then $2uv \in U$ for all $u, v \in U$. An additive mapping $d: R \longrightarrow R$ is called a derivation, if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. An additive mapping $F: R \longrightarrow R$ is called a generalized derivation of R, if there exists a derivation $d: R \longrightarrow R$ such that $F(xy) = F(x)y + xd(y)$ holds for any $x, y \in R$. Moreover, a mapping $f: R \longrightarrow R$ is called commuting on a subset S if $[f(x), x] = 0$ for all $x \in S$. The study of such mappings was initiated by E.C. Posner [17], which states that the existence of a non zero commuting derivation d on a prime ring R forces that R is commutative. For the development of the theory of commuting mappings and their applications on ring (see [6] where further references can be found).

The notion of a multiplicative derivation was introduced by Daif [7] and it was motivated by the work of Martindale [16]. According to Daif [7]: a map $d: R \longrightarrow R$ is called a multiplicative derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. These maps are not additive. Then the complete description of those maps was given by Goldmann and semrl in [12]. Further, Daif and Thammam-El-Sayiad in [11] extended the notion of multiplicative derivation to multiplicative generalized derivation if there exists a derivation d such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$, where $d: R \longrightarrow R$ is not necessarily additive. Dhara and Ali [10], made a slight generalization of Daif and Tamman El-Sayiad's definition of multiplicative(generalized)-derivation by considering d as any map. In [10], Dhara and Ali defined that a mapping $F: R \longrightarrow R$(not necessarily additive) is said to be multiplicative(generalized)-derivation if $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$, where d is any mapping(not necessarily a derivation nor an additive map). Obviously, every generalized derivation is a multiplicative generalized derivation on R but converse need not be true in general. Several authors have studied commutativity in prime, and semiprime rings with the help of derivations, generalized derivations satisfying appropriate algebraic conditions on some suitable subsets of the ring R (see [1], [2], [4] ,[9], [19]). Ashraf and Rehman [3], proved that a prime ring R, with a nonzero derivation d on R must be commutative. Dhara et al. [9], studied a result on generalized derivations on Lie ideals in prime rings. Recently, in [15] Koc and Golbasi proved multiplicative generalized derivations on Lie ideals of semiprime rings. In this line of investigation, it is more interesting to study the identities involving multiplicative(generalized)-derivations and lie ideals in prime and semiprime rings. The main objective of the present paper is to study commuting map on U if any one of the following holds: $(i)F([x,y]) = \pm[d(x),y], (ii)F(xoy) = \pm(d(x)oy), (iii)F([x,y]) = \pm(d(x)oy), (iv)F(xoy) = \pm[d(x)oy], (v)F([x,y]) = \pm[F(x),y], (vi)F(xoy) = \pm[F(x)oy], (vii)F([x,y]) = \pm[F(x)oy], (viii)F(xoy) = \pm[F(x),y]$ for all $x,y \in U$. Throughout this paper, R will be a 2-torsion free semiprime ring, U a square closed Lie ideal of R.

2. preliminary Results

Throughout the paper, we shall frequently use the following basic commutator and anti-commutator identities. For any $x,y,z \in R$.

$[x,yz] = y[x,z]+[x,y]z.$

$[xy,z] = [x,z]y+x[y,z].$

$xoyz = (xoy)z - y[x,z] = y(xoz)+[x,y]z.$

$xyoz = x(yoz) - [x,z]y = (xoz)y+x[y,z].$

The following lemmas will be used in the main results.

Lemma 2.1. $[5, Lemma\ 4]$ *If $U \nsubseteq Z(R)$ is a Lie ideal of a 2-torsion free semiprime ring R and $a,b \in R$ such that $aUb = 0$, then $a = 0$ or $b = 0$.*

Lemma 2.2. $[5, Lemma\ 5]$ *Let R be a prime ring with characteristic different from two and U a Lie ideal of R. If d is a nonzero derivation of R such that $d(U) = 0$, then $U \subseteq Z(R)$.*

Lemma 2.3. $[13, Lemma\ 1]$ *Let R be a semiprime ring with characteristic different from two, U a Lie ideal of R such that $[U,U] \subseteq Z(R)$. Then $U \subseteq Z(R)$.*

Corollary 2.4. $[14, Corollary\,(2.1)]$ *Let R be a 2-torsion free semiprime ring. U a Lie ideal of R such that $U \nsubseteq Z(R)$ and $a,b \in U$.*
(i) If $aUa = 0$, then $a = 0$.
(ii) If $aU = (0)$ (or Ua=(0)), then $a = 0$.
(iii) If U is a square closed and $aUb = (0)$, then $ab = 0$ and $b = 0$.

3. Main Results

Theorem 3.1. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative(generalized)-derivation F associated with a nonzero map d such that $d(U) \subseteq U$ and $F([x,y]) = \pm[d(x),y]$ for all $x,y \in U$, then d is commuting on U.*

Proof. By the assumption, we have

$$F[x,y] = [d(x),y] \ \ for\ all\ x,y \in U. \tag{3.1}$$

Replacing y by $2yx$ in (3.1) and using the fact that $char(R) \neq 2$, we get

$$F([x,y])x+[x,y]d(x) = y[d(x),x]+[d(x),y]x\ for\ all\ x,y \in U. \tag{3.2}$$

Using (3.1) in (3.2), we obtain

$$[x,y]d(x) = y[d(x),x] \ \ for\ all\ x,y \in U. \tag{3.3}$$

Using the fact $d(U) \subseteq U$, we replace y by $2d(x)y$ in (3.3) and use (3.3), we get

$$[x,d(x)]yd(x) = 0\ for\ all\ x,y \in U. \tag{3.4}$$

Writing $2yx$ for y in (3.4), we get

$$[x,d(x)]yxd(x) = 0\ for\ all\ x,y \in U. \tag{3.5}$$

Multiplying (3.4) by x on the right, we find that

$$[x,d(x)]yd(x)x = 0\ for\ all\ x,y \in U. \tag{3.6}$$

Subtracting (3.5) from (3.6), we get

$$[x,d(x)]y[x,d(x)] = 0\ for\ all\ x,y \in U.$$

That is

$$[x,d(x)]U[x,d(x)] = 0\ for\ all\ x \in U.$$

By Corollary(2.4) $[x,d(x)] = 0$ for all $x \in U$ and hence d is commuting on U. In a similar manner, we can prove that the same conclusion is true for $F[x,y] = -[d(x),y] = 0$ for all $x,y \in U$.Therefore the proof of the theorem is completed. □

Corollary 3.2. *Let R be a prime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero derivation d and $F[x,y] = \pm[d(x),y] = 0$ for all $x,y \in U$, then $U \subseteq Z(R)$.*

Proof. Assume that $U \nsubseteq Z(R)$. By the same technique in the proof of Theorem(3.1), we obtain that

$$[x,y]d(x) = y[d(x),x] \ \ for\ all\ x,y \in U. \tag{3.7}$$

Replacing y by $2yz$ in (3.7) and using (3.7), we have

$$[x,y]zd(x) = 0 \text{ for all } x,y \in U. \quad (3.8)$$

This implies that

$$[x,y]Ud(x) = 0 \text{ for all } x,y \in U.$$

By Lemma(2.1), we have either $[x,y] = 0$ or $d(x) = 0$ for all $x,y \in U$. Let $U_1 = \{x \in U/[x,y] = 0\}$ for all $y \in U$ and $U_2 = \{y \in U/d(x) = 0\}$. Then U_1 and U_2 are both additive subgroups of U such that U is set theoretic union of U_1 *and* U_2. By Brauer's trick, either $U_1 = U$ or $U_2 = U$. In the former case, $[U,U] = 0$. Then lemma(2.3) yields that $U \subseteq Z(R)$, a contradiction. In the latter case, $d(U) = 0$. By using lemma(2.2) we have $U \subseteq Z(R)$, again a contradiction. This completes the proof. □

Theorem 3.3. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative(generalized)-derivation F associated with a nonzero derivation d such that $d(U) \subseteq U$ and $F(xoy) = \pm(d(x)oy)$ for all $x,y \in U$, then d is commuting on U.*

Proof. By the assumption, we have

$$F(xoy) = d(x)oy \text{ for all } x,y \in U. \quad (3.9)$$

Replacing y by $2yx$ in (3.9) and using the fact that $char(R) \neq 2$

$$F(xoy)x + (xoy)d(x) = (d(x)oy)x - y[d(x),x] \text{ for all } x,y \in U. \quad (3.10)$$

Using (3.9), we arrive at

$$(xoy)d(x) = -y[d(x),x] \text{ for all } x,y \in U. \quad (3.11)$$

Substituting $2d(x)y$ for y in (3.11) , we get

$$[x,d(x)]yd(x) = 0 \text{ for all } x,y \in U. \quad (3.12)$$

This equation is same as (3.4) in the proof of $Theorem(3.1)$ and we get required result.
Similar proof shows that the same conclusion hold as $F(xoy) = -(d(x)oy)$ for all $x,y \in U$. □

Corollary 3.4. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero derivation $F(xoy) = \pm(d(x)oy)$ for all $x,y \in U$, then $U \subseteq Z(R)$.*

Proof. By the same technique in the proof of $Theorem(3.3)$, we obtain that

$$(xoy)d(x) = -y[d(x),x] \text{ for all } x,y \in U. \quad (3.13)$$

Taking y by $2yz$ in (3.13) and using (3.13), we arrive at

$$[x,y]zd(x) = 0 \text{ for all } x,y,z \in U.$$

Using Corollary(2.4), we get the required result. □

Theorem 3.5. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative(generalized)-derivation F such that $d(U) \subseteq U$ and $F[x,y] = \pm(d(x)oy)$ for all $x,y \in U$, then d is commuting on U.*

Proof. By the assumption, we have

$$F(xoy) = d(x)oy \text{ for all } x,y \in U. \quad (3.14)$$

Replacing y by $2yx$ in (3.14) and using the fact that $char(R) \neq 2$, we obtain

$$F(xoy)x + (xoy)d(x) = (d(x)oy)x - y[d(x),x] \text{ for all } x,y \in U. \quad (3.15)$$

Using (3.14), we arrive at

$$(xoy)d(x) = -y[d(x),x] \text{ for all } x,y \in U. \quad (3.16)$$

Substituting $2d(x)y$ for y in (3.16), we get

$$[x,d(x)]yd(x) = 0 \text{ for all } x,y \in U. \quad (3.17)$$

This equation is same as (3.4) in the proof of $Theorem(3.1)$ and we get required result.
The same argument can be adopted in case $F[x,y] = -(d(x)oy)$ for all $x,y \in U$. This proves the theorem completely. □

Corollary 3.6. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero derivation $F[x,y] = \pm(d(x)oy)$ for all $x,y \in U$, then $U \subseteq Z(R)$.*

Theorem 3.7. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative(generalized)-derivation F such that $d(U) \subseteq U$ and $F(xoy) = \pm[d(x),y]$ for all $x,y \in U$, then d is commuting on U.*

Proof. By the assumption, we have

$$F(xoy) = [d(x),y] \text{ for all } x,y \in U. \quad (3.18)$$

Replacing y by $2yx$ in (3.18) and using the fact that $char(R) \neq 2$, we get

$$F((xoy)x) + (xoy)d(x) = y[d(x),x] + [d(x),y]x \text{ for all } x,y \in U. \quad (3.19)$$

Using (3.18), we get

$$(xoy)d(x) = y[d(x),x] \text{ for all } x,y \in U. \quad (3.20)$$

Substituting $2d(x)y$ for y in (3.20), we get

$$[x,d(x)]yd(x) = 0 \text{ for all } x,y \in U. \quad (3.21)$$

This equation is same as (3.4) in the proof of Theorem (3.1) and we get required result.
Similar proof shows that the same conclusion hold as $F(xoy) = -[d(x),y]$ for all $x,y \in U$. □

Corollary 3.8. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero derivation $F[x,y] = \pm[d(x),y]$ for all $x,y \in U$, then $U \subseteq Z(R)$.*

Theorem 3.9. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative(generalized)-derivation F such that $d(U) \subseteq U$ and $F[x,y] = \pm[F(x),y]$ for all $x,y \in U$, then d is commuting on U.*

Proof. By the assumption, we have

$$F[x,y] = [d(x),y] \ for\ all\ x,y \in U. \tag{3.22}$$

Replacing y by $2yx$ in (3.22) and using the fact that $char(R) \neq 2$, we get

$$F([x,y])x + [x,y]d(x) = y[F(x),x] + [F(x),y]x \ for\ all\ x,y \in U. \tag{3.23}$$

Using (3.22), we obtain

$$[x,y]d(x) = y[F(x),x] \ for\ all\ x,y \in U. \tag{3.24}$$

Using the fact $d(U) \subseteq U$, we replace y by $2d(x)y$ in (3.21) and use (3.24), we get

$$[x,d(x)y]d(x) = d(x)y[F(x),y] \ for\ all\ x,y \in U.$$

So that

$$[x,d(x)]yd(x) = 0 \ for\ all\ x,y \in U. \tag{3.25}$$

Writing $2yx$ for y in (3.25), we get

$$[x,d(x)]yxd(x) = 0 \ for\ all\ x,y \in U. \tag{3.26}$$

Multiplying (3.25) by x on the right, we find that

$$[x,d(x)]yd(x)x = 0 \ for\ all\ x,y \in U. \tag{3.27}$$

Subtracting (3.26) from (3.27), we get

$$[x,d(x)]y[x,d(x)] = 0 \ for\ all\ x,y \in U.$$

That is $[x,d(x)]U[x,d(x)] = 0$ for all $x \in U$. Application of corollary(2.4) gives that $[x,d(x)] = 0$ for all $x \in U$ and hence d is commuting on U.
In a similar manner, we can prove that the same conclusion holds for $F[x,y] = -[F(x),y]$ for all $x,y \in U$. □

Corollary 3.10. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero derivation $F[x,y] = \pm[F(x),y]$ for all $x,y \in U$, then $U \subseteq Z(R)$.*

Theorem 3.11. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative(generalized)-derivation F satisfying $d(U) \subseteq U$ and $F(xoy) = \pm(F(x)oy)$ for all $x,y \in U$, then d is commuting on U.*

Proof. By the assumption, we have

$$F(xoy) = (F(x)oy) \ for\ all\ x,y \in U. \tag{3.28}$$

Replacing y by $2yx$ in (3.28) and using the fact that $char(R) \neq 2$, we get

$$F(xoy)x + (xoy)d(x) = (F(x)oy)x - y[F(x),x] \ for\ all\ x,y \in U. \tag{3.29}$$

Using (3.28), we arrive at

$$(xoy)d(x) = -y[F(x),x] \ for\ all\ x,y \in U. \tag{3.30}$$

Substituting $2d(x)y$ for y in (3.32), we get

$$[x,d(x)]yd(x) = 0 \ for\ all\ x,y \in U.$$

This equation is same as (3.30) in the proof of $Theorem(3.1)$ and we get required result.
By using similar argument we can get the result for the case $F(xoy) = -(F(x)oy)$ for all $x,y \in U$. This proves the theorem completely. □

Corollary 3.12. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero derivation $F(xoy) = \pm(F(x)oy)$ for all $x,y \in U$, then $U \subseteq Z(R)$.*

Theorem 3.13. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative(generalized)-derivation F satisfying $d(U) \subseteq U$ and $F[x,y] = \pm(F(x)oy)$ for all $x,y \in U$, then d is commuting on U.*

Proof. By the assumption, we have

$$F[x,y] = (F(x)oy) \ for\ all\ x,y \in U. \tag{3.31}$$

Replacing y by $2yx$ in (3.31) and using the fact that $char(R) \neq 2$, we get

$$F(xoy)x + (xoy)d(x) = y[F(x),x] + [F(x),y]x \ for\ all\ x,y \in U. \tag{3.32}$$

Using (3.31), we arrive at

$$(xoy)d(x) = -y[F(x),x] \ for\ all\ x,y \in U. \tag{3.33}$$

Substituting $2d(x)y$ for y in (3.33), we get

$$[x,d(x)]yd(x) = 0 \ for\ all\ x,y \in U. \tag{3.34}$$

This equation is same as (3.25) in the proof of $Theorem\,(3.9)$ and we get required result.
By repeating the arguments with necessary variations, we can get the same conclusion for the identity $F[x,y] = -(F(x)oy)$ for all $x,y \in U$. □

Corollary 3.14. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero derivation $F[x,y] = \pm(F(x)oy)$ for all $x,y \in U$, then $U \subseteq Z(R)$.*

Theorem 3.15. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative(generalized)-derivation F satisfying $d(U) \subseteq U$ and $F(xoy) = \pm[F(x),y]$ for all $x,y \in U$, then d is commuting on U.*

Proof. By the assumption, we have

$$F(xoy) = [F(x),y] \text{ for all } x,y \in U. \tag{3.35}$$

Replacing y by $2yx$ in (3.35) and using the fact that $char(R) \neq 2$, we get

$$F(xoy)x + (xoy)d(x) = [F(x),y]x + y[F(x),x] \text{ for all } x,y \in U. \tag{3.36}$$

Using (3.35), we arrive at

$$(xoy)d(x) = y[F(x),x] \text{ for all } x,y \in U. \tag{3.37}$$

Substituting $2d(x)y$ for y in (3.37), we get

$$[x,d(x)]yd(x) = 0 \text{ for all } x,y \in U. \tag{3.38}$$

This equation is same as (3.25) in the proof of $Theorem\,(3.9)$ and we get required result.
In a similar manner, we can prove that the same conclusion holds for $F(xoy) = -[F(x),y]$ for all $x,y \in U$. □

Corollary 3.16. *Let R be a semiprime ring with characteristic different from two and U be a square closed Lie ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero derivation $F(xoy) = \pm[F(x),y]$ for all $x,y \in U$, then $U \subseteq Z(R)$.*

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