Malaya Journal of Matematik, Vol. 9, No. 1, 993-995, 2021

https://doi.org/10.26637/MJM0901/0174

Color class dominating sets in ladder and grid graphs

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Abstract

Let $G=(V,E)$ be a graph. A color class dominating set of G is a proper coloring $\mathscr{C}$ of G with the extra property that every color class in $\mathscr{C}$ is dominated by a vertex in G. A color class dominating set is said to be a minimal color class dominating set if no proper subset of $\mathscr{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_x(G)$. In this paper, we obtain $\gamma_x(G)$ for Ladder graph and Grid graph.

Keywords

Chromatic number, domination number, color class dominating set, color class domination number.

AMS Subject Classification

05C15, 05C69.

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Article History: Received **24** November **2020**; Accepted **18** February **2021**

Contents

1. Introduction

All graphs considered in this paper are finite, undirected graphs and we follow standard definitions of graph theory as found in [3].

Let $G=(V,E)$ be a graph of order p. The open neighborhood of a vertex $v \in V(G)$ is $N(v)=\{u \in V(G)/uv \in E(G)\}\ldots$ The closed neighborhood of $vi_is_N[v]=N(v)\cup\{v\}$. For a set $S\subseteq V$, the open neighborhood $N(S)$ is defined to be $\bigcup_{v\in S}N(v)$, and the closed neighborhood of S is $N[S]=N(S)\cup S$.

A subset S of V is called a dominating set if every vertex in $V-S$ is adjacent to some vertex in S. A dominating set is minimal dominating set if no proper subset of S is a dominating set of G. The domination number $\gamma(G)$ is the minimum cardinality taken over all minimal dominating sets of G. A γ -set is any minimal dominating set with cardinality γ.

A proper coloring of G is an assignment of colors to the vertices of G, such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$.A color class dominating set of G is a proper coloring $\mathscr{C}$ of G with the extra property that every color classes in $\mathscr{C}$ is dominated by a vertex in G.A color class dominating set is said to be a minimal color class dominating set if no proper subset of C is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_\chi(G)$. This concept was introduced by A. Vijayalekshmi et all [2].

A cartesian product of two subgraphs G_1 and G_2 is the graph $G_1\times G_2$ such that its vertex set is

$$V(G_1\times G_2)=\{(x,y)/x\in V(G_1), y\in V(G_2)\}$$

and the edge set is $E(G_1\times G_2)=\{((x_1,x_2),(y_1,y_2))/x_1=y_1$ and $(x_2,y_2)\in E(G_2)$ or $x_2=y_2$ and $(x_1,y_1)\in E(G_1)\}$.The ladder graph is defined by $L_{2n}=P_2\times P_n$, where P_n is a path graph with n vertices.A two dimensional grid graph G_m^n is the Cartesian product of path graphs P_m and P_n.

2. Main Results

Theorem 2.1. *The ladder graph L_p has $\gamma_\chi(L_p)=\lfloor\frac{p}{3}\rfloor+r$ if $p\equiv r(\bmod 3)$ where $r=0,1,2$.*

Proof. Let $L_p = L_{2n} = P_2 \times P_n$ and let

$$V(L_p) = \{u_1, u_2, \ldots, u_{n+1}, \ldots, u_{2n}\}$$

with $\deg(u_i) = 2$ for $i = 1, n, (n+1), 2n$ and $\deg(u_j) = 3$ for all $j \neq i$. We take $\mathrm{N}(u_i) = \{u_{i-1}, u_{i+1}, u_{i+n}\}$ for $i = 2, 3, \ldots, n-1$ and $\mathrm{N}(u_j) = \{u_{j-1}, u_{j+1}, u_{j-n}\}$ for $j = (n+2), (n+3), \ldots, (2n-1)$. We consider three cases.

Case (1). $p \equiv 0(\bmod 3)$.

Decompose L_p into $\frac{p}{3}$ copies of L_6. Assign new colors, say, $2i = 1, 2i\left(1 \leq i \leq \frac{p}{3}\right)$ to the vertices $N(u_i)$ for $i = 2, 5, \ldots, (n-1)$ and $i = (n+2), (n+5), \ldots, (2n-1)$, We get a γ_χ -coloring of L_p. Thus $\gamma_\chi(L_p) = \frac{p}{3}$.

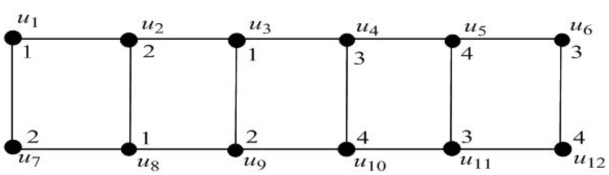

Figure- 1.

$$\gamma_\chi(L_{12}) = \left\lceil \frac{12}{3} \right\rceil + 0 = 4$$

Case (2). $p \equiv 1(\bmod 3)$.

In this case L_p is obtained by L_{p-4} followed by L_4.Since $p-4 \equiv 0(\bmod 3)$ and by case (1), $\gamma_\chi(L_{p-4}) = \frac{p-4}{3}$. Assign two new colors, say, $\left(\frac{p-4}{3}\right)+1$ and $\left(\frac{p-4}{3}\right)+2$ to the vertices $\{u_{n-1}, u_{2n}\}$ and $\{u_n, u_{2n-1}\}$ respectively, we get a γ_χ -coloring of L_p. So $\gamma_\chi(L_p) = \gamma_\chi(L_{p-4}) + 2 = \frac{p-4}{3} + 2 = \frac{p+2}{3} = \left\lfloor \frac{p}{3} \right\rfloor + 1$

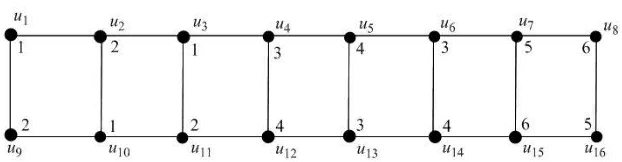

Figure- 2.

$$\gamma_\chi(L_{16}) = \left| \frac{16}{3} \right| + 1 = 6$$

Case (3). $p \equiv 2(\bmod 3)$.

In this case L_p is obtained by L_{p-2} followed by L_2. Since $p-2 \equiv 0(\bmod 3)$ and by case (1), $\gamma_\chi(L_{p-2}) = \frac{p-2}{3}$. Assign two new colors, say, $\frac{p-2}{3}+1$ and $\frac{p-2}{3}+2$ to the vertices $\{u_n\}$ and $\{u_{2n}\}$ respectively, we get a γ_χ -coloring of L_p. So $\gamma_\chi(L_p) = \gamma_\chi(L_{p-2}) + 2 = \frac{p-2}{3} + 2 = \frac{p+4}{3} = \left[\frac{p}{3}\right] + 2$.

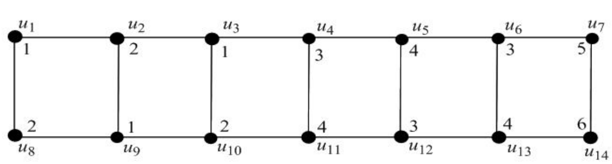

Figure- 3.

$$\gamma_\chi(L_{14}) = \left\lfloor \frac{14}{3} \right\rfloor + 1 = 6$$

□

Theorem 2.2. *The Grid* graph $G_m^n = P_m \times P_n$ *has* $\gamma_\chi(G_m^n) = \left\lfloor \frac{mn}{3} \right\rfloor + r$ *if* $mn \equiv r(\bmod 3)$ *where* $r = 0, 1, 2$.

Proof. Let $G_m^n = P_m \times P_n$ and

$$V(G_m^n) = \left\{ v_j^i / 1 \leq i \leq m, 1 \leq j \leq n \right\}.$$

We consider three cases.

Case (1). $mn \equiv 0(\bmod 3)$.

We have two subcases.

Subcase (1.1). $m \equiv 0(\bmod 3)$ and $n \equiv 0(\bmod 3)$.

Decompose G_m^n into $\frac{n}{3}$ copies of G_m^3. Assign $m-2$ distinct colors, say, $1, 3, 4, \ldots, m-1$ to the vertices $\left\{v_1^i, v_3^i, v_2^{i+1}\right\}$ for $i = 1, 3, 4, \ldots, n-1$ and assign colors 2 and m to the vertices $\left\{v_2^1, v_1^2, v_3^2, v_2^3\right\}$and $\left\{v_1^n, v_3^n\right\}$ respectively,we get a γ_χ -coloring of G_m^3. Similarly assign m distinct colors, say, $m+1, m+2, \ldots, 2m$ to the identical vertices of second copy of G_m^3 and so on. So $\gamma_\chi(G_m^n) = \frac{n}{3} \times m = \frac{mn}{3}$.

Subcase (1.2). $m \equiv 0(\bmod 3)$ and $n \equiv 1(\bmod 3)$.

In this case, G_m^n is obtained by G_m^{n-4} followed by G_m^4. Since $n-4 \equiv 0(\bmod 3)$, by subcase(1.1), $\gamma_\chi(G_m^n) = \gamma_\chi\left(G_m^{n-4}\right) \times \gamma_\chi\left(G_m^4\right) = \frac{m(n-4)}{3} + 2\gamma_\chi(L_{2m}) = \frac{m(n-4)}{3} + 2 \times \left(\frac{2m}{3}\right) = \frac{mn}{3}$.

Subcase (1.3). $m \equiv 0(\bmod 3)$ and $n \equiv 2(\bmod 3)$.

In this case, G_m^n is obtained by G_m^{n-2} followed by G_m^2. Since $n-2 \equiv 0(\bmod 3)$, by subcase (1.1), $\gamma_\chi(G_m^n) = \gamma_\chi\left(G_m^{n-2}\right) \times \gamma_\chi(G_m^n) = \frac{m(n-2)}{3} + \gamma_\chi(L_{2m}) = \frac{m(n-2)}{3} + \left(\frac{2m}{3}\right) = \frac{mn}{3}$.

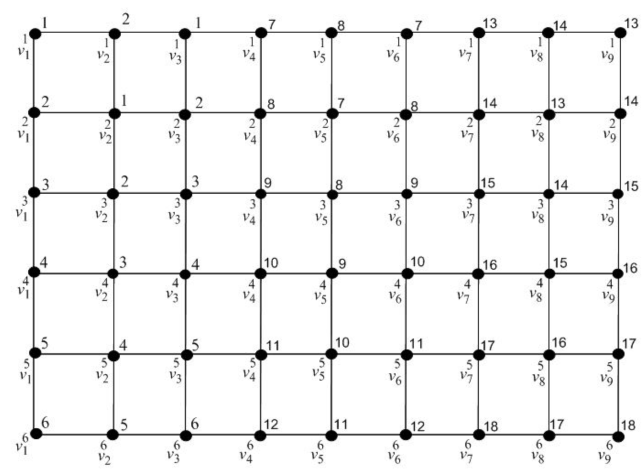

Figure- 4.

$$\gamma_\chi\left(G_9^6\right) = \left\lceil \frac{9 \times 6}{3} \right\rceil + 0 = 18$$

Case (2). $mn \equiv 1(\bmod 3)$.

We have two subcases.

Subcase (2.1). $m \equiv 1(\bmod 3)$ and $n \equiv 1(\bmod 3)$.

In this case G_m^n is obtained by G_m^{n-4} followed by G_m^4.Since $G_m^4 \cong 2(L_{2n})$, in the γ_χ - coloring of first copy of L_{2n} two unique vertices $\left\{v_{m-3}^n\right\}$ and $\left\{v_{m-1}^n\right\}$ have two distinct colors, say, $\lfloor \frac{mn}{3} \rfloor$ and $\lceil \frac{mn}{3} \rceil + 1$ respectively. In the γ_χ -coloring of second copy of L_{2n}, the same unique colors say, $\lceil \frac{mn}{3}$ and $\lceil \frac{mn}{3} \rceil + 1$ are assigned to the identical vertices, say, $\{v_{m-2}^n\}$ and $\{v_m^n\}$ respectively. So$\gamma_\chi(G_m^4) = 2\left\lceil \frac{2(n-1)}{3} \right\rceil + 2$. Thus $\gamma_\chi(G_m^n) = \left\lceil \frac{(m-4)n}{3} \right\rceil + 2\left\lceil \frac{2(n-1)}{3} \right\rceil + 2 = \lceil \frac{mn}{3} \rceil + 1$.

Subcase (2.2). $m \equiv 2(\bmod 3)$ and $n \equiv 2(\bmod 3)$.
Since $n-2 \equiv 0(\bmod 3)$ by subcase (1.2), $\gamma_\chi(G_m^n) = \gamma_\chi(G_m^{n-2}) \times \gamma_\chi(G_m^2) = \frac{m(n-2)}{3} + \lfloor \frac{2m}{3} \rceil + 1 = \frac{mn}{3} + 1$.

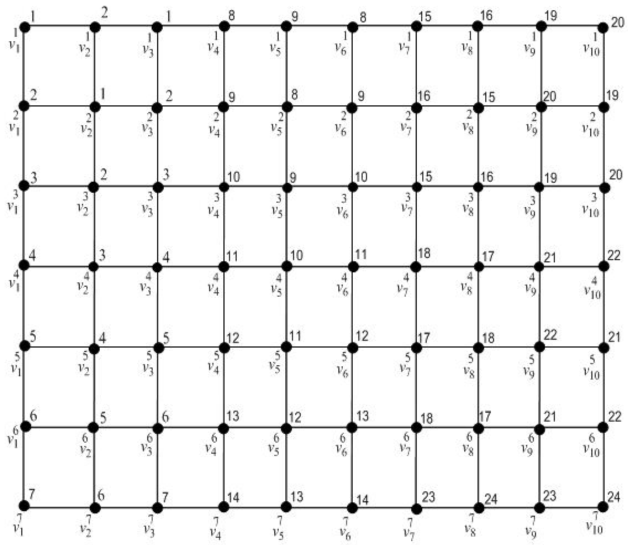

Figure- 5.

$$\gamma_\chi(G_{10}^7) = \left\lfloor \frac{10 \times 7}{3} \right\rceil + 1 = 24$$

Case (3). $mn \equiv 2(\bmod 3)$.
We have two subcases.

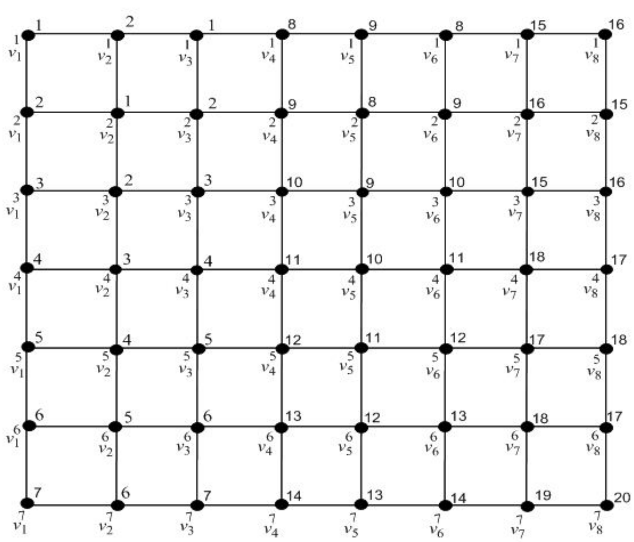

Figure- 6.

Subcase (3.1). $m \equiv 1(\bmod 3)$ and $n \equiv 2(\bmod 3)$.
In this case G_m^n is obtained by G_m^{n-2} followed by G_m^2.Since $n-2 \equiv 0(\bmod 3)$ by subcase(1.2) $\gamma_\chi(G_m^n) = \gamma_\chi(G_m^{n-2}) \times \gamma_\chi(G_m^2) = \frac{m(n-2)}{3} + \lfloor \frac{2m}{3} \rceil + 2 = \frac{mn}{3} + 2$.

Subcase (3.2). $m \equiv 2(\bmod 3)$ and $n \equiv 1(\bmod 3)$. Interchanging m and n in subcase(3.1) $\gamma_\chi(G_m^n) = \frac{(m-2)n}{3} + \lfloor \frac{2n}{3} \rceil + 2 = \frac{mn}{3} + 2$.

$$\gamma_\chi(G_8^7) = \left\lfloor \frac{8 \times 7}{3} \right\rceil + 2 = 20$$

□

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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