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Dominator color class dominating sets on fire cracker, gear and flower graphs

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Abstract

Let $G = (V,E)$ be a graph. Let $\mathscr{C} = \{\mathscr{C}_1, \mathscr{C}_2, \mathscr{C}_3 \dots \mathscr{C}_\chi\}$ be a proper coloring of $G.\mathscr{C}$ is called a dominator color class dominating set if each vertex v in G is dominated by a color class $C_i \in \mathscr{C}$ and each color class $C_i \in \mathscr{C}$ is dominated by a vertex v in G. The dominator color class domination number is the minimum cardinality taken over all dominator color class dominating sets in G and is denoted by $\gamma_\chi^d(G)$. In this paper, we obtain $\gamma_\chi^d(G)$ for Fire cracker graph, Gear graph, Flower graph and Sunflower graph.

Keywords

Chromatic number, Domination number, color class dominating set, dominator color class dominating set, color class domination number, dominator color class domination number.

AMS Subject Classification

05C15, 05C69.

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1. Introduction

All graphs considered in this paper are finite, undirected graphs and we follow standard definitions of graph theory as found in [4].

Let $G = (V,E)$ be a graph of order p. The open neighborhood $N(v)$ of a vertex $v \in V(G)$ consists of the set of all vertices adjacent to v. The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood $N(S)$ is defined to be $U_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. For any set H of vertices of G, the induced sub graph $\langle H \rangle$ is the maximal sub graph of G with vertex set H. A subset S of V is called a dominating set if every vertex in $V - S$ is adjacent to some vertex in S. A dominating set is a minimal dominating set if no proper subset of S is a dominating set of G. The domination number $\gamma(G)$ is the minimum cardinality taken over all minimal dominating sets of G. A $\gamma-$ set of G is any minimal dominating set with cardinality γ. A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$. A dominator coloring of G is a proper coloring of G in which every vertex of G dominates at least one color class. The dominator chromatic number is denoted by $\chi_d(G)$ and is defined by the minimum number of colors needed in a dominator coloring of G.

A dominator color class dominating set of G is a proper coloring of G with the extra property that each vertex v in G is dominated by a color class $C_i \in \mathscr{C}$ and every color class in $C_i \in \mathscr{C}$ is dominated by a vertex in G. A dominator color class dominating set is said to be a minimal dominator color class dominating set if no proper subset of $\mathscr{C}$ is a dominator color class dominating set of G. The dominator color class domination number of G is the minimum cardinality taken over all minimal dominator color class dominating sets of G and is denoted by $\gamma_\chi^d(G)$. This notation was introduced by A.Vijayalekshmi et.al in [3].

The join $G_1 + G_2$ of graphs G_1 and G_2 with disjoint vertex set V_1 and V_2 and edge sets E_1 and E_2 is the graph union

$G_1 \cup G_2$ together with each vertex in V_1 is adjacent to every vertices in V_2. The Fire cracker graph $F(n,k)$ is the graph obtained by the concatenation of nk -stars by linking one leaf from each. The $F(n,k)$ has order nk and size $nk-1$. A Gear graph is a graph obtained by inserting an extra vertex between each pair of adjacent vertices on the perimeter of a Wheel graph $W_{1,n}$. The flower graph Fl_n is the graph obtained from a helm graph by joining each pendant vertex to the central vertex of the helm. The sunflower graph Sf_n is the resultant graph obtained from the flower graph of wheel $W_{1,n}$ by adding pendant edges to the central vertex.

2. Main Results

Definition 2.1. *Let $G=(V,E)$ be a graph and let $\mathscr{C}=\{\mathscr{C}_1,\mathscr{C}_2,\ldots\mathscr{C}_\chi\}$ be a proper coloring of $G.\mathscr{C}$ is called a dominator color class dominating set if each vertex v in G is dominated by a color class $\mathscr{C}_i \in \mathscr{C}$ and each color class $\mathscr{C}_i \in \mathscr{C}$ is dominated by a vertex v in G. The dominator color class domination number is the minimum cardinality taken over all dominator color class dominating sets in G and is denoted by $\gamma_x^d(G)$.*

Theorem 2.2. *For the Fire cracker graph $F(n,k)$,*

$$\gamma_\chi^d(F_{n,k}) = \begin{cases} 3 & \text{if } n=k=2 \\ 4 & \text{if } n=2 \text{ and } k\geq 3 \\ & \text{and } n=3 \text{ and } k=2 \\ \lceil \frac{4n}{3} \rceil & \text{if } n\geq 4 \text{ and } k=2 \\ 2n & \text{otherwise} \end{cases}$$

Proof. Let

$$V(F_{n,k}) = \left\{ u_{i,j} / {}^{1\leq i\leq n}_{1\leq j\leq k} \right\}$$

with deg $u_{i1}=k-1, 1\leq i\leq n$ and u_{i1} is adjacent to $u_{1j}, 2\leq j\leq k$. If $n=2$ and $k\geq 2$, the proof is obvious. We consider two cases.

Case (1): When $n\geq 4$ and $k=2$

We have three subcases,

Subcase 1.1: When $n\equiv 0(\text{mod}\,3)$. For, $m=1,2,\ldots\ldots\frac{n}{3}$. Let $H_m=\langle u_{3m-2,1},u_{3m-2,2},u_{3m-1,1},u_{3m-1,2},u_{3m,1},u_{3m,2}\rangle$ be the vertex induced subgroup of $F_{n,k}$. Assign four distinct colors $1,2,3$ and 4 to the vertices $\{u_{3m-1,2}\},\{u_{3m-2,2},u_{3m-1,1},u_{3m,2}\}$, $\{u_{3m-2,1}\}$ and $\{u_{3m,1}\}$ respectively.

This dominator coloring satisfies one requirement and $F_{n,k}\approx \frac{n}{3}H_m$, Then $\gamma_\chi^d(F_{n,k})=\frac{4n}{3}$.

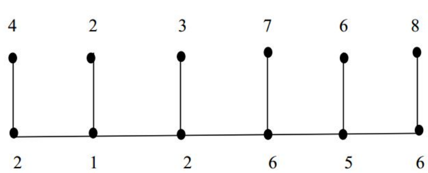

Figure 1. $\gamma_\chi^d(F_{6,2})=8$

Subcase 1.2: When $n\equiv 1(\,\text{mod}\,3)$

Since $n-1\equiv 0(\,\text{mod}\,3), F_{n,k}$ is obtained by $F_{n-1,k}$ followed by $F_{1,k}$.

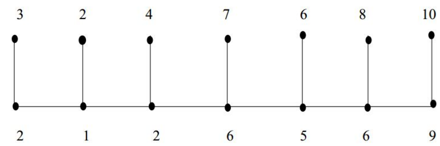

Figure 2. $\gamma_\chi^d(F_{7,2})=10$

So $\gamma_\chi^d(F_{n,k})=\gamma_\chi^d(F_{n-1,k})+\gamma_\chi^d(F_{1,k})=\frac{4n}{3}$

Subcase 1.3: When $n\equiv 2(\,\text{mod}\,3)$,

Since $n-2\equiv 0(\,\text{mod}\,3), F_{n,k}$ is obtained from $F_{n-2,k}$ followed by $F_{2,k}$. So, $\gamma_\chi^d(F_{n,k})=\gamma_\chi^d(F_{n-2,k})+\gamma_\chi^d(F_{2,k})=\frac{4n}{3}$

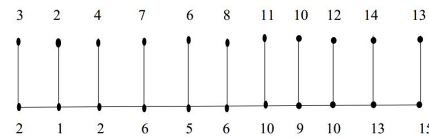

Figure 3. $\gamma_\chi^d(F_{11,2})=15$

Case (2): When $n=3,k\geq 3$ and $n\geq 5,k\geq 3$

In this case, we assign n distinct colors say $1,3,5,\ldots\ldots(2n-1)$ to the vertices $\{u_{11}\},\{u_{21}\}\ldots\ldots\ldots\{u_{n1}\}$ respectively. Also assign n distinct colors say $2,4,6,\ldots\ldots 2n$ to the vertices $\{u_{1j}/2\leq j\leq k\},\{u_{2j}/2\leq j\leq k\}\ldots\ldots\{u_{nj}/2\leq j\leq k\}$ respectively, we get a γ_χ^d - coloring of $F_{n,k}$. Thus $\gamma_\chi^d(F_{n,k})=2n$

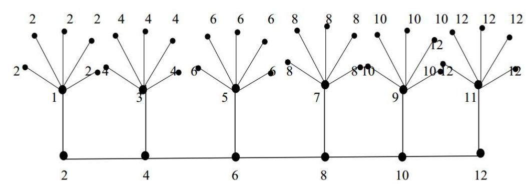

Figure 4. $\gamma_\chi^d(F_{6,7})=12$

□

Theorem 2.3. *For the Gear graph G_n, when $n\geq 3$*

$$\gamma_\chi^d(G_n) = \begin{cases} \frac{4n}{3} & \text{if } n\equiv 0(\,\text{mod}\,3) \\ 4\lfloor\frac{n}{3}\rfloor+2 & \text{if } n\equiv 1(\,\text{mod}\,3) \\ 4\lfloor\frac{n}{3}\rfloor+3 & \text{if } n\equiv 2(\,\text{mod}\,3) \end{cases}$$

Proof. Let $V(G_n)=\{v\}\cup\{v_1,v_2,\ldots v_n\}\cup\{u_1,u_2,\ldots u_n\}$ where u is the central vertex and deg $u_i=3$ and deg $v_{i=}2, 1\leq i\leq n$. We have 3 cases.

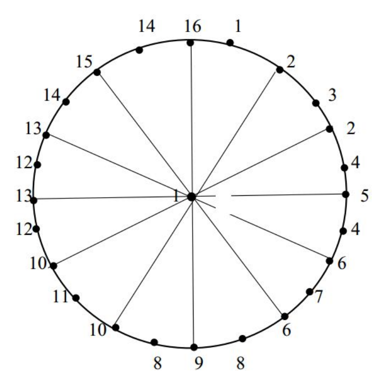

Figure 5. $\gamma_\chi^d(G_{12}) = 16$

Case (i): When $n \equiv 0(\bmod 3)$
Assign colors 1 and 2 to the vertices $\{u_1\}$ and,$\{u, v_1, v_{12}\}$ respectively. Assign distinct colors say $3,5,7\ldots\left(\frac{4n}{3}-1\right)$ to the vertices $\{u_2,u_3\},\{v_3,v_4\},\{u_5,u_6\},\{v_6,v_9\},\ldots\{v_{n-3},v_{n-2}\}$, $\{u_{n-1},u_n\}$ and colors $4,6,8,\ldots,\frac{4n}{3}$.
Case (ii): When $n \equiv 1(\bmod 3)$
As in case (i), the same coloring together with two additional colors say $\frac{4n}{3}+1$ and $\frac{4n}{3}+2$ assigned to the vertices $\{u_n\}$ and $\{v_n\}$ respectively. We obtain the γ_χ^d coloring of G_n. So $\gamma_\chi^d(G_n) = \frac{4n}{3}+2$.

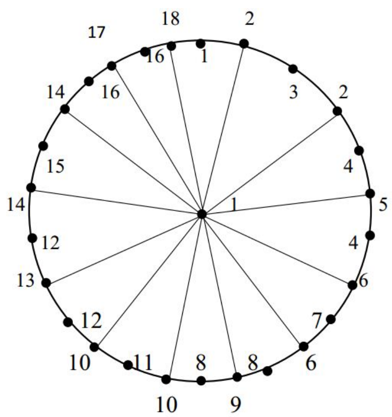

Figure 6. $\gamma_\chi^d(G_{13}) = 18$

Case (iii): When $n \equiv 2(\bmod 3)$
Apply the same coloring in case (i) and together with three distinct colors say $\frac{4n}{3}+1, \frac{4n}{3}+2$ and $\frac{4n}{3}+3$ to the vertices $\{v_{n-2},v_{n-1}\},\{u_{n-1}\}$ and $\{u_n\}$ respectively, we get the required coloring. Thus $\gamma_\chi^d(G_n) = \frac{4n}{3}+3$

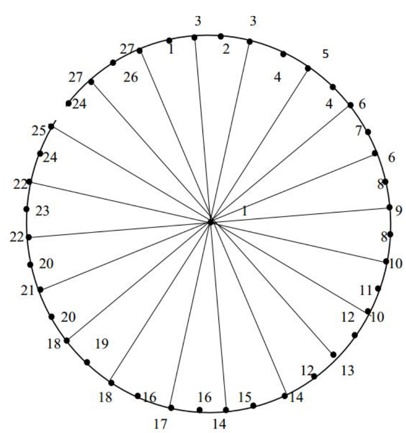

Figure 7. $\gamma_\chi^d(G_{20}) = 27$

□

Theorem 2.4. *If G is a Flower graph Fl_n,*

$$\gamma_\chi^d(Fl_n) = \begin{cases} 4 & \text{if } n \text{ is even} \\ 5 & \text{if } n \text{ is odd} \end{cases}$$

Proof. The Flower graph Fl_n is obtained from a helm graph by joining each pendant vertex to the central vertex. Let $V(Fl_n) = \{u_1,u_2,\ldots\ldots.u_{2n+1}\}$ where u_1 is the central vertex, $u_i(2 \le i \le n+1)$ is the vertices on the cycle C_n and $u_j(n+2 \le j \le 2n+1)$ is adjacent to $uu_i(2 \le i \le n+1)$ and u_1. We have two cases.
Case (i): When n is even.
Let $\mathscr{C} = \{\mathscr{C}_1,\mathscr{C}_2,\mathscr{C}_3,\mathscr{C}_4\}$ be a dominator coloring of Fl_n, with $\mathscr{C}_1 = \{u_1\}, \mathscr{C}_2 = \{u_2,u_4,\ldots u_n\}, \mathscr{C}_3 = \{u_3,u_5,\ldots u_{n+1}\}$ and $\{u_{n+2},\ldots,u_{2n+1}\}$ respectively.

Then the color classes $\mathscr{C}_1,\mathscr{C}_2,\mathscr{C}_3,\mathscr{C}_4$ are dominated by the vertex u_1 and each vertex is dominated by the color class $\mathscr{C}_1$. In this case $\gamma_\chi^d(Fl_n) = 4$.

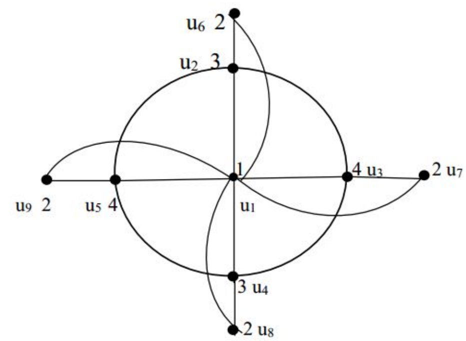

Figure 8. $\gamma_\chi^d(Fl_n) = 4$

Case (ii): When n is odd. Let $\mathscr{C} = \{\mathscr{C}_1,\mathscr{C}_2,\mathscr{C}_3,\mathscr{C}_4,\mathscr{C}_5\}$ be a dominator coloring of Fl_n in which $\mathscr{C}_1 = \{u_1\}\mathscr{C}_2 = \{u_2,u_4,\ldots u_{n-1}\}\mathscr{C}_3 = \{u_3,u_5\ldots\ldots u_n\}\ \mathscr{C}_4 = \{u_{n+1}\}$ and $\mathscr{C}_5 = \{u_{n+2},\ldots u_{2n+1}\}$ respectively. As in case (i) $\gamma_\chi^d(Fl_n) = 5$.

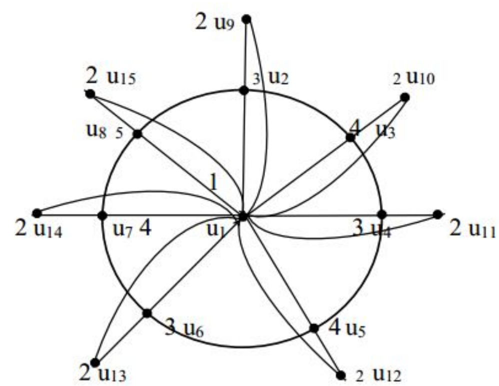

Figure 9. $\gamma_\chi^d(Fl_7) = 5$

□

Theorem 2.5. *For the sunflower graph Sf_n,*

$$n \ge 3, \gamma_\chi^d(Sf_n) = \begin{cases} 4 & \text{if } n \text{ is even} \\ 5 & \text{if } n \text{ is odd} \end{cases}$$

Proof. Let G be a flower graph with pendant edges attached to the central vertex. Then G is a sunflower graph Sf_n. By Theorem 2.9 we assign the same dominator coloring of Fl_n

in the color 2 to the pendant vertices $\{v_{2n+2}, v_{2n+3}, \ldots v_{3n+1}\}$ we obtain the $\gamma_\chi^d -$ coloring of Sf_n. Hence,

$$\gamma_\chi^d(Sf_n) = \begin{cases} 4 \text{ if } n \text{ is even} \\ 5 \text{ if } n \text{ is odd} \end{cases}$$

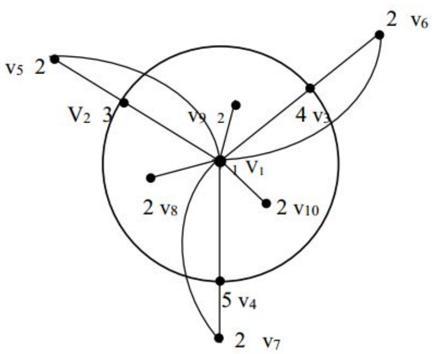

Figure 10. n is odd $\gamma_\chi^d(Sf_3) = 5$

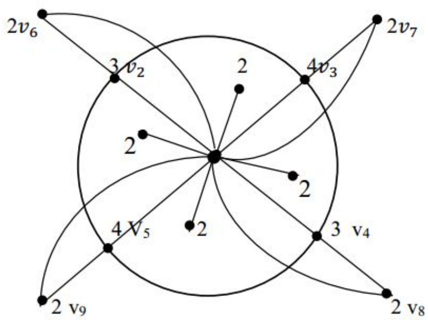

Figure 11. n is even $\gamma_\chi^d(Sf_4) = 4$

□

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