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New forms of open and closed sets using (1,2)S_β-open sets in bitopological spaces

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Abstract

The aim of this paper is to define new forms of open and closed sets known as $\underset{(1,2)S_\beta}{\wedge}$-set and $\underset{(1,2)S_\beta}{\vee}$-set using (1,2)$S_\beta$-open sets in bitopological spaces and study some of their properties.

Keywords

(1,2)semi-open sets, (1,2)S_β-open sets, (1,2)S_β-Interior, (1,2)β-closed sets, (1,2)S_β-Closure, $\underset{(1,2)S_\beta}{\wedge}$-set, $\underset{(1,2)S_\beta}{\vee}$-set.

AMS Subject Classification

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Contents

1. Introduction

In the year 1963, Kelly introduced the systematic study of bitopology which is a triple (X,τ,σ), where X is a non-empty set together with two distinct topologies τ, σ on X. Levine initiated semi-open sets and their properties in 1963. In 1983, Abd-El-monsef introduced the notion of β - open sets and β - continuity in topological spaces. In 1986, Maki introduced some forms of open and closed sets known as $\wedge$-sets and $\vee$-sets. In 2013, Alias B.Khalaf and Nehmat K.Ahmed introduced and defined a class of semi-open sets called S_β - open sets in topological spaces. The aim of this paper is to define new forms of open and closed sets known as $\underset{(1,2)S_\beta}{\wedge}$-set and $\underset{(1,2)S_\beta}{\vee}$-set using (1,2)$S_\beta$-open sets in bitopological spaces and study some of their properties.

2. Preliminaries

Definition 2.1 ([5]). *Let A be a subset of a bitopological space $(X,\ \tau_1,\ \tau_2)$. Then A is said to be*

(i) $\tau_1\tau_2$ *- open if* $A \in \tau_1 \cup \tau_2$,

(ii) $\tau_1\tau_2$ *- closed if* $A^c \in \tau_1 \cup \tau_2$,

(iii) (1,2)β-open if $A \subseteq \tau_1\tau_2 - cl(\tau_1 - int(\tau_1\tau_2 - cl(A)))$, *where τ_1-Int(A) is the interior of A with respect to the topology τ_1 and $\tau_1\tau_2$-Cl(A) is the intersection of all $\tau_1\tau_2$-closed sets containing A.*

(iv) (1,2)β-Int(A) is the union of all (1,2)β-open sets contained in A.

(v) (1,2)β-Cl(A) is the intersection of all (1,2)β-closed sets containing A.

Definition 2.2 ([5]). *A subset A of X is said to be*

(i) (1,2)semi-open if $A \subseteq \tau_1\tau_2\text{-}Cl(\tau_1\text{-}Int(A))$,

(ii) (1,2)regular-open if $A = \tau_1\text{-}Int(\tau_1\tau_2\text{-}Cl(A))$,

(iii) *(1,2)β-open if* $A \subseteq \tau_1\tau_2\text{-}Cl(\tau_1\text{-}Int(\tau_1\tau_2\text{-}Cl(A)))$.

The set of all (1,2)semi-open, (1,2)regular-open, (1,2)β-open are denoted as (1,2)SO(X,τ_1,τ_2)*, (1,2)RO*(X,τ_1,τ_2)*, (1,2)β O*(X,τ_1,τ_2) *or simply, (1,2)SO(X), (1,2)RO(X), (1,2)βO(X) respectively.*

Definition 2.3 ([4])**.** *A subset A of X is said to be*

(i) *(1,2)semi-closed if* $\tau_1\tau_2\text{-}Int(\tau_1\text{-}Cl(A)) \subseteq A$.

(ii) *(1,2)regular-closed if* $A = \tau_1\text{-}Cl(\tau_1\tau_2\text{-}Int(A))$

(iii) *(1,2)β- closed if* $\tau_1\tau_2\text{-}Int(\tau_1 - Cl(\tau_1\tau_2\text{-}Int(A))) \subseteq A$.

The set of all (1,2)semi-closed, (1,2)regular-closed, (1,2)β-closed are denoted as (1,2)SCL(X,τ_1,τ_2)*, (1,2)RCL*(X,τ_1,τ_2)*, (1,2)βCL*(X,τ_1,τ_2) *or simply, (1,2)SCL(X), (1,2)RCL(X), (1,2) βCL(X) respectively.*

Remark 2.4 ([5])**.** *For any subset A of X,*

(i) $\tau_1\text{-}Int(A) \subseteq \tau_1\tau_2\text{-}Int(A)$ *and* $\tau_2\text{-}Int(A) \subseteq \tau_1\tau_2\text{-}Int(A)$.

(ii) $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_1\text{-}Cl(A)$ *and* $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_2\text{-}Cl(A)$.

(iii) $\tau_1\tau_2\text{-}Cl(A\cap B) \subseteq \tau_1\tau_2\text{-}Cl(A) \cap \tau_1\tau_2\text{-}Cl(B)$.

(iv) $\tau_1\tau_2\text{-}Int(A) \cup \tau_1\tau_2\text{-}Int(B) \subseteq \tau_1\tau_2\text{-}Int(A\cup B)$.

Theorem 2.5 ([3])**.** *Let* (X,τ_1,τ_2) *be a bitopological space. If* $A \in \tau_1$ *and* $B \in (1,2)SO(X)$*, then* $A\cap B \in$ *(1,2)SO(X).*

Theorem 2.6 ([2])**.** *Let* $A \subset Y \subset (X,\tau_1,\tau_2)$ *and if A is* τ_i*-semi open in X, then A is* τ_i*-semi open in Y.*

Definition 2.7 ([8])**.** *A (1,2)semi-open subset A of a bitopological space* (X,τ_1,τ_2) *is said to be (1,2)*S_β*-open if for each* $x \in A$ *there exists a (1,2)β-closed set F such that* $x \in F \subseteq A$.

Theorem 2.8 ([8])**.** *Let* $\{A_\alpha : \alpha \in \Delta\}$ *be a family of (1,2)*S_β*-open sets in a bitopological space* (X,τ_1,τ_2)*.Then* $\bigcup_{\alpha\in\Delta} A_\alpha$ *is also a (1,2)*S_β*-open set.*

3. $\underset{(1,2)S_\beta}{\wedge}$ - set and $\underset{(1,2)S_\beta}{\vee}$ - set

Definition 3.1. *In a bitopological space X, a subset B of X is said to be (1,2)*S_β*-∧-set (*$\underset{(1,2)S_\beta}{\wedge}$*-set) if* $B = B^{\overset{\wedge}{(1,2)S_\beta}}$*, where*

$B^{\overset{\wedge}{(1,2)S_\beta}} = \cap\{G/G \supseteq B$ *and* $G \in (1,2)S_\beta\text{-}O(X)\}$.

Definition 3.2. *In a bitopological space X, a subset B of X is said to be (1,2)*S_β*-∨-set (*$\underset{(1,2)S_\beta}{\vee}$*-set) if* $B = B^{\overset{\vee}{(1,2)S_\beta}}$*, where*

$B^{\overset{\vee}{(1,2)S_\beta}} = \cup\{F/F \subseteq B$ *and* $F \in (1,2)S_\beta CL(X)\}$.

The family of all (1,2)S_β-∧-sets (resp.(1,2)S_β-∨-sets) is denoted by $\underset{(1,2)S_\beta}{\wedge}$ - O(X) (resp. $\underset{(1,2)S_\beta}{\vee}$ - O(X)).

Example 3.3. *Let X = {a, b, c, d } with two topologies* τ_1 *= {ϕ, X, {b}, {a, c}, {a, b, c}, {a, c, d}} and* τ_2 *= {ϕ, X}. Then, (1,2)*S_β*-O(X) = {ϕ, X, {b}, {a, c}, {a, b, c}, {a, c, d}} and (1, 2)*S_β*-CL(X) = {ϕ, X, {b}, { d } , {b, d}, {a,c,d} }. Therefore,* $\underset{(1,2)S_\beta}{\wedge}$ *-O(X) = {ϕ, X, {b}, {a, c}, {a, b, c}, {a, c, d}} and* $\underset{(1,2)S_\beta}{\vee}$ *- O(X) = {ϕ, X, {b}, { d } , {b, d}, {a,c,d} }.*

Proposition 3.4. *Let A and B be two subsets of a bitopological space X. Then the following properties are hold.*

(i) $B \subseteq B^{\overset{\wedge}{(1,2)S_\beta}}$.

(ii) $B^{\overset{\vee}{(1,2)S_\beta}} \subseteq B$.

(iii) *If* $A \subseteq B$*, then* $A^{\overset{\wedge}{(1,2)S_\beta}} \subseteq B^{\overset{\wedge}{(1,2)S_\beta}}$.

(iv) $(B^{\overset{\wedge}{(1,2)S_\beta}})^{\overset{\wedge}{(1,2)S_\beta}} = B^{\overset{\wedge}{(1,2)S_\beta}}$.

(v) *If* $A \in (1,2)S_\beta O(X)$*, then* $A = A^{\overset{\wedge}{(1,2)S_\beta}}$.

(vi) $(B^c)^{\overset{\wedge}{(1,2)S_\beta}} = (B^{\overset{\vee}{(1,2)S_\beta}})^c$*, (i.e)*$(X–B)^{\overset{\wedge}{(1,2)S_\beta}} = X – B^{\overset{\vee}{(1,2)S_\beta}}$

Proof. (i) and (ii) are obvious from the definitions 3.1 and 3.2.
(iii) Let $x \notin B^{\overset{\wedge}{(1,2)S_\beta}}$. Then there exists a (1,2)S_β-open set G containing B such that $x \notin G$. That is, $x \notin G \supseteq B \supseteq A$. Hence, $x \notin A^{\overset{\wedge}{(1,2)S_\beta}}$.
(iv)By using (i) and (iii) we have, $B \subseteq (B^{\overset{\wedge}{(1,2)S_\beta}})$ and $B^{\overset{\wedge}{(1,2)S_\beta}} \subseteq (B^{\overset{\wedge}{(1,2)S_\beta}})^{\overset{\wedge}{(1,2)S_\beta}}$.
Also By definition 3.1, $(B^{\overset{\wedge}{(1,2)S_\beta}})^{\overset{\wedge}{(1,2)S_\beta}} \subseteq (B^{\overset{\wedge}{(1,2)S_\beta}})$. Hence, $(B^{\overset{\wedge}{(1,2)S_\beta}})^{\overset{\wedge}{(1,2)S_\beta}} = B^{\overset{\wedge}{(1,2)S_\beta}}$.
(v)By definition 3.1 and as $A \in$ (1,2)S_βO(X). We have, $A^{\overset{\wedge}{(1,2)S_\beta}} \subseteq A$. By (i)
$A \subseteq A^{\overset{\wedge}{(1,2)S_\beta}}$. Therefore, $A^{\overset{\wedge}{(1,2)S_\beta}} = A$.
(vi) $(B^{\overset{\vee}{(1,2)S_\beta}})^c = \cup\{F/F \subseteq B$ and $F \in (1,2)S_\beta\text{-CL(X)}\}^c = \cap\{F^c/F^c \supseteq B^c$ *and* $F^c \in (1,2)S_\beta\text{-O(X)}\}$.
Therefore, $X–B^{\overset{\vee}{(1,2)S_\beta}} = (B^c)^{\overset{\wedge}{(1,2)S_\beta}} = (X–B)^{\overset{\wedge}{(1,2)S_\beta}}$. □

Proposition 3.5. *Let* $B_i : i \in I$ *be subsets of a bitopological space X. Then we have the following properties.*

(i) $(\bigcup_{i\in I} B_i)^{\overset{\wedge}{(1,2)S_\beta}} = \bigcup_{i\in I} B_i^{\overset{\wedge}{(1,2)S_\beta}}$.

(ii) $(\bigcap_{i\in I} B_i)^{\overset{\wedge}{(1,2)S_\beta}} \subseteq \bigcap_{i\in I} B_i^{\overset{\wedge}{(1,2)S_\beta}}$.

(iii) $(\bigcup_{i\in I} B_i)^{\overset{\vee}{(1,2)S_\beta}} \supseteq \bigcup_{i\in I} B_i^{\overset{\vee}{(1,2)S_\beta}}$ *for any index set I.*

Proof. (i) Let $x \in X$ such that $x \notin (\bigcup_{i\in I} B_i)^{\overset{\wedge}{(1,2)S_\beta}}$. Then there exists a (1,2) S_β-open set G such that $x \notin G$ and $\bigcup_{i\in I} B_i \subseteq G$. Therefore, for each $i \in$ I, $B_i \subseteq G$ and $x \notin G$. So $x \notin B_i^{\overset{\wedge}{(1,2)S_\beta}}$. Then $x \notin \bigcup_{i\in I} B_i^{\overset{\wedge}{(1,2)S_\beta}}$. Therefore, $\bigcup_{i\in I} B_i^{\overset{\wedge}{(1,2)S_\beta}} \subseteq (\bigcup_{i\in I} B_i)^{\overset{\wedge}{(1,2)S_\beta}}$.

Conversely, suppose that for each $x \in$ X, there exists a point $x \in$ X such that $x \notin \bigcup_{i\in I} B_i^{\overset{\wedge}{(1,2)S_\beta}}$. Then $x \notin B_i^{\overset{\wedge}{(1,2)S_\beta}}$ for each $i \in$ I. Thus there exist (1,2)S_β - open sets G_i such that $B_i \subseteq G_i$ and $x \notin G_i$ for each $i \in$ I. Let $G = \bigcup_{i\in I} G_i$. By theorem 2.8, G is a (1,2)S_β-open set. Now, $\bigcup_{i\in I} B_i \subseteq G$ and $x \notin G$. This implies $x \notin (\bigcup_{i\in I} B_i)^{\overset{\wedge}{(1,2)S_\beta}}$. So $(\bigcup_{i\in I} B_i)^{\overset{\wedge}{(1,2)S_\beta}} \subseteq (\bigcup_{i\in I} B_i)^{\overset{\wedge}{(1,2)S_\beta}}$.

(ii) Suppose that there exists a point $x \in$ X such that $x \notin \bigcap_{i\in I} B_i^{\overset{\wedge}{(1,2)S_\beta}}$. Then there exists a $i \in$ I such that $x \notin B_i^{\overset{\wedge}{(1,2)S_\beta}}$ and so there exists $G_i \in$ (1,2)S_β O(X) such that $x \notin G_i$ and $B_i \subseteq G_i$. Thus $x \notin G_i$ and $\bigcap_{i\in I} B_i \subseteq G_i \Rightarrow x \notin (\bigcap_{i\in I} B_i)^{\overset{\wedge}{(1,2)S_\beta}}$.

(iii)$(\bigcup_{i\in I} B_i)^{\overset{\vee}{(1,2)S_\beta}} = (((\bigcup_{i\in I} B_i)^c)^{\overset{\wedge}{(1,2)S_\beta}})^c = ((\bigcap_{i\in I} B_i^c)^{\overset{\wedge}{(1,2)S_\beta}})^c \supseteq (\bigcap_{i\in I} (B_i^c)^{\overset{\wedge}{(1,2)S_\beta}})^c = ((\bigcap_{i\in I} (B_i)^{\overset{\wedge}{(1,2)S_\beta}})^c)^c = \bigcup_{i\in I} B_i^{\overset{\vee}{(1,2)S_\beta}}$ □

Remark 3.6. *In general,* $(B_1 \cap B_2)^{\overset{\wedge}{(1,2)S_\beta}} \neq B_1^{\overset{\wedge}{(1,2)S_\beta}} \cap B_2^{\overset{\wedge}{(1,2)S_\beta}}$.

This can be shown in the following example.

Example 3.7. *As in example 3.3, let* $B_1 = \{ d \}$ *and* $B_2 = \{a, b, c\}$. *Then* $B_1^{\overset{\wedge}{(1,2)S_\beta}} = \{a,c,d\}$, $B_2^{\overset{\wedge}{(1,2)S_\beta}} = \{a, b, c\}$ *and* $B_1^{\overset{\wedge}{(1,2)S_\beta}} \cap B_2^{\overset{\wedge}{(1,2)S_\beta}} = \{a, c\}$. *But* $(B_1 \cap B_2) = \phi$.

Proposition 3.8. *For any bitopological space X,*

(i) The sets ϕ *and X are both* $\underset{(1,2)S_\beta}{\wedge}$*-sets and* $\underset{(1,2)S_\beta}{\vee}$*-sets.*

(ii) Every union of $\underset{(1,2)S_\beta}{\wedge}$*-sets is a* $\underset{(1,2)S_\beta}{\wedge}$*-set.*

(iii) Every intersection of $\underset{(1,2)S_\beta}{\vee}$*-sets is a* $\underset{(1,2)S_\beta}{\vee}$*-set.*

4. Conclusion

In this work, we have defined and studied some properties of $\underset{(1,2)S_\beta}{\wedge}$-set and $\underset{(1,2)S_\beta}{\vee}$-set using (1,2)$S_\beta$-open sets in bitopological spaces. This work will lead to the generalization of corresponding sets. Also, these findings will help to carry out more theoretical research for future researchers.

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