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Graph identification of different operations of vertex edge neighborhood prime

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Abstract
Let $G(V(G),E(G))$ be a graph. Vertex edge neighborhood prime labeling is a function $h: V(G)\cup E(G) \rightarrow \{1,2,...,|V(G)\cup E(G)|\}$ with one to one correspondence and if **(i)** for $u \in V(G)$ with $deg(u)=1$, gcd$(h(v),h(uv): v \in N_V(u))=1$. **(ii)** for $u\in V(G)$ with $deg(u)\geq 2$, gcd$(h(v): v\in N_V(u))=1$. gcd$(h(e): e \in N_E(u))=1$. A graph admits such labeling is called *vertex edge neighborhood prime graph*. In the present work we investigate with some families of graphs are vertex edge neighborhood prime graph.

Keywords
Vertex edge neighborhood prime graphs, operations of graphs, one point union of graphs, m fold types of graphs.

AMS Subject Classification
05C78.

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Contents

1. Introduction

Consider graphs are connected, undirected, finite and simple graphs. For standard terminology and notations, we refer [1]. $V(G)$ and $E(G)$ denote the vertices and edges of G. Let p and q be the cardinality of vertex and edge set is called the order and size of a graph G. See the dynamic graph labeling survey [2] by Gallian is regularly updated. The following definitions are taken from [8] "A *prime labeling* is an assignment of the integers 1 to p as labels of the vertices such that each pair of labels from adjacent vertices is relatively prime. A graph that has such a labeling is called *prime graph*. A *neighborhood prime labeling* of a graph G with p vertices is a labeling of the vertex set with the integers 1 to p in which for each vertex $v\in V(G)$ of degree greater than 1, the gcd of the labels of the vertices in $N(v)$ is 1. A graph which admits neighborhood prime labeling is called a *neighborhood prime graph*. This concept was introduced by Patel and Shrimali [4]. A bijection $f: V(G)\cup E(G)\rightarrow \{1,2,3,...,p+q\}$ is said to be *total neighborhood prime labeling* if it satisfies the following two conditions: **(i)** for each vertex of degree at least two, the gcd of labeling on its neighborhood vertices is one; **(ii)** for each vertex of degree at least two, the gcd of labeling on the induced edges is one. A graph which admits total neighborhood prime labeling is called *total neighborhood prime graph*. This concept was introduced by Rajeshkumar, et. al., [5]. Motivated by neighborhood prime graph and total neighborhood prime graph, Pandya and Shrimali [3] defined the concept of vertex edge neighborhood prime labeling.They observed that **(i)** every vertex edge neighborhood prime graph is total neighborhood prime graph, but converse is not true.
(ii) the graph which is not having degree one, if it is total neighborhood prime graph, then it is vertex edge neighborhood prime graph.
Let $G=(V(G),E(G))$ be a graph, $u\in V(G)$

$$N_V(u)=\{w\in V(G)/uw \text{ is an edge}\}$$
$$N_E(u)=\{e\in E(G)/e=uv \text{ for some } v\in V(G)\}$$

Vertex edge neighborhood prime labeling is a function $f: V(G)\cup E(G)\rightarrow\{1,2,3,...,p+q\}$ with the property that if degree of vertex is exactly one, then that neighborhood vertex and its incident edge are relatively prime and if the degree of vertex is at least two then their neighborhood vertices are relatively prime and its incident edges are also relatively prime. A graph which admits vertex edge neighborhood prime labeling is called *vertex edge neighborhood prime graph*."

The following definitions are taken from [6], [7], [8], [9], [10]. "An *armed crown* is a graph in which path P_n is attached at each vertex of cycle C_n by an edge. The *butterfly graph* $BF(m,n)$ is a graph obtained from two cycles C_n of the same order, sharing a common vertex with an arbitrary number m of pendant edges attached at the common vertex. An *octopus graph* $O_s, (s \geq 2)$ can be constructed by a fan graph $f_s, (s \geq 2)$ joining a star graph $K_{1,s}$ with sharing a common vertex. i.e, $O_s = f_s \odot K_{1,s}$. A *shell graph* is defined as a cycle C_n with $C(n,n-3)$ chords sharing a common end point called the apex. Shell graph are denoted as S_n. A shell S_n is also called fan f_{n-1}. The *planter graph* $R_z, (z \geq 3)$ is a graph obtained by joining a fan graph $f_z, (z \geq 2)$ and cycle graph $C_z, (z \geq 3)$ with sharing a common vertex. i.e, $R_z = f_z \odot C_z$. The Petersen graph $P(n,2)$ is a graph with vertex set $(u_0, u_1, ..., u_{n-1}, v_0, v_1, , ..., v_{n-1})$ and edge set $(u_i u_{i+1}, u_i v_i, v_i v_{i+2} : 0 \leq i \leq n-1)$ where subscripts are to be taken modulo n and $2 < \frac{n}{2}$. The *quadrilateral snake* Q_n is obtained from the path P_n by replacing each edge of the path by a quadrilateral C_4. The *triangular snake* T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3. An *alternate triangular snake* $A(T_n)$, where $n = 4, 6, 8, 10, ...$ from a path $u_1, u_2, u_3, ..., u_n$ by joining u_i and u_{i+1} (alternately) to a new vertex v_i. That is every alternate edge of a path is replaced by C_3. A *double triangular snake* $D(T_n)$, where $n > 1$ consists of two triangular snakes that have a common path. The graph lotus inside a circle LC_n is obtained from the cycle $C_n : w_1 w_2 w_3 ... w_n w_1$ and a star $K_{1,n}$ with central vertex u and the end vertices $u_1, u_2, u_3, ..., u_n$ by joining each u_i to w_i and $w_{i+1}(\text{mod } n)$. A *closed helm* CH_n is a graph obtained from a helm by joining each pendant vertex to form a cycle. A *prism graph* Y_m is cartesian product graph $C_m \times P_2$, where C_m is cycle graph of order m and P_2 is path of order 2. A m- sided *anti-prism* A_m is polyhedron composed of two parallel copies of some particular m- sided polygon connected by alternating band of triangle." The Mycielskian graph $\mu(G)$ of G is defined as follows: The vertex set $V(\mu(G))$ of $\mu(G)$ is the disjoint union $V \cup V' \cup u$, where $V' = \{x' : x \in V\}$ and the edge set of $\mu(G)$ is $E(\mu(G)) = E \cup \{x'y : xy \in E\} \cup \{x'u : x' \in V'\}$. If G_1 and G_2 are two connected graphs, then the graph obtained by superimposing any selected vertex of G_2 on any selected vertex of G_1 is denoted by $G_1 \odot G_2$.

In section 2, 3, 4, we prove that for operations of graphs, one point union of graphs, m fold types of graphs are vertex edge neighborhood prime graphs.

2. Graph identification of operations of graphs

In this section, We investigate operations of graphs.

Theorem 2.1. *If $G_1(p_1, q_1)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_1 \odot$ [combination of graphs complete graph K_z, prism Y_z and antiprism A_z] that admits vertex edge neighborhood prime.*

Proof. Let $G_1(p_1, q_1)$ be vertex edge neighborhood prime graph with bijection $g_1 : V(G_1) \cup E(G_1) \rightarrow \{1, 2, ..., |V(G_1) \cup E(G_1)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_1 be the combination of graphs complete graph K_z, prism Y_z and antiprism A_z with
$V(H_1) = \left\{a''_k, b''_k, c''_k : 1 \leq k \leq z\right\}$ and
$E(H_1) = \left\{a''_k b''_k, b''_k c''_k : 1 \leq k \leq z\right\} \cup \left\{b''_1 b''_z\right\} \cup \left\{b''_1 c''_z\right\} \cup \left\{c''_1 c''_z\right\} \cup \left\{b''_k b''_{k+1}, b''_{k+1} c''_k, c''_k c''_{k+1} : 1 \leq k \leq z-1\right\} \cup \left\{a''_k a''_{z-(l-1)} : 1 \leq k \leq z-1, 1 \leq l \leq z-k\right\}$.
We overlay one of the vertex say c''_1 of H_1 on selected vertex of s_1 in G_1 with $g_1(s_1) = 1$.
Also, $G_1^* = G_1 \odot H_1$ with $V(G_1^*) = V(G_1) \cup V(H_1)$ and $E(G_1^*) = E(G_1) \cup E(H_1)$.
$|V(G_1^*)| = p_1 + 3z - 1$ and $|E(G_1^*)| = q_1 + 5z + \frac{z(z-1)}{2}$.
Define
$$h_1 : V(G_1^*) \cup E(G_1^*) \rightarrow \left\{1, 2, ..., p_1 + q_1 + 8z + \frac{z(z-1)}{2} - 1\right\}$$
as follows:
$g_1(z_1) = h_1(z_1)$ for all $z_1 \in V(G_1)$ and $g_1(e_1) = h_1(e_1)$ for all $e_1 \in E(G_1)$.
$h_1(c''_1) = h_1(s_1) = 1, h_1(c''_1 c''_z) = p_1 + q_1 + 4z - 1, h_1(b''_1 c''_z) = p_1 + q_1 + 6z - 1, h_1(b''_1 b''_z) = p_1 + q_1 + 7z - 1$.
For each $1 \leq k \leq z, h_1(a''_k) = p_1 + q_1 + 2z + k - 1, h_1(b''_k) = p_1 + q_1 + 2k - 1, h_1(b''_k c''_k) = p_1 + q_1 + 4z + 2k - 2, h_1(a''_k b''_k) = p_1 + q_1 + 7z + k - 1$.
For each $1 \leq k \leq z-1, h_1(c''_k c''_{k+1}) = p_1 + q_1 + 3z + k - 1$,
$h_1(b''_{k+1} c''_k) = p_1 + q_1 + 4z + 2k - 1, h_1(b''_k b''_{k+1}) = p_1 + q_1 + 6z + k - 1$,
$h_1(c''_{k+1}) = p_1 + q_1 + 2k, .$
$h_1(a''_k a''_{z-(l-1)}) = p_1 + q_1 + 8z + l + \{(z-1) + (z-2) + ... + [z-(k-1)]\} - 1$ for $1 \leq k \leq z-1$ and $1 \leq l \leq z-k$.
Clearly, G_1 is vertex edge neighborhood prime graph. We claim that H_1 is vertex edge neighborhood prime graph. Let x_1 be any vertex of H_1.
For $x_1 = a''_k, b''_k, c''_k$ for $1 \leq k \leq z$ with $deg(x_1) \geq 2$. Here,
$\gcd\{h_1(w_1) : w_1 \in N_V(x_1)\} = 1$ and
$\gcd\{h_1(e_1) : e_1 \in N_E(x_1)\} = 1$.
Hence $G_1^* = G_1 \odot H_1$ is vertex edge neighborhood prime graph. □

Theorem 2.2. *If $G_2(p_2, q_2)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_2 \odot$ [combination of graphs barycentric cycle BC_s, prism Y_s and antiprism A_s] that admits vertex edge neighborhood prime.*

Proof. Let $G_2(p_2, q_2)$ be vertex edge neighborhood prime graph with bijection $g_2 : V(G_2) \cup E(G_2) \rightarrow \{1, 2, ..., |V(G_2) \cup E(G_2)|\}$ satisfying the property of vertex edge neighborhood prime graph.

Consider H_2 be the combination of graphs barycentric cycle BC_s, prism Y_s and antiprism A_s with
$V(H_2) = \{u'_t, v'_t, w'_t, x'_t : 1 \le t \le s\}$ and
$E(H_2) = \{u'_t v'_t, v'_t w'_t, w'_t x'_t : 1 \le t \le s\} \cup \{u'_1 u'_s\} \cup \{u'_1 v'_s\} \cup \{w'_1 w'_s\} \cup \{w'_1 x'_s\} \cup \{x'_1 x'_s\} \cup \{u'_t u'_{t+1}, u'_{t+1} v'_t, w'_t w'_{t+1}, w'_{t+1} x'_t, x'_t x'_{t+1} : 1 \le t \le s-1\}$.
We overlay one of the vertex say x'_1 of H_2 on selected vertex of t_1 in G_2 with $g_2(t_1) = 1$.
Note that $G_2^* = G_2 \odot H_2$ with $V(G_2^*) = V(G_2) \cup V(H_2)$ and $E(G_2^*) = E(G_2) \cup E(H_2)$.
$|V(G_2^*)| = p_2 + 4s - 1$ and $|E(G_2^*)| = q_2 + 8s$.
Define $h_2 : V(G_2^*) \cup E(G_2^*) \to \{1, 2, ..., p_2 + q_2 + 12s - 1\}$ as follows:
$g_2(z_2) = h_2(z_2)$ for all $z_2 \in V(G_2)$ and $g_2(e_2) = h_2(e_2)$ for all $e_2 \in E(G_2)$.
$h_2(x'_1) = h_2(t_1) = 1, h_2(x'_1 x'_s) = p_2 + q_2 + 5s - 1, h_2(w'_1 x'_s) = p_2 + q_2 + 7s - 1, h_2(w'_1 w'_s) = p_2 + q_2 + 8s - 1, h_2(u'_1 v'_s) = p_2 + q_2 + 11s - 1, h_2(u'_1 u'_s) = p_2 + q_2 + 12s - 1$.
For each $1 \le t \le s, h_2(w'_t x'_t) = p_2 + q_2 + 5s + 2t - 2, h_2(v'_t w'_t) = p_2 + q_2 + 8s + t - 1, h_2(u'_t v'_t) = p_2 + q_2 + 9s + 2t - 2$.
For each $1 \le t \le s-1, h_2(x'_t x'_{t+1}) = p_2 + q_2 + 4s + t - 1$, $h_2(w'_{t+1} x'_t) = p_2 + q_2 + 5s + 2t - 1, h_2(w'_t w'_{t+1}) = p_2 + q_2 + 7s + t - 1, h_2(u'_{t+1} v'_t) = p_2 + q_2 + 9s + 2t - 1, h_2(u'_t u'_{t+1}) = p_2 + q_2 + 11s + t - 1$.
Consider the following two cases.
Case 1. $p_2 + q_2$ is odd
For each $1 \le t \le s, h_2(v'_t) = p_2 + q_2 + 2t - 1, h_2(u'_t) = p_2 + q_2 + 2t, h_2(w'_t) = p_2 + q_2 + 2s + 2t - 1$.
$h_2(x'_{t+1}) = p_2 + q_2 + 2s + 2t$ for $1 \le t \le s - 1$.
Case 2. $p_2 + q_2$ is even
For each $1 \le t \le s, h_2(u'_t) = p_2 + q_2 + 2t - 1, h_2(v'_t) = p_2 + q_2 + 2t$.
For each $1 \le t \le s-1, h_2(w'_t) = p_2 + q_2 + 2s + 2t, h_2(x'_{t+1}) = p_2 + q_2 + 2s + 2t - 1$.
$h_2(w'_s) = p_2 + q_2 + 4s - 1$.
Already, G_2 is vertex edge neighborhood prime graph. Now we have to prove H_2 is vertex edge neighborhood prime graph. Let a_2 be any vertex of H_2.
For $a_2 = u'_t, v'_t, w'_t, x'_t$ for $1 \le t \le s$ with $deg(a_2) \ge 2$. Here, $\gcd\{h_2(b_2) : b_2 \in N_V(a_2)\} = 1$ and
$\gcd\{h_2(d_2) : d_2 \in N_E(a_2)\} = 1$.
Hence $G_2^* = G_2 \odot H_2$ admits vertex edge neighborhood prime graph. □

Theorem 2.3. *If $G_3(p_3, q_3)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_3 \odot$ [combination of graphs closed helm CH_a, prism Y_a and antiprism A_a] that admits vertex edge neighborhood prime.*

Proof. Let $G_3(p_3, q_3)$ be vertex edge neighborhood prime graph with bijection $g_3 : V(G_3) \cup E(G_3) \to \{1, 2, ..., |V(G_3) \cup E(G_3)|\}$ satisfying the property of vertex edge neighborhood prime graph.
Consider H_3 be the combination of graphs closed helm CH_a, prism Y_a and antiprism A_a with
$V(H_3) = \{r_0\} \cup \{r_t, r'_t, s_t, s'_t : 1 \le t \le a\}$ and
$E(H_3) = \{r_0 r_t, r_t r'_t, r'_t s_t, s_t s'_t : 1 \le t \le a\} \cup \{r_1 r_a\} \cup \{r'_1 r'_a\} \cup \{s_1 s_a\} \cup \{s_1 s'_a\} \cup \{s'_1 s'_a\}$
$\cup \{r_t r_{t+1}, r'_t r'_{t+1}, s_t s_{t+1}, s'_t s_{t+1}, s'_t s'_{t+1} : 1 \le t \le a-1\}$.
We overlay one of the vertex say r_0 of H_3 on selected vertex of u_1 in G_3 with $g_3(u_1) = 1$.
Also, $G_3^* = G_3 \odot H_3$ with $V(G_3^*) = V(G_3) \cup V(H_3)$ and $E(G_3^*) = E(G_3) \cup E(H_3)$.
$|V(G_3^*)| = p_3 + 4a$ and $|E(G_3^*)| = q_3 + 9a$.
Define $h_3 : V(G_3^*) \cup E(G_3^*) \to \{1, 2, ..., p_3 + q_3 + 13a\}$ as follows:
$g_3(z_3) = h_3(z_3)$ for all $z_3 \in V(G_3)$ and $g_3(e_3) = h_3(e_3)$ for all $e_3 \in E(G_3)$.
$h_3(r_0) = h_3(u_1) = 1, h_3(s'_1 s'_a) = p_3 + q_3 + 5a, h_3(s_1 s'_a) = p_3 + q_3 + 7a, h_3(s_1 s_a) = p_3 + q_3 + 8a, h_3(r'_1 r'_a) = p_3 + q_3 + 10a$, $h_3(r_1 r_a) = p_3 + q_3 + 12a$.
For each $1 \le t \le a, h_3(s_t s'_t) = p_3 + q_3 + 5a + 2t - 1, h_3(r'_t s_t) = p_3 + q_3 + 8a + t, h_3(r_t r'_t) = p_3 + q_3 + 10a + t, h_3(r_0 r_t) = p_3 + q_3 + 12a + t$.
For each $1 \le t \le a-1, h_3(s'_t s'_{t+1}) = p_3 + q_3 + 4a + t$, $h_3(s'_t s_{t+1}) = p_3 + q_3 + 5a + 2t, h_3(s_t s_{t+1}) = p_3 + q_3 + 7a + t, h_3(r'_t r'_{t+1}) = p_3 + q_3 + 9a + t, h_3(r_t r_{t+1}) = p_3 + q_3 + 11a + t$.
We consider the following two cases.
Case 1. $p_3 + q_3$ is odd
For each $1 \le t \le a, h_3(r_t) = p_3 + q_3 + 2t, h_3(r'_t) = p_3 + q_3 + 2t - 1, h_3(s_t) = p_3 + q_3 + 2a + 2t - 1, h_3(s'_t) = p_3 + q_3 + 2a + 2t$.
Case 2. $p_3 + q_3$ is even
For each $1 \le t \le a, h_3(r_t) = p_3 + q_3 + 2t - 1, h_3(r'_t) = p_3 + q_3 + 2t, h_3(s_t) = p_3 + q_3 + 2a + 2t - 1, h_3(s'_t) = p_3 + q_3 + 2a + 2t$.
For proving G_3^* is vertex edge neighborhood prime graph. In earlier, G_3 is vertex edge neighborhood prime graph. Now we have to prove H_3 is vertex edge
neighborhood prime graph. Let a_3 be any vertex of H_3.
For $a_3 = r_0, r_t, r'_t, s_t, s'_t$ for $1 \le t \le a$ with $deg(a_3) \ge 2$. Here, $\gcd\{h_3(b_3) : b_3 \in N_V(a_3)\} = 1$ and
$\gcd\{h_3(d_3) : d_3 \in N_E(a_3)\} = 1$.
Hence $G_3^* = G_3 \odot H_3$ admits vertex edge neighborhood prime graph. □

Theorem 2.4. *If G_4 has vertex edge neighborhood prime graph, then there exists a graph from the class $G_4 \odot$ [combination of graphs lotus inside a circle LC_k, prism Y_k and antiprism A_k] that admits vertex edge neighborhood prime.*

Proof. Let $G_4(p_4, q_4)$ be vertex edge neighborhood prime graph with labeling $g_4 : V(G_4) \cup E(G_4) \to \{1, 2, ..., |V(G_4) \cup E(G_4)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Let H_4 be the combination of graphs lotus inside a circle LC_k, prism Y_k and antiprism A_k with
$V(H_4) = \{r_0\} \cup \{r'_c, s'_c, t'_c, u'_c : 1 \le c \le k\}$ and
$E(H_4) = \{r_0 r'_c, r'_c s'_c, s'_c t'_c, t'_c u'_c : 1 \le c \le k\} \cup \{s'_1 s'_k\} \cup \{t'_1 t'_k\} \cup \{r'_{c+1} s'_c, s'_c s'_{c+1}, t'_c t'_{c+1}, t'_c u'_{c+1}, u'_c u'_{c+1} : 1 \le c \le k-1\} \cup \{u'_1 u'_k\}$

$\cup\{r'_1s'_k\}\cup\{u'_1t'_k\}$.
We overlay one of the vertex say r_0 of H_4 on selected vertex of b_1 in G_4 with $g_4(b_1)=1$.
Note that $G_4^*=G_4\odot H_4$ with $V(G_4^*)=V(G_4)\cup V(H_4)$ and $E(G_4^*)=E(G_4)\cup E(H_4)$.
$|V(G_4^*)|=p_4+4k$ and $|E(G_4^*)|=q_4+9k$.
Define $h_4:V(G_4^*)\cup E(G_4^*)\to\{1,2,...,p_4+q_4+13k\}$ as follows:
$g_4(z_4)=h_4(z_4)$ for all $z_4\in V(G_4)$ and $g_4(e_4)=h_4(e_4)$ for all $e_4\in E(G_4)$.
$h_4(r_0)=h_4(b_1)=1, h_4(u'_1u'_k)=p_4+q_4+5k, h_4(u'_1t'_k)=p_4+q_4+5k+1, h_4(t'_1t'_k)=p_4+q_4+8k, h_4(s'_1s'_k)=p_4+q_4+10k, h_4(r'_1s'_k)=p_4+q_4+12k$.
For each $1\le c\le k, h_4(r'_c)=p_4+q_4+3k+c, h_4(s'_c)=p_4+q_4+2k+c, h_4(t'_c)=p_4+q_4+k+c, h_4(u'_c)=p_4+q_4+c$, $h_4(r'_cs'_c)=p_4+q_4+10k+2c-1, h_4(r_0r'_c)=p_4+q_4+12k+c, h_4(s'_ct'_c)=p_4+q_4+8k+c, h_4(t'_cu'_c)=p_4+q_4+5k+2c$.
For each $1\le c\le k-1, h_4(u'_cu'_{c+1})=p_4+q_4+4k+c$, $h_4(t'_cu'_{c+1})=p_4+q_4+5k+2c+1, h_4(t'_ct'_{c+1})=p_4+q_4+7k+c, h_4(s'_cs'_{c+1})=p_4+q_4+9k+c, h_4(r'_{c+1}s'_c)=p_4+q_4+10k+2c$.
Clearly, G_4 is vertex edge neighborhood prime graph. In order to show that H_4 is vertex edge neighborhood prime graph. Let a_4 be any vertex of H_4.
For $a_4=r_0,r'_c,s'_c,t'_c,u'_c$ for $1\le c\le k$ with $deg(a_4)\ge 2$. Here, $\gcd\{h_4(w_4):w_4\in N_V(a_4)\}=1$ and
$\gcd\{h_4(d_4):d_4\in N_E(a_4)\}=1$.
Hence $G_4^*=G_4\odot H_4$ is vertex edge neighborhood prime graph. □

Theorem 2.5. *If G_5 has vertex edge neighborhood prime graph, then there exists a graph from the class $G_5\odot$[composed of graphs lotus inside a circle LC_n,prism Y_n and antiprism A_n connected by an alternating band of triangle] that admits vertex edge neighborhood prime.*

Proof. Let $G_5(p_5,q_5)$ is vertex edge neighborhood prime graph with bijection $g_5:V(G_5)\cup E(G_5)\to$ $\{1,2,...,|V(G_5)\cup E(G_5)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_5 be the composed of graphs lotus inside a circle LC_n,prism Y_n and antiprism A_n connected by an alternating band of triangle with
$V(H_5)=\{c_0\}\cup\{c_i,d_i,c'_i,d'_i,c''_i,d''_i:1\le i\le n\}$ and
$E(H_5)=\{c_0c_i,c_id_i,c'_id_i,c'_id'_i,c''_id'_i,c''_id''_i:1\le i\le n\}\cup\{d_1d_n\}\cup\{d_1c'_n\}\cup\{c'_1c'_n\}\cup\{c''_nd'_1\}\cup\{d'_1d'_n\}\cup\{c''_1c''_n\}\cup\{c''_nd''_1\}\cup\{c_{i+1}d_i,d_id_{i+1},d_{i+1}c'_i,c'_ic'_{i+1},c''_id'_{i+1},d'_id'_{i+1}:1\le i\le n-1\}\cup\{c''_ic''_{i+1},c''_id''_{i+1},d''_id''_{i+1}:1\le i\le n-1\}\cup\{d''_1d''_n\}\cup\{c_1d_n\}$.
We overlay one of the vertex say c_0 of H_5 on selected vertex of r_1 in G_5 with $g_5(r_1)=1$.
Note that $G_5^*=G_5\odot H_5$ with $V(G_5^*)=V(G_5)\cup V(H_5)$ and $E(G_5^*)=E(G_5)\cup E(H_5)$.
$\left|V(G_5^*)\right|=p_5+6n$ and $\left|E(G_5^*)\right|=q_5+15n$.
Define $h_5:V(G_5^*)\cup E(G_5^*)\to\{1,2,...,p_5+q_5+21n\}$ as follows:
$g_5(u_5)=h_5(u_5)$ for all $u_5\in V(G_5)$ and $g_5(e_5)=h_5(e_5)$ for all $e_5\in E(G_5)$.
$h_5(c_0)=h_5(r_1)=1, h_5(d''_1d''_n)=p_5+q_5+7n, h_5(d''_1c''_n)=p_5+q_5+7n+1, h_5(c''_1c''_n)=p_5+q_5+10n, h_5(c''_nd'_1)=p_5+q_5+12n, h_5(d'_1d'_n)=p_5+q_5+13n, h_5(c'_1c'_n)=p_5+q_5+15n, h_5(d_1c'_n)=p_5+q_5+17n, h_5(d_1d_n)=p_5+q_5+18n, h_5(c_1d_n)=p_5+q_5+20n$.
For each $1\le i\le n, h_5(c_i)=p_5+q_5+5n+i, h_5(d_i)=p_5+q_5+4n+i, h_5(c'_i)=p_5+q_5+3n+i, h_5(d'_i)=p_5+q_5+2n+i, h_5(c''_i)=p_5+q_5+n+i, h_5(d''_i)=p_5+q_5+i, h_5(c''_id''_i)=p_5+q_5+7n+2i, h_5(c''_id'_i)=p_5+q_5+10n+2i-1, h_5(c'_id'_i)=p_5+q_5+13n+i, h_5(c'_id_i)=p_5+q_5+15n+2i-1, h_5(c_id_i)=p_5+q_5+18n+2i-1, h_5(c_0c_i)=p_5+q_5+20n+i$.
For each $1\le i\le n-1, h_5(d''_id''_{i+1})=p_5+q_5+6n+i$, $h_5(c''_id''_{i+1})=p_5+q_5+7n+2i+1, h_5(c''_ic''_{i+1})=p_5+q_5+9n+i, h_5(c''_id'_{i+1})=p_5+q_5+10n+2i, h_5(d'_id'_{i+1})=p_5+q_5+12n+i, h_5(c'_ic'_{i+1})=p_5+q_5+14n+i, h_5(d_{i+1}c'_i)=p_5+q_5+15n+2i, h_5(d_id_{i+1})=p_5+q_5+17n+i, h_5(c_{i+1}d_i)=p_5+q_5+18n+2i$.
Clearly, G_5 is vertex edge neighborhood prime graph. We need to prove H_5 is vertex edge neighborhood prime graph. Let u_5 be any vertex of H_5.
For $u_5=c_0,c_i,d_i,c'_i,d'_i,c''_i,d''_i$ for $1\le i\le n$ with $deg(u_5)\ge 2$. Here, $gcd\{h_5(w_5):w_5\in N_V(u_5)\}=1$ and
$\gcd\{h_5(d_5):d_5\in N_E(u_5)\}=1$.
Hence $G_5^*=G_5\odot H_5$ admits vertex edge neighborhood prime graph. □

Theorem 2.6. *If G_6 has vertex edge neighborhood prime graph, then there exists a graph from the class $G_6\odot$[composed of graphs closed helm CH_n, prism Y_n and antiprism A_n connected by an alternating band of triangle] that admits vertex edge neighborhood prime .*

Proof. Let $G_6(p_6,q_6)$ be vertex edge neighborhood prime graph with bijection $g_6:V(G_6)\cup E(G_6)\to$ $\{1,2,...,|V(G_6)\cup E(G_6)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_6 be the composed of graphs closed helm CH_n, prism Y_n and antiprism A_n connected by an alternating band of triangle with
$V(H_6)=\{a'_0\}\cup\{a'_i,b'_i,c'_i,d'_i,e'_i,f'_i:1\le i\le n\}$ and
$E(H_6)=\{a'_0a'_i,a'_ib'_i,b'_ic'_i,c'_id'_i,d'_ie'_i,e'_if'_i:1\le i\le n\}\cup\{a'_1a'_n\}\cup\{b'_1b'_n\}\cup\{b'_1c'_n\}\cup\{c'_1c'_n\}\cup\{d'_1d'_n\}\cup\{d'_1e'_n\}\cup\{e'_1e'_n\}\cup\{f'_1e'_n\}\cup\{f'_1f'_n\}\cup\{a'_ia'_{i+1},b'_ib'_{i+1},b'_{i+1}c'_i,c'_ic'_{i+1},d'_id'_{i+1}:1\le i\le n-1\}\cup\{e'_ie'_{i+1},e'_if'_{i+1},f'_if'_{i+1},d'_{i+1}e'_i:1\le i\le n-1\}$.
We overlay one of the vertex say a'_0 of H_6 on selected vertex of z_1 in G_6 with $g_6(z_1)=1$.
Also, $G_6^*=G_6\odot H_6$ with $V(G_6^*)=V(G_6)\cup V(H_6)$ and $E(G_6^*)=E(G_6)\cup E(H_6)$
$\left|V(G_6^*)\right|=p_6+6n$ and $\left|E(G_6^*)\right|=q_6+15n$.
Define $h_6:V(G_6^*)\cup E(G_6^*)\to\{1,2,...,p_6+q_6+21n\}$ as follows:
$g_6(z_6)=h_6(z_6)$ for all $z_6\in V(G_6)$ and $g_6(d_6)=h_6(d_6)$ for all $d_6\in E(G_6)$.

$h_6(a'_0) = h_6(z_1) = 1, h_6(f'_1 f'_n) = p_6 + q_6 + 7n, h_6(f'_1 e'_n) = p_6 + q_6 + 7n + 1, h_6(e'_1 e'_n) = p_6 + q_6 + 10n, h_6(d'_1 e'_n) = p_6 + q_6 + 12n, h_6(d'_1 d'_n) = p_6 + q_6 + 13n, h_6(c'_1 c'_n) = p_6 + q_6 + 15n,$
$h_6(b'_1 c'_n) = p_6 + q_6 + 17n, h_6(b'_1 b'_n) = p_6 + q_6 + 18n,$
$h_6(a'_1 a'_n) = p_6 + q_6 + 20n.$
For each $1 \le i \le n, h_6(e'_i f'_i) = p_6 + q_6 + 7n + 2i, h_6(d'_i e'_i) = p_6 + q_6 + 10n + 2i - 1, h_6(c'_i d'_i) = p_6 + q_6 + 13n + i, h_6(b'_i c'_i) = p_6 + q_6 + 15n + 2i - 1, h_6(a'_i b'_i) = p_6 + q_6 + 18n + i, h_6(a'_0 a'_i) = p_6 + q_6 + 20n + i.$
For each $1 \le i \le n-1, h_6(f'_i f'_{i+1}) = p_6 + q_6 + 6n + i,$
$h_6(e'_i f'_{i+1}) = p_6 + q_6 + 7n + 2i + 1, h_6(e'_i f'_i) = p_6 + q_6 + 9n + i, h_6(d'_{i+1} e'_i) = p_6 + q_6 + 10n + 2i, h_6(d'_i d'_{i+1}) = p_6 + q_6 + 12n + i, h_6(c'_i c'_{i+1}) = p_6 + q_6 + 14n + i, h_6(b'_{i+1} c'_i) = p_6 + q_6 + 15n + 2i, h_6(b'_i b'_{i+1}) = p_6 + q_6 + 17n + i, h_6(a'_i a'_{i+1}) = p_6 + q_6 + 19n + i.$
Consider the following cases.
Case 1. $p_6 + q_6$ is odd
For each $1 \le i \le n, h_6(a'_i) = p_6 + q_6 + 4n + 2i, h_6(b'_i) = p_6 + q_6 + 4n + 2i - 1, h_6(c'_i) = p_6 + q_6 + 2n + 2i, h_6(d'_i) = p_6 + q_6 + 2n + 2i - 1, h_6(e'_i) = p_6 + q_6 + 2i, h_6(f'_i) = p_6 + q_6 + 2i - 1.$
Case 2. $p_6 + q_6$ is even
For each $1 \le i \le n, h_6(a'_i) = p_6 + q_6 + 4n + 2i - 1, h_6(b'_i) = p_6 + q_6 + 4n + 2i, h_6(c'_i) = p_6 + q_6 + 2n + 2i - 1, h_6(d'_i) = p_6 + q_6 + 2n + 2i, h_6(e'_i) = p_6 + q_6 + 2i - 1, h_6(f'_i) = p_6 + q_6 + 2i.$
Already, G_6 is vertex edge neighborhood prime graph. It's enough to prove H_6 is vertex edge neighborhood prime graph.
Let a_6 be any vertex of H_6.
For $a_6 = a'_0, a'_i, b'_i, c'_i, d'_i, e'_i, f'_i$ for $1 \le i \le n$ with $deg(a_6) \ge 2$. Here,
$\gcd\{h_6(w_6) : w_6 \in N_V(a_6)\} = 1$ and
$\gcd\{h_6(d_6) : d_6 \in N_E(a_6)\} = 1.$
Hence $G_6 \odot H_6$ admits vertex edge neighborhood prime graph. □

Theorem 2.7. *If $G_7(p_7, q_7)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_7 \odot$ armed crown graph AC_t that admits vertex edge neighborhood prime for all t.*

Proof. Let $G_7(p_7, q_7)$ be vertex edge neighborhood prime graph with bijection $g_7 : V(G_7) \cup E(G_7) \to \{1, 2, ..., |V(G_7) \cup E(G_7)|\}$ satisfying the property of vertex edge neighborhood prime graph.
Consider H_7 be armed crown graph AC_t with
$V(H_7) = \{u'_x, v'_x, w'_x : 1 \le x \le t\}$ and
$E(H_7) = \{u'_x u'_{x+1} : 1 \le x \le t-1\} \cup \{u'_1 u'_t\} \cup \{u'_x v'_x, v'_x w'_x : 1 \le x \le t\}.$
We overlay one of the vertex say u'_1 of H_7 on selected vertex of a_1 in G_7 with $g_7(a_1) = 1$.
Note that $G^*_7 = G_7 \odot H_7$ with $V(G^*_7) = V(G_7) \cup V(H_7)$ and $E(G^*_7) = E(G_7) \cup E(H_7)$.
$|V(G^*_7)| = p_7 + 3t - 1$ and $|E(G^*_7)| = q_7 + 3t$.
Define $h_7 : V(G^*_7) \cup E(G^*_7) \to \{1, 2, ..., p_7 + q_7 + 6t - 1\}$ as follows:
$g_7(z_7) = h_7(z_7)$ for all $z_7 \in V(G_7)$ and $g_7(d_7) = h_7(d_7)$ for all $d_7 \in E(G_7)$.
$h_7(u'_1) = h_7(a_1) = 1, h_7(u'_1 u'_t) = p_7 + q_7 + 5t.$
For each $1 \le x \le t, h_7(v'_x) = p_7 + q_7 + 2t + 3x - 3, h_7(u'_x v'_x) = p_7 + q_7 + 2t + 3x - 1, h_7(v'_x w'_x) = p_7 + q_7 + 2t + 3x - 2.$
$h_7(u'_x u'_{x+1}) = p_7 + q_7 + 5t + x$ for $1 \le x \le t-1$.
We consider the following two cases.
Case 1. $p_7 + q_7$ is odd
$h_7(u'_{x+1}) = p_7 + q_7 + 2x$ for $1 \le x \le t-1$.
$h_7(w'_x) = p_7 + q_7 + 2x - 1$ for $1 \le x \le t$.
Case 2. $p_7 + q_7$ is even
$h_7(w'_1) = p_7 + q_7 + 1.$
$h_7(u'_x) = p_7 + q_7 + 2x - 1$ for $2 \le x \le t$.
$h_7(w'_{x+1}) = p_7 + q_7 + 2x$ for $1 \le x \le t-1$.
We claim that G^*_7 is vertex edge neighborhood prime graph. Clearly, G_7 is vertex edge neighborhood prime graph. We have to prove H_7 is vertex edge neighborhood prime graph.
Let a_7 be any vertex of H_7.
For $a_7 = u'_x, v'_x, w'_x$ for $1 \le x \le t$ with $deg(a_7) \ge 2$. Here,
$\gcd\{h_7(y_7) : y_7 \in N_V(a_7)\} = 1$ and
$\gcd\{h_7(d_7) : d_7 \in N_E(a_7)\} = 1.$
Hence $G_7 \odot H_7$ is vertex edge neighborhood prime graph. □

Theorem 2.8. *If $G_8(p_8, q_8)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_8 \odot$ [x copies of prism $C_y \times K_2$ connected by an alternating band of triangle] that admits vertex edge neighborhood prime when y is even.*

Proof. Let $G_8(p_8, q_8)$ be vertex edge neighborhood prime graph with bijection $g_8 : V(G_8) \cup E(G_8) \to \{1, 2, ..., |V(G_8) \cup E(G_8)|\}$ satisfying the property of vertex edge neighborhood prime graph.
Consider H_8 be x copies of prism $C_y \times K_2$ connected by alternating band of triangle, where y is even with
$V(H_8) = \{k'_{bc}, l'_{bc} : 1 \le b \le x, 1 \le c \le y\}$ and
$E(H_8) = \{k'_{bc} l'_{bc} : 1 \le b \le x, 1 \le c \le y\}$
$\cup \left\{l'_{b1} k'_{(b+1)y} : 1 \le b \le x-1\right\}$
$\cup \left\{k'_{bc} k'_{b(c+1)}, l'_{bc} l'_{b(c+1)} : 1 \le b \le x, 1 \le c \le y-1\right\} \cup$
$\left\{k'_{b1} k'_{by}, l'_{b1} l'_{by} : 1 \le b \le x\right\}$
$\cup \left\{l'_{bc} k'_{(b+1),c} : 1 \le b \le x-1, 1 \le c \le y\right\}$
$\cup \left\{l'_{b(c+1)} k'_{(b+1)c} : 1 \le b \le x-1, 1 \le c \le y-1\right\}.$
We overlay one of the vertex say l_{x1} of H_8 on selected vertex of f_1 in G_8 with $g_8(f_1) = 1$.
Also, $G^*_8 = G_8 \odot H_8$ with $V(G^*_8) = V(G_8) \cup V(H_8)$ and $E(G^*_8) = E(G_8) \cup E(H_8)$
$\left|V(G^*_8)\right| = p_8 + 2xy - 1$ and $\left|E(G^*_8)\right| = q_8 + 3xy + 2(x-1)y.$
Define

$$h_8 : V(G^*_8) \cup E(G^*_8) \to \{1, 2, ..., p_8 + q_8 + 5xy + 2(x-1)y - 1\}$$

as follows:
$g_8(z_8) = h_8(z_8)$ for all $z_8 \in V(G_8)$ and $g_8(d_8) = h_8(d_8)$ for all $d_8 \in E(G_8)$.
$h_8(l_{x1}) = h_8(f_1) = 1, h_8(k_{xy}) = p_8 + q_8 + (2x-2)y + 2y - 1.$

For each $1 \leq b \leq x-1$ and $1 \leq c \leq y, h_8(k'_{bc}) = p_8 + q_8 + (2b-2)y + 2c, h_8(l'_{bc}) = p_8 + q_8 + (2b-2)y + 2c - 1$.
For each $1 \leq c \leq y-1, h_8(k_{xc}) = p_8 + q_8 + (2x-2)y + 2c$, $h_8(l_{x(c+1)}) = p_8 + q_8 + (2x-2)y + 2c - 1$.
For each $1 \leq b \leq x$ and $1 \leq c \leq y-1, h_8(l'_{bc}l'_{b(c+1)}) = p_8 + q_8 + 2xy + 5(x-b)y + c - 1, h_8(k'_{bc}k'_{b(c+1)}) = p_8 + q_8 + 2xy + [5(x-b)+2]y + c - 1$.
For each $1 \leq b \leq x, h_8(l'_{b1}l'_{by}) = p_8 + q_8 + 2xy + [5(x-b)+1]y - 1, h_8(k'_{b1}k'_{by}) = p_8 + q_8 + 2xy + [5(x-b)+3]y - 1$.
$h_8(l'_{bc}k'_{(b+1)c}) = p_8 + q_8 + 2xy + [5(x-b-1)+3]y + 2c - 2$ for $1 \leq b \leq x-1$ and $1 \leq c \leq y$.
$h_8(l'_{b(c+1)}k'_{(b+1)c}) = p_8 + q_8 + 2xy + [5(x-b-1)+3]y + 2c - 1$ for $1 \leq b \leq x-1$ and $1 \leq c \leq y-1$.
$h_8(l'_{b1}k'_{(b+1)y}) = p_8 + q_8 + 2xy + 5(x-b)y - 1$ for $1 \leq b \leq x-1$.
In earlier, G_8 is vertex edge neighborhood prime graph. It's enough to prove H_8 is vertex edge neighborhood prime graph.
Let a_8 be any vertex of H_8.
For $a_8 = k'_{bc}, l'_{bc}$ for $1 \leq b \leq x$ and $1 \leq c \leq y$ with $deg(a_8) \geq 2$.
Here, $\gcd\{h_8(z_8) : z_8 \in N_V(a_8)\} = 1$ and
$\gcd\{h_8(e_8) : e_8 \in N_E(a_8)\} = 1$.
Hence $G_8^* = G_8 \odot H_8$ is vertex edge neighborhood prime graph. □

Theorem 2.9. *If $G_9(p_9, q_9)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_9 \odot [s$ copies of antiprism A_t connected by an alternating band of triangle$]$ that admits vertex edge neighborhood prime.*

Proof. Let $G_9(p_9, q_9)$ be vertex edge neighborhood prime graph with bijection $g_9 : V(G_9) \cup E(G_9) \to \{1, 2, ..., |V(G_9) \cup E(G_9)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_9 be s copies of antiprism A_t connected by an alternating band of triangle with
$V(H_9) = \{d'_{uv}, e'_{uv} : 1 \leq u \leq s, 1 \leq v \leq t\}$ and
$E(H_9) = \{d'_{uv}e'_{uv} : 1 \leq u \leq s, 1 \leq v \leq t\} \cup$
$\left\{d'_{u+1,v}e'_{u(v+1)} : 1 \leq u \leq s-1, 1 \leq v \leq t-1\right\} \cup$
$\left\{d'_{u+1,v}e'_{uv} : 1 \leq u \leq s-1, 1 \leq v \leq t\right\} \cup$
$\{d'_{u1}d'_{ut}, e'_{u1}e'_{ut}, d'_{ut}e'_{u1} : 1 \leq u \leq s\} \cup$
$\left\{d'_{uv}d'_{u(v+1)}, e'_{uv}e'_{u(v+1)}, d'_{uv}e'_{u(v+1)} : 1 \leq u \leq s, 1 \leq v \leq t-1\right\}$
$\cup \left\{d'_{(u+1)t}e'_{u1} : 1 \leq u \leq s-1\right\}$.
We overlay one of the vertex say e'_{s1} of H_9 on selected vertex of v_1 in G_9 with $g_9(v_1) = 1$.
Note that, $G_9^* = G_9 \odot H_9$ with $V(G_9^*) = V(G_9) \cup V(H_9)$ and $E(G_9^*) = E(G_9) \cup E(H_9)$
$\left|V(G_9^*)\right| = p_9 + 2st - 1$ and $\left|E(G_9^*)\right| = q_9 + 4st + 2(s-1)t$.
Define $h_9 : V(G_9^*) \cup E(G_9^*) \to$
$\{1, 2, ..., p_9 + q_9 + 6st + 2(s-1)t - 1\}$ as follows:
$g_9(z_9) = h_9(z_9)$ for all $z_9 \in V(G_9)$ and $g_9(c_9) = h_9(c_9)$ for all $c_9 \in E(G_9)$.
$h_9(e'_{s1}) = h_9(v_1) = 1$.
For each $1 \leq u \leq s$ and $1 \leq v \leq t, h_9(d'_{uv}) = p_9 + q_9 + (2s-2u)t + 2v - 1, h_9(e'_{uv}) = p_9 + q_9 + (2s-2u)t + 2v - 2$,
$h_9(d'_{uv}e'_{uv}) = p_9 + q_9 + 2st + [6(s-u)+1]t + 2v - 1$.
For each $1 \leq u \leq s$ and $1 \leq v \leq t-1, h_9(e'_{uv}e'_{u(v+1)}) = p_9 + q_9 + 2st + 6(s-u)t + v - 1, h_9(d'_{uv}d'_{u(v+1)}) = p_9 + q_9 + 2st + [6(s-u)+3]t + v - 1, h_9(d'_{u,v}e'_{u(v+1)}) = p_9 + q_9 + 2st + [6(s-u)+1]t + 2v$.
For each $1 \leq u \leq s, h_9(e'_{u1}e'_{ut}) = p_9 + q_9 + 2st + [6(s-u)+1]t - 1, h_9(d'_{u1}d'_{ut}) = p_9 + q_9 + 2st + [6(s-u)+4]t - 1$,
$h_9(d'_{ut}e'_{u1}) = p_9 + q_9 + 2st + [6(s-u)+1]t$.
$h_9(d'_{(u+1)v}e'_{uv}) = p_9 + q_9 + 2st + [6(s-1-u)+4]t - 2 + 2v$ for $1 \leq u \leq s-1$ and $1 \leq v \leq t$.
$h_9(d'_{(u+1)v}e'_{u(v+1)}) = p_9 + q_9 + 2st + [6(s-1-u)+4]t + 2v - 1$ for $1 \leq u \leq s-1$ and $1 \leq v \leq t-1$.
$h_9(d'_{(u+1)t}e'_{u1}) = p_9 + q_9 + 2st + [6(s-1-u)+6]t - 1$ for $1 \leq u \leq s-1$.
Already, G_9 is vertex edge neighborhood prime graph. It's enough to prove H_9 is vertex edge neighborhood prime graph.
Let a_9 be any vertex of H_9.
For $a_9 = d'_{uv}, e'_{uv}$ for $1 \leq u \leq s$ and $1 \leq v \leq t$ with $deg(a_9) \geq 2$.
Here, $\gcd\{h_9(z_9) : z_9 \in N_V(a_9)\} = 1$ and
$\gcd\{h_9(c_9) : c_9 \in N_E(a_9)\} = 1$.
Hence $G_9^* = G_9 \odot H_9$ admits vertex edge neighborhood prime graph. □

Theorem 2.10. *If $G_{10}(p_{10}, q_{10})$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_{10} \odot [P_y + zK_1]$ that admits vertex edge neighborhood prime.*

Proof. Let $G_{10}(p_{10}, q_{10})$ be vertex edge neighborhood prime graph with bijection $g_{10} : V(G_{10}) \cup E(G_{10}) \to \{1, 2, ..., |V(G_{10}) \cup E(G_{10})|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_{10} be the graph of $P_y + zK_1$ with
$V(H_{10}) = \{c'_a : 1 \leq a \leq y\} \cup \{d'_b : 1 \leq b \leq z\}$ and
$E(H_{10}) = \{c'_a c'_{a+1} : 1 \leq a \leq y-1\}$
$\cup \{c'_a d'_b : 1 \leq a \leq y, 1 \leq b \leq z\}$.
We overlay one of the vertex say d'_1 of H_{10} on selected vertex of s_1 in G_{10} with $g_{10}(s_1) = 1$.
Also, $G_{10}^* = G_{10} \odot H_{10}$ with $V(G_{10}^*) = V(G_{10}) \cup V(H_{10})$ and $E(G_{10}^*) = E(G_{10}) \cup E(H_{10})$.
$\left|V(G_{10}^*)\right| = p_{10} + y + z - 1$ and $\left|E(G_{10}^*)\right| = q_{10} + yz + y - 1$.
Define $h_{10} : V(G_{10}^*) \cup E(G_{10}^*) \to$
$\{1, 2, ..., p_{10} + q_{10} + yz + 2y + z - 2\}$ as follows:
$g_{10}(z_{10}) = h_{10}(z_{10})$ for all $z_{10} \in V(G_{10})$ and $g_{10}(e_{10}) = h_1(e_{10})$ for all $e_{10} \in E(G_{10})$.
$h_{10}(d'_1) = h_{10}(s_1) = 1, h_{10}(c'_1 d'_1) = p_{10} + q_{10} + y + z, h_{10}(c'_y d'_1) = p_{10} + q_{10} + 2y + z$.
$h_{10}(c'_{2a-1}) = p_{10} + q_{10} + a$ for $1 \leq a \leq \left\lceil \frac{Y}{2} \right\rceil$.
$h_{10}(c'_{2a}) = p_{10} + q_{10} + \left\lceil \frac{y}{2} \right\rceil + a$ for $1 \leq a \leq \left\lfloor \frac{y}{2} \right\rfloor$.
$h_{10}(d'_b) = p_{10} + q_{10} + y + b - 1$ for $2 \leq b \leq z$.
$h_{10}(c'_a c'_{a+1}) = p_{10} + q_{10} + y + z + a$ for $1 \leq a \leq y-1$.
$h_{10}(c'_a d'_1) = p_{10} + q_{10} + 2y + z + a - 1$ for $2 \leq a \leq y-1$.
$h_{10}(c'_a d'_b) = p_{10} + q_{10} + (b+1)y + z - 2 + a$ for $2 \leq b \leq z$ and

$1 \leq a \leq y$.
Clearly, G_{10} is vertex edge neighborhood prime graph. We claim that H_{10} is vertex edge neighborhood prime graph. Let x_{10} be any vertex of H_{10}.
For $x_{10} = c'_a, d'_b$ for $1 \leq a \leq y$ and $1 \leq b \leq z$ with $deg(x_{10}) \geq 2$.
Here,
$\gcd\{h_{10}(w_{10}) : w_{10} \in N_V(x_{10})\} = 1$ and
$\gcd\{h_{10}(e_{10}) : e_{10} \in N_E(x_{10})\} = 1$.
Hence $G^*_{10} = G_{10} \odot H_{10}$ is vertex edge neighborhood prime graph. □

Theorem 2.11. *If $G_{11}(p_{11}, q_{11})$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_{11} \odot [C_u + tK_1]$ that admits vertex edge neighborhood prime.*

Proof. Let $G_{11}(p_{11}, q_{11})$ be vertex edge neighborhood prime graph with bijection $g_{11} : V(G_{11}) \cup E(G_{11}) \to \{1, 2, ..., |V(G_{11}) \cup E(G_{11})|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_{11} be the graph of $C_u + tK_1$ with
$V(H_{11}) = \{c''_a : 1 \leq a \leq u\} \cup \{d''_b : 1 \leq b \leq t\}$ and
$E(H_{11}) = \{c''_a c''_{a+1} : 1 \leq a \leq u-1\} \cup \{c''_1 c''_u\} \cup \{c''_a d''_b : 1 \leq a \leq u, 1 \leq b \leq t\}$.
We overlay one of the vertex say d''_1 of H_{11} on selected vertex of s_1 in G_{11} with $g_{11}(s_1) = 1$.
Note that $G^*_{11} = G_{11} \odot H_{11}$ with $V(G^*_{11}) = V(G_{11}) \cup V(H_{11})$ and $E(G^*_{11}) = E(G_{11}) \cup E(H_{11})$.
$|V(G^*_{11})| = p_{11} + u + t - 1$ and $|E(G^*_{11})| = q_{11} + u + ut$.
Define $h_{11} : V(G^*_{11}) \cup E(G^*_{11}) \to \{1, 2, ..., p_{11} + q_{11} + 2u + ut + t - 1\}$ as follows:
$g_{11}(z_{11}) = h_{11}(z_{11})$ for all $z_{11} \in V(G_{11})$ and $g_{11}(e_{11}) = h_{11}(e_{11})$ for all $e_{11} \in E(G_{11})$.
$h_{11}(d''_1) = h_{11}(s_1) = 1, h_{11}(c''_1 c''_u) = p_{11} + q_{11} + 2u + t - 1$.
$h_{11}(c''_a) = p_{11} + q_{11} + a$ for $1 \leq a \leq u$.
$h_{11}(d''_b) = p_{11} + q_{11} + u + b - 1$ for $2 \leq b \leq t$.
$h_{11}(c''_a c''_{a+1}) = p_{11} + q_{11} + u + t + a - 1$ for $1 \leq a \leq u-1$.
$h_{11}(c''_a d''_b) = p_{11} + q_{11} + (b+1)u + t + a - 1$ for $1 \leq a \leq u$ and $1 \leq b \leq t$.
Clearly, G_{11} is vertex edge neighborhood prime graph. We claim that H_{11} is vertex edge neighborhood prime graph. Let x_{11} be any vertex of H_{11}.
For $x_{11} = c''_a, d''_b$ for $1 \leq a \leq u$ and $1 \leq b \leq t$ with $deg(x_{11}) \geq 2$.
Here, $\gcd\{h_{11}(w_{11}) : w_{11} \in N_V(x_{11})\} = 1$ and
$\gcd\{h_{11}(e_{11}) : e_{11} \in N_E(x_{11})\} = 1$.
Hence $G^*_{11} = G_{11} \odot H_{11}$ is vertex edge neighborhood prime graph. □

Theorem 2.12. *If $G_{12}(p_{12}, q_{12})$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_{12} \odot$ [Mycielskian graph $\mu(C_x)$, of cycle C_x] that admits vertex edge neighborhood prime for all x is odd.*

Proof. Let $G_{12}(p_{12}, q_{12})$ be vertex edge neighborhood prime graph with bijection $g_{12} : V(G_{12}) \cup E(G_{12}) \to \{1, 2, ..., |V(G_{12}) \cup E(G_{12})|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_{12} be Mycielskian graph $\mu(C_x)$ of cycle C_x(x is odd) with
$V(H_{12}) = \{r_0\} \cup \{r_z, s_z : 1 \leq z \leq x\}$ and
$E(H_{12}) = \{r_0 r_z : 1 \leq z \leq x\} \cup \{r_{z+1} s_z, r_z s_{z+1} : 1 \leq z \leq x-1\} \cup \{r_1 s_x\} \cup \{r_x s_1\}$.
We overlay one of the vertex say r_1 of H_{12} on selected vertex of s_1 in G_{12} with $g_{12}(s_1) = 1$.
Also, $G^*_{12} = G_{12} \odot H_{12}$ with $V(G^*_{12}) = V(G_{12}) \cup V(H_{12})$ and $E(G^*_{12}) = E(G_{12}) \cup E(H_{12})$.
$|V(G^*_{12})| = p_{12} + 2x$ and $|E(G^*_{12})| = q_{12} + 3x$.
Define $h_{12} : V(G^*_{12}) \cup E(G^*_{12}) \to \{1, 2, ..., p_{12} + q_{12} + 5x\}$ as follows:
$g_{12}(z_{12}) = h_{12}(z_{12})$ for all $z_{12} \in V(G_{12})$ and $g_{12}(e_{12}) = h_{12}(e_{12})$ for all $e_{12} \in E(G_{12})$.
$h_{12}(r_1) = h_{12}(s_1) = 1, h_{12}(r_0) = p_{12} + q_{12} + x, h_{12}(s_1 r_x) = p_{12} + q_{12} + 2x + 2\left\lceil \frac{x}{2} \right\rceil - 1, h_{12}(s_1 r_2) = p_{12} + q_{12} + 2x + 2\left\lceil \frac{x}{2} \right\rceil$,
$h_{12}(r_1 s_x) = p_{12} + q_{12} + 4x$.
$h_{12}(r_{2z-1}) = p_{12} + q_{12} + z - 1$ for $2 \leq z \leq \left\lceil \frac{x}{2} \right\rceil$.
$h_{12}(s_{2z-1}) = p_{12} + q_{12} + x + z$ for $1 \leq z \leq \left\lceil \frac{x}{2} \right\rceil$.
For each $1 \leq z \leq \left\lfloor \frac{x}{2} \right\rfloor, h_{12}(r_{2z}) = p_{12} + q_{12} + \left\lceil \frac{x}{2} \right\rceil + z - 1$,
$h_{12}(s_{2z}) = p_{12} + q_{12} + x + \left\lceil \frac{x}{2} \right\rceil + z, h_{12}(r_{2z-1} s_{2z}) = p_{12} + q_{12} + 2x + 2z - 1, h_{12}(r_{2z} s_{2z+1}) = p_{12} + q_{12} + 2x + 2\left\lceil \frac{x}{2} \right\rceil + 2z - 1$,
$h_{12}(r_{2z+1} s_{2z}) = p_{12} + q_{12} + 2x + 2z$.
$h_{12}(r_0 r_z) = p_{12} + q_{12} + 4x + z$ for $1 \leq z \leq x$.
$h_{12}(r_{2z} s_{2z-1}) = p_{12} + q_{12} + 2x + 2\left\lceil \frac{x}{2} \right\rceil + 2z - 2$ for $2 \leq z \leq \left\lfloor \frac{x}{2} \right\rfloor$.
Clearly, G_{12} is vertex edge neighborhood prime graph. We claim that H_{12} is vertex edge neighborhood prime graph. Let x_{12} be any vertex of H_{12}.
For $x_{12} = r_0, r_z, s_z$ for $1 \leq z \leq x$ with $deg(x_{12}) \geq 2$. Here,
$\gcd\{h_{12}(w_{12}) : w_{12} \in N_V(x_{12})\} = 1$ and
$\gcd\{h_{12}(e_{12}) : e_{12} \in N_E(x_{12})\} = 1$.
Hence $G^*_{12} = G_{12} \odot H_{12}$ is vertex edge neighborhood prime graph. □

3. Graph identification of one point union of graphs

$G^{(k)}$ is one point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p,q) graph then $\left|V(G^{(k)})\right| = k(p-1)+1$ and $\left|E(G^{(k)})\right| = kq$. In this section, We discuss about one point union of graphs.

Theorem 3.1. *If $G_1(p_1, q_1)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_1 \odot$ [one point union of different copies of triangular snake graphs $T_{s_y}(1 \leq y \leq t)$] that admits vertex edge neighborhood prime.*

Proof. Let $G_1(p_1, q_1)$ be vertex edge neighborhood prime graph with bijection $g_1 : V(G_1) \cup E(G_1) \to \{1, 2, ..., |V(G_1) \cup E(G_1)|\}$ satisfying the condition of vertex

edge neighborhood prime graph.
Consider H_1 be the one point union of different copies of triangular snake graphs $T_{s_y}(1 \leq y \leq t)$ with
$V(H_1) = \{u'_{10}\} \cup \{u'_{yz}, v'_{yz} : 1 \leq y \leq t, 1 \leq z \leq s_y - 1\}$ and
$E(H_1) = \Big\{u'_{10}u'_{y1}, u'_{10}v'_{y1} : 1 \leq y \leq t\Big\} \cup$
$\{u'_{yz}v'_{yz} : 1 \leq y \leq t, 1 \leq z \leq s_y - 1\} \cup$
$\Big\{u'_{yz}u'_{yz+1}, u'_{yz}v'_{yz+1} : 1 \leq y \leq t, 1 \leq z \leq s_y - 2\Big\}$.
We superimposing one of the vertex say u_{10} of H_1 on selected vertex of s_1 in G_1 with $g_1(s_1) = 1$.
Also, $G_1^* = G_1 \odot H_1$ with $V(G_1^*) = V(G_1) \cup V(H_1)$ and $E(G_1^*) = E(G_1) \cup E(H_1)$.
$|V(G_1^*)| = p_1 + 2(s_1 + s_2 + ... + s_t) - 2t$ and $|E(G_1^*)| = q_1 + 3(s_1 + s_2 + ... + s_t) - 3t$.
Define $h_1 : V(G_1^*) \cup E(G_1^*) \to$
$\{1, 2, ..., p_1 + q_1 + 5(s_1 + s_2 + ... + s_t) - 5t\}$ as follows:
$g_1(z_1) = h_1(z_1)$ for all $z_1 \in V(G_1)$ and $g_1(e_1) = h_1(e_1)$ for all $e_1 \in E(G_1)$.
$h_1(u_{10}) = h_1(s_1) = 1$.
For each $1 \leq y \leq t, h_1(u'_{10}u'_{y1}) = p_1 + q_1 + 2\sum_{c=1}^{t} s_c - 2t + 3\sum_{c=1}^{y-1} s_c - 3(y-1) + 1, h_1(u'_{10}v'_{y1}) = p_1 + q_1 + 2\sum_{c=1}^{t} s_c - 2t + 3\sum_{c=1}^{y-1} s_c - 3(y-1) + 2, h_1(u'_{ys_y-1}v'_{ys_y-1}) = p_1 + q_1 + 2\sum_{c=1}^{t} s_c - 2t + 3\sum_{c=1}^{y} s_c - 3(y-1) - 4, h_1(u'_{ys_y-2}v'_{ys_y-1}) = p_1 + q_1 + 2\sum_{c=1}^{t} s_c - 2t + 3\sum_{c=1}^{y} s_c - 3(y-1) - 3$.
For each $1 \leq y \leq t$ and $1 \leq z \leq s_y - 2, h_1(u'_{yz}u'_{yz+1}) = p_1 + q_1 + 2\sum_{c=1}^{t} s_c - 2t + 3\sum_{c=1}^{y-1} s_c - 3(y-1) + 1 + 3z, h_1(u'_{yz}v'_{yz}) = p_1 + q_1 + 2\sum_{c=1}^{t} s_c - 2t + 3\sum_{c=1}^{y-1} s_c - 3(y-1) + 3z$.
$h_1(u'_{yz}v'_{yz+1}) = p_1 + q_1 + 2\sum_{c=1}^{t} s_c - 2t + 3\sum_{c=1}^{y-1} s_c - 3(y-1) + 2 + 3z$ for $1 \leq y \leq t$ and $1 \leq z \leq s_y - 3$.
Consider the followig cases.
Case 1. $p_1 + q_1$ is odd
For each $1 \leq y \leq t$ and $1 \leq z \leq s_y - 1, h_1(u'_{y,z}) = p_1 + q_1 + 2\sum_{c=1}^{y-1} s_c - 2(y-1) + 2z, h_1(v'_{yz}) = p_1 + q_1 + 2\sum_{c=1}^{y-1} s_c - 2(y-1) + 2z - 1$.
Case 1. $p_1 + q_1$ is even
For each $2 \leq y \leq t$ and $1 \leq z \leq s_y - 1, h_1(u'_{y,z}) = p_1 + q_1 + 2\sum_{c=1}^{y-1} s_c - 2(y-1) + 2z - 1, h_1(v'_{yz}) = p_1 + q_1 + 2\sum_{c=1}^{y-1} s_c - 2(y-1) + 2z - 2$.
$h_1(v'_{11}) = p_1 + q_1 + 2\sum_{c=1}^{y-1} s_c - 2(y-1) + 2(s_y - 1)$.
$h_1(u'_{1z}) = p_1 + q_1 + 2z - 1$ for $1 \leq z \leq s_1 - 1$.
Clearly, G_1 is vertex edge neighborhood prime graph. We claim that H_1 is vertex edge neighborhood prime graph. Let x_1 be any vertex of H_1.
For $x_1 = u'_{10}, u'_{yz}, v'_{yz}$ for $1 \leq y \leq t$ and $1 \leq z \leq s_y - 1$ with $deg(x_1) \geq 2$. Here, $\gcd\{h_1(w_1) : w_1 \in N_V(x_1)\} = 1$ and $\gcd\{h_1(e_1) : e_1 \in N_E(x_1)\} = 1$.
Hence $G_1^* = G_1 \odot H_1$ is vertex edge neighborhood prime graph. □

Theorem 3.2. *If $G_2(p_2, q_2)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_2 \odot$ [one point union of different copies of quadrilateral snake graphs $Q_{t_a}(1 \leq a \leq r)$] that admits vertex edge neighborhood prime.*

Proof. Let $G_2(p_2, q_2)$ be vertex edge neighborhood prime graph with bijection $g_2 : V(G_2) \cup E(G_2) \to$
$\{1, 2, ..., |V(G_2) \cup E(G_2)|\}$ satisfying the property of vertex edge neighborhood prime graph.
Consider H_2 be the one point union of different copies of quadrilateral snake graphs $Q_{t_a}(1 \leq a \leq r)$ with
$V(H_2) = \{x'_{10}\} \cup \{x'_{ab}, y'_{ab}, z'_{ab} : 1 \leq a \leq r, 1 \leq b \leq t_a - 1\}$ and
$E(H_2) = \{x'_{10}x'_{a1}, x'_{10}y'_{a1} : 1 \leq a \leq r\} \cup$
$\{y'_{ab}z'_{ab}, x'_{ab}z'_{ab} : 1 \leq a \leq r, 1 \leq b \leq t_a - 1\} \cup$
$\{x'_{ab}y'_{ab+1}, x'_{ab}x'_{ab+1} : 1 \leq a \leq r, 1 \leq b \leq t_a - 2\}$.
We superimposing one of the vertex say x'_{10} of H_2 on selected vertex of t_1 in G_2 with $g_2(t_1) = 1$.
Note that $G_2^* = G_2 \odot H_2$ with $V(G_2^*) = V(G_2) \cup V(H_2)$ and $E(G_2^*) = E(G_2) \cup E(H_2)$.
$|V(G_2^*)| = p_2 + 3(t_1 + t_2 + ... + t_r) - 3r$ and $|E(G_2^*)| = q_2 4(t_1 + t_2 + ... + t_r) - 4r$.
Define $h_2 : V(G_2^*) \cup E(G_2^*) \to$
$\{1, 2, ..., p_2 + q_2 + 7(t_1 + t_2 + ... + t_r) - 7r\}$ as follows:
$g_2(z_2) = h_2(z_2)$ for all $z_2 \in V(G_2)$ and $g_2(e_2) = h_2(e_2)$ for all $e_2 \in E(G_2)$.
$h_2(x'_{10}) = h_2(t_1) = 1$.
For each $1 \leq a \leq r$ and $1 \leq b \leq t_a - 1, h_2(x'_{ab}) = p_2 + q_2 + 3\sum_{s=1}^{a-1} t_s - 3(a-1) + 3b, h_2(y'_{ab}) = p_2 + q_2 + 3\sum_{s=1}^{a-1} t_s - 3(a-1) + 3b - 1, h_2(z'_{ab}) = p_2 + q_2 + 3\sum_{s=1}^{a-1} t_s - 2 - 3(a-1) + 3b, h_2(y'_{ab}z'_{ab}) = p_2 + q_2 + 3\sum_{s=1}^{r} t_s - 3r + 4\sum_{s=1}^{a-1} t_s - 4(a-1) + 4b - 1$.
For each $1 \leq a \leq r, h_2(x'_{10}x'_{a1}) = p_2 + q_2 + 3\sum_{s=1}^{r} t_s - 3r + 4\sum_{s=1}^{a-1} t_s + 1 - 4(a-1), h_2(x'_{10}y'_{a1}) = p_2 + q_2 + 3\sum_{s=1}^{r} t_s - 3r + 4\sum_{s=1}^{a-1} t_s + 2 - 4(a-1), h_2(x'_{at_a-2}y'_{at_a-1}) = p_2 + q_2 + 3\sum_{s=1}^{r} t_s - 3r + 4\sum_{s=1}^{a} t_s - 4 - 4(a-1), h_2(x'_{at_a-1}z'_{at_a-1}) = p_2 + q_2 + 3\sum_{s=1}^{r} t_s - 3r + 4\sum_{s=1}^{a} t_s - 6 - 4(a-1)$.
For each $1 \leq a \leq r$ and $1 \leq b \leq t_a - 2, h_2(x'_{ab}z'_{ab}) = p_2 + q_2 + 3\sum_{s=1}^{r} t_s - 3r + 4\sum_{s=1}^{a-1} t_s - 4(a-1) + 4b, h_2(x'_{ab}x'_{ab+1}) = p_2 + q_2 + 3\sum_{s=1}^{r} t_s - 3r + 4\sum_{s=1}^{a-1} t_s + 1 - 4(a-1) + 4b$.
$h_2(x'_{ab}y'_{ab+1}) = p_2 + q_2 + 3\sum_{s=1}^{r} t_s - 3r + 4\sum_{s=1}^{a-1} t_s + 2 - 4(a-1) + 4b$ for $1 \leq a \leq r$ and $1 \leq b \leq t_a - 3$.
Already, G_2 is vertex edge neighborhood prime graph. Now we have to prove H_2 is vertex edge neighborhood prime graph.
Let a_2 be any vertex of H_2.
For $a_2 = x'_{10}, x'_{ab}, y'_{ab}, z'_{ab}$ for $1 \leq a \leq r$ and $1 \leq b \leq t_a - 1$ with $deg(a_2) \geq 2$. Here, $\gcd\{h_2(b_2) : b_2 \in N_V(a_2)\} = 1$ and $\gcd\{h_2(d_2) : d_2 \in N_E(a_2)\} = 1$.
Hence $G_2^* = G_2 \odot H_2$ admits vertex edge neighborhood prime graph. □

Theorem 3.3. *If $G_3(p_3, q_3)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_3 \odot$ [one point union of different copies of butterfly graphs BF_{c_r,d_r} $(1 \leq r \leq z)$] that admits vertex edge neighborhood prime.*

Proof. Let $G_3(p_3, q_3)$ be vertex edge neighborhood prime

graph with bijection $g_3 : V(G_3) \cup E(G_3) \to \{1, 2, ..., |V(G_3) \cup E(G_3)|\}$ satisfying the property of vertex edge neighborhood prime graph.
Consider H_3 be the one point union of different copies of butterfly graphs $BF_{c_r,d_r} (1 \le r \le z)$ with
$V(H_3) = \{u''_0\} \cup \{u''_{rs}, v''_{rs} : 1 \le r \le z, 1 \le s \le d_r - 1\} \cup \{w''_{rs} : 1 \le r \le z, 1 \le s \le c_r\}$ and
$E(H_3) = \left\{u''_0u''_{r1}, u''_0u''_{rd_r-1}, u''_0v''_{r1}, u''_0v''_{rd_r-1} : 1 \le r \le z\right\} \cup \{u''_0w''_{rs} : 1 \le r \le z, 1 \le s \le c_r\}$
$\cup \{u''_{rs}u''_{rs+1}, v''_{rs}v''_{rs+1} : 1 \le r \le z, 1 \le s \le d_r - 2\}$.
We superimposing one of the vertex say u''_0 of H_3 on selected vertex of u_1 in G_3 with $g_3(u_1) = 1$.
Also, $G^*_3 = G_3 \odot H_3$ with $V(G^*_3) = V(G_3) \cup V(H_3)$ and $E(G^*_3) = E(G_3) \cup E(H_3)$.
$|V(G^*_3)| = p_3 + \sum_{y=1}^{z} c_y + 2\sum_{y=1}^{z} d_y - 2z$ and $|E(G^*_3)| = q_3 + \sum_{y=1}^{z} c_y + 2\sum_{y=1}^{z} d_y$.
Define $h_3 : V(G^*_3) \cup E(G^*_3) \to \left\{1, 2, ..., p_3 + q_3 + 2\sum_{y=1}^{z} c_y + 4\sum_{y=1}^{z} d_y - 2z\right\}$ as follows:
$g_3(z_3) = h_3(z_3)$ for all $z_3 \in V(G_3)$ and $g_3(e_3) = h_3(e_3)$ for all $e_3 \in E(G_3)$.
$h_3(u''_0) = h_3(u_1) = 1$.
For each $1 \le r \le z$ and $1 \le s \le c_r, h_3(w''_{rs}) = p_3 + q_3 + \sum_{y=1}^{r-1} c_y + s, h_3(u''_0w''_{rs}) = p_3 + q_3 + \sum_{y=1}^{z} c_y + 2\sum_{y=1}^{z} d_y - 2z + \sum_{y=1}^{r-1} c_y + s$.
For each $1 \le r \le z, h_3(u''_0u''_{r1}) = p_3 + q_3 + 2\sum_{y=1}^{z}(c_y + d_y) + \sum_{y=1}^{r-1} d_y - 2z + 1, h_3(u''_0u''_{rd_r-1})$
$= p_3 + q_3 + 2\sum_{y=1}^{z}(c_y + d_y) + \sum_{y=1}^{r} d_y - 2z, h_3(u''_0v''_{r1}) = p_3 + q_3 + 2\sum_{y=1}^{z} c_y + 3\sum_{y=1}^{z} d_y + \sum_{y=1}^{r-1} d_y - 2z + 1, h_3(u''_0v''_{rd_r-1}) = p_3 + q_3 + 2\sum_{y=1}^{z} c_y + 3\sum_{y=1}^{z} d_y + \sum_{y=1}^{r} d_y - 2z$.
For each $1 \le r \le z$ and $1 \le s \le d_r - 2, h_3(u''_{rs}u''_{rs+1}) = p_3 + q_3 + 2\sum_{y=1}^{z}(c_y + d_y) + \sum_{y=1}^{r-1} d_y - 2z + 1 + s, h_3(v''_{rs}v''_{rs+1}) = p_3 + q_3 + 2\sum_{y=1}^{z} c_y + 3\sum_{y=1}^{z} d_y + \sum_{y=1}^{r-1} d_y - 2z + 1 + s$.
For each $1 \le r \le z$ and $1 \le s \le \left\lfloor \frac{d_r}{2} \right\rfloor, h_3(u''_{r2s-1}) = p_3 + q_3 + \sum_{y=1}^{z} c_y + \sum_{y=1}^{r-1} d_y + (2 - r) + s - 1, h_3(v''_{r2s-1}) = p_3 + q_3 + \sum_{y=1}^{z}(c_y + d_y) + \sum_{y=1}^{r-1} d_y - z + (2 - r) + s - 1$.
For each $1 \le r \le z$ and $1 \le s \le \left\lceil \frac{d_r}{2} \right\rceil - 1, h_3(u''_{r2s}) = p_3 + q_3 + \sum_{y=1}^{z} c_y + \sum_{y=1}^{r-1} d_y + \left\lfloor \frac{d_r}{2} \right\rfloor + (2 - r) + s - 1, h_3(v''_{r2s}) = p_3 + q_3 + \sum_{y=1}^{z}(c_y + d_y) + \sum_{y=1}^{r-1} d_y + \left\lfloor \frac{d_r}{2} \right\rfloor - z + (2 - r) + s - 1$.
For proving G^*_3 is vertex edge neighborhood prime graph. In earlier, G_3 is vertex edge neighborhood prime graph. Now we have to prove H_3 is vertex edge neighborhood prime graph. Let a_3 be any vertex of H_3.
For $a_3 = u''_0, u''_{rs}, v''_{rs}$ for $1 \le r \le z$ and $1 \le s \le d_r$ with $deg(a_3) \ge 2$. Here, $\gcd\{h_3(b_3) : b_3 \in N_V(a_3)\} = 1$ and $\gcd\{h_3(d_3) : d_3 \in N_E(a_3)\} = 1$.
Hence $G^*_3 = G_3 \odot H_3$ admits vertex edge neighborhood prime graph. □

Theorem 3.4. *If G_4 has vertex edge neighborhood prime graph, then there exists a graph from the class $G_4 \odot$ [one point union of different copies of shell graphs $S_{a_c} (1 \le c \le b)$] that admits vertex edge neighborhood prime.*

Proof. Let $G_4(p_4, q_4)$ be vertex edge neighborhood prime graph with labeling $g_4 : V(G_4) \cup E(G_4) \to \{1, 2, ..., |V(G_4) \cup E(G_4)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_4 be the one point union of different copies of shell graphs $S_{a_c} (1 \le c \le b)$ when $a_c \ge 5$ with
$V(H_4) = \{z_0\} \cup \{z_{cd} : 1 \le c \le b, 1 \le d \le a_c - 1\}$ and
$E(H_4) = \{z_0z_{c1}, z_0z_{ca_c-1} : 1 \le c \le b\} \cup \{z_{cd}z_{cd+1} : 1 \le c \le b, 1 \le d \le a_c - 2\} \cup \{z_0z_{cd+1} : 1 \le c \le b, 1 \le d \le a_c - 3\}$.
We superimposing one of the vertex say z_0 of H_4 on selected vertex of b_1 in G_4 with $g_4(b_1) = 1$.
Note that $G^*_4 = G_4 \odot H_4$ with $V(G^*_4) = V(G_4) \cup V(H_4)$ and $E(G^*_4) = E(G_4) \cup E(H_4)$.
$|V(G^*_4)| = p_4 + (a_1 + a_2 + ... + a_b) - b$ and $|E(G^*_4)| = q_4 + 2(a_1 + a_2 + ... + a_b) - 3b$.
Define $h_4 : V(G^*_4) \cup E(G^*_4) \to \{1, 2, ..., p_4 + q_4 + 3(a_1 + a_2 + ... + a_b) - 4b\}$ as follows:
$g_4(z_4) = h_4(z_4)$ for all $z_4 \in V(G_4)$ and $g_4(e_4) = h_4(e_4)$ for all $e_4 \in E(G_4)$.
$h4(z_0) = h_4(b_1) = 1$.
$h_4(z_{cd}) = p_4 + q_4 + \sum_{r=1}^{c-1} a_r + (2 - c) + d - 1$ for $1 \le c \le b$ and $1 \le d \le a_c - 1$.
For each $1 \le c \le b, h_4(z_0z_{c1}) = p_4 + q_4 + \sum_{r=1}^{b} a_r - b + \sum_{r=1}^{c-1} a_r + 1, h_4(z_0z_{ca_c-1}) = p_4 + q_4 + \sum_{r=1}^{b} a_r - b + \sum_{r=1}^{c} a_r$.
$h_4(z_{cd}z_{cd+1}) = p_4 + q_4 + \sum_{r=1}^{b} a_r - b + \sum_{r=1}^{c-1} a_r + 1 + d$ for $1 \le c \le b$ and $1 \le d \le a_c - 2$.
$h_4(z_0z_{cd+1}) = p_4 + q_4 + 2\sum_{r=1}^{b} a_r - b + \sum_{r=1}^{c-1} a_r - 3(c - 1) + d$ for $1 \le c \le b$ and
$1 \le d \le a_c - 3$.
Clearly, G_4 is vertex edge neighborhood prime graph. In order to show that H_4 is vertex edge neighborhood prime graph. Let a_4 be any vertex of H_4.
For $a_4 = z_0, z_{cd}$ for $1 \le c \le b$ and $1 \le d \le a_c - 1$ with $deg(a_4) \ge 2$. Here, $\gcd\{h_4(w_4) : w_4 \in N_V(a_4)\} = 1$ and $\gcd\{h_4(d_4) : d_4 \in N_E(a_4)\} = 1$.
Hence $G^*_4 = G_4 \odot H_4$ is vertex edge neighborhood prime graph. □

Theorem 3.5. *If G_5 has vertex edge neighborhood prime graph, then there exists a graph from the class $G_5 \odot$ [one point union of different copies of fan graphs $F_{n_i} (1 \le i \le k)$] that admits vertex edge neighborhood prime.*

Proof. Let $G_5(p_5, q_5)$ is vertex edge neighborhood prime graph with bijection $g_5 : V(G_5) \cup E(G_5) \to \{1, 2, ..., |V(G_5) \cup E(G_5)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_5 be the one point union of different copies of fan graphs $F_{n_i} (1 \le i \le k)$ with
$V(H_5) = \{s_0\} \cup \{s_{ij} : 1 \le i \le k, 1 \le j \le m_i\}$ and
$E(H_5) = \{s_0s_{ij} : 1 \le i \le k, 1 \le j \le m_i\} \cup$

$\{s_{ij}s_{ij+1} : 1 \leq i \leq k, 1 \leq j \leq m_i - 1\}$.
We superimposing one of the vertex say s_0 of H_5 on selected vertex of r_1 in G_5 with $g_5(r_1) = 1$.
Also, $G_5^* = G_5 \odot H_5$ with $V(G_5^*) = V(G_5) \cup V(H_5)$ and $E(G_5^*) = E(G_5) \cup E(H_5)$.
$|V(G_5^*)| = p_5 + (m_1 + m_2 + ... + m_k)$ and $|E(G_5^*)| = q_5 + 2(m_1 + m_2 + ... + m_k) - k$.
Define $h_5 : V(G_5^*) \cup E(G_5^*) \rightarrow$
$\{1, 2, ..., p_5 + q_5 + 3(m_1 + m_2 + ... + m_k) - k\}$ as follows:
$g_5(u_5) = h_5(u_5)$ for all $u_5 \in V(G_5)$ and $g_5(e_5) = h_5(e_5)$ for all $e_5 \in E(G_5)$.
$h_5(s_0) = h_5(r_1) = 1$.
$h_5(s_{ij}) = p_5 + q_5 + \sum_{c=1}^{i-1} m_c + j$ for $1 \leq i \leq k$ and $1 \leq j \leq m_i$.
$h_5(s_{ij}s_{ij+1}) = p_5 + q_5 + \sum_{c=1}^{k} m_c + 2\sum_{c=1}^{i-1} m_c + j + (3-i) - 1$ for $1 \leq i \leq k$ and $1 \leq j \leq m_i - 1$.
$h_5(s_0s_{ij}) = p_5 + q_5 + \sum_{c=1}^{k} m_c + 2\sum_{c=1}^{i-1} m_c + m_i + (3-i) + j - 2$ for $1 \leq i \leq k$ and
$2 \leq j \leq m_i - 1$.
For each $1 \leq i \leq k, h_5(s_0s_{i1}) = p_5 + q_5 + \sum_{c=1}^{k} m_c + 2\sum_{c=1}^{i-1} m_c + (3-i) - 1, h_5(s_0s_{im_i}) = p_5 + q_5 + \sum_{c=1}^{k} m_c + 2\sum_{c=1}^{i-1} m_c + m_i + (3-i) - 1$.
Clearly, G_5 is vertex edge neighborhood prime graph. We need to prove H_5 is vertex edge neighborhood prime graph.
Let u_5 be any vertex of H_5.
For $u_5 = s_0, s_{ij}$ for $1 \leq i \leq k$ and $1 \leq j \leq m_i$ with $deg(u_5) \geq 2$.
Here, $gcd\{h_5(w_5) : w_5 \in N_V(u_5)\} = 1$ and
$\gcd\{h_5(d_5) : d_5 \in N_E(u_5)\} = 1$.
Hence $G_5^* = G_5 \odot H_5$ admits vertex edge neighborhood prime graph. □

Theorem 3.6. *If G_6 has vertex edge neighborhood prime graph, then there exists a graph from the class $G_6 \odot$[one point union of different copies of octopus graphs $O_{s_y}(1 \leq y \leq v)$] that admits vertex edge neighborhood prime.*

Proof. Let $G_6(p_6, q_6)$ be vertex edge neighborhood prime graph with bijection $g_6 : V(G_6) \cup E(G_6) \rightarrow$
$\{1, 2, ..., |V(G_6) \cup E(G_6)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_6 be the one point union of different copies of octopus graphs $O_{s_y}(1 \leq y \leq v)$ with
$V(H_6) = \{u'_0\} \cup \{u'_{yz}, v'_{yz} : 1 \leq y \leq v, 1 \leq z \leq s_y\}$ and
$E(H_6) = \{u'_0u'_{yz}, u'_0v'_{yz} : 1 \leq y \leq v, 1 \leq z \leq s_y\} \cup$
$\{u'_{yz}u'_{yz+1} : 1 \leq y \leq v, 1 \leq z \leq s_y - 1\}$.
We superimposing one of the vertex say u'_0 of H_6 on selected vertex of z_1 in G_6 with $g_6(z_1) = 1$.
Note that $G_6^* = G_6 \odot H_6$ with $V(G_6^*) = V(G_6) \cup V(H_6)$ and $E(G_6^*) = E(G_6) \cup E(H_6)$
$|V(G_6^*)| = p_6 + 2(s_1 + s_2 + ... + s_v)$ and $|E(G_6^*)| = q_6 + 3(s_1 + s_2 + ... + s_v) - v$.
Defrine $h_6 : V(G_6^*) \cup E(G_6^*) \rightarrow$
$\{1, 2, ..., p_6 + q_6 + 5(s_1 + s_2 + ... + s_v) - v\}$ as follows:
$g_6(z_6) = h_6(z_6)$ for all $z_6 \in V(G_6)$ and $g_6(d_6) = h_6(d_6)$ for all $d_6 \in E(G_6)$.
$h_6(u'_0) = h_6(z_1) = 1$.
For each $1 \leq y \leq v$ and $1 \leq z \leq s_y, h_6(u'_{yz}) = p_6 + q_6 + \sum_{c=1}^{y-1} s_c + z, h_6(v'_{yz}) = p_6 + q_6 + \sum_{c=1}^{v} s_c + \sum_{c=1}^{y-1} s_c + z, h_6(u'_0v'_{yz}) = p_6 + q_6 + 2\sum_{c=1}^{v} s_c + \sum_{c=1}^{y-1} s_c + z, h_6(u'_0u'_{yz}) = p_6 + q_6 + 3\sum_{c=1}^{v} s_c + 2\sum_{c=1}^{y-1} s_c + 2z + (1-y) - 1$.
$h_6(u'_{yz}u'_{yz+1}) = p_6 + q_6 + 3\sum_{c=1}^{v} s_c + 2\sum_{c=1}^{y-1} s_c + 2z + (2-y) - 1$ for $1 \leq y \leq v$ and $1 \leq z \leq s_y - 1$.
Already, G_6 is vertex edge neighborhood prime graph. It's enough to prove H_6 is vertex edge neighborhood prime graph.
Let a_6 be any vertex of H_6.
For $a_6 = u'_0, u'_{yz}$ for $1 \leq y \leq v$ and $1 \leq z \leq s_y$ with $deg(a_6) \geq 2$.
Here, $\gcd\{h_6(w_6) : w_6 \in N_V(a_6)\} = 1$ and
$\gcd\{h_6(d_6) : d_6 \in N_E(a_6)\} = 1$.
Hence $G_6 \odot H_6$ admits vertex edge neighborhood prime graph. □

Theorem 3.7. *If $G_7(p_7, q_7)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_7 \odot$[one point union of different copies of planter graphs $R_{z_a}(1 \leq a \leq y)$] that admits vertex edge neighborhood prime for all t.*

Proof. Let $G_7(p_7, q_7)$ be vertex edge neighborhood prime graph with bijection $g_7 : V(G_7) \cup E(G_7) \rightarrow$
$\{1, 2, ..., |V(G_7) \cup E(G_7)|\}$ satisfying the property of vertex edge neighborhood prime graph.
Consider H_7 be the one point union of different copies of planter graphs $R_{z_a}(1 \leq a \leq y)$ with
$V(H_7) = \{u''_0\} \cup \{u''_{ab} : 1 \leq a \leq y, 1 \leq b \leq z_a - 1\} \cup$
$\{v''_{ab} : 1 \leq a \leq y, 1 \leq b \leq z_a\}$ and
$E(H_7) = \{u''_0v''_{ab} : 1 \leq a \leq y, 1 \leq b \leq z_a\} \cup$
$\{v''_{ab}v''_{ab+1} : 1 \leq a \leq y, 1 \leq b \leq z_a - 1\} \cup$
$\{u''_{ab}u''_{ab+1} : 1 \leq a \leq y, 1 \leq b \leq z_a - 2\} \cup$
$\{u''_0u''_{a1}, u''_0u''_{az_a-1} : 1 \leq a \leq y\}$.
We superimposing one of the vertex say u''_0 of H_7 on selected vertex of a_1 in G_7 with $g_7(a_1) = 1$.
Also, $G_7^* = G_7 \odot H_7$ with $V(G_7^*) = V(G_7) \cup V(H_7)$ and $E(G_7^*) = E(G_7) \cup E(H_7)$.
Here, $|V(G_7^*)| = p_7 + (z_1 + z_2 + ... + z_y) - y$ and $|E(G_7^*)| = q_7 + 3(z_1 + z_2 + ... + z_y) - y$.
Define $h_7 : V(G_7^*) \cup E(G_7^*) \rightarrow$
$\{1, 2, ..., p_7 + q_7 + 5(z_1 + z_2 + ... + z_y) - 2y\}$ as follows:
$g_7(z_7) = h_7(z_7)$ for all $z_7 \in V(G_7)$ and $g_7(d_7) = h_7(d_7)$ for all $d_7 \in E(G_7)$.
$h_7(u''_0) = h_7(a_1) = 1$.
For each $1 \leq a \leq y$ and $1 \leq b \leq z_a, h_7(v''_{ab}) = p_7 + q_7 + \sum_{c=1}^{a-1} z_c + b, h_7(u''_0v''_{ab}) = p_7 + q_7 + 2\sum_{c=1}^{y} z_c - y + 3\sum_{c=1}^{a-1} z_c + (1-a) + 2b - 1$.
For each $1 \leq a \leq y, h_7(u''_0u''_{a1}) = p_7 + q_7 + 2\sum_{c=1}^{y} z_c - y + 3\sum_{c=1}^{a-1} z_c + 2z_a + (2-a) - 1, h_7(u''_0u''_{az_a-1}) = p_7 + q_7 + 2\sum_{c=1}^{y} z_c - y + 3\sum_{c=1}^{a} z_c + (1-a) - 1$.
$h_7(u''_{a2b-1}) = p_7 + q_7 + \sum_{c=1}^{y} z_c + \sum_{c=1}^{a-1} z_c + (2-a) + b - 1$ for $1 \leq a \leq y$ and $1 \leq b \leq \lfloor \frac{z_a}{2} \rfloor$.
$h_7(u''_{a2b}) = p_7 + q_7 + \sum_{c=1}^{y} z_c + \sum_{c=1}^{a-1} z_c + \lfloor \frac{z_a}{2} \rfloor + (2-a) + b -$

1 for $1 \leq a \leq y$ and $1 \leq b \leq \lceil \frac{z_a}{2} \rceil - 1$.
$h_7(v''_{ab}v''_{ab+1}) = p_7 + q_7 + p_7 + q_7 + 2\sum_{c=1}^{y} z_c - y + 3\sum_{c=1}^{a-1} z_c + (2-a) + 2b - 1$ for $1 \leq a \leq y$ and $1 \leq b \leq z_a - 1$.
$h_7(u''_{ab}u''_{ab+1}) = p_7 + q_7 + 2\sum_{c=1}^{y} z_c - y + 3\sum_{c=1}^{a-1} z_c + 2z_a + (2-a) + b - 1$ for $1 \leq a \leq y$ and $1 \leq b \leq z_a - 2$.
We claim that G_7^* is vertex edge neighborhood prime graph. Clearly, G_7 is vertex edge neighborhood prime graph. We have to prove H_7 is vertex edge neighborhood prime graph. Let a_7 be any vertex of H_7.
For $a_7 = u''_0, v''_{ab}$ for $1 \leq a \leq y$ and $1 \leq b \leq z_a$ and u''_{ab} for $1 \leq a \leq y$ and $1 \leq b \leq z_a - 1$ with $deg(a_7) \geq 2$. Here,
$\gcd\{h_7(y_7) : y_7 \in N_V(a_7)\} = 1$ and
$\gcd\{h_7(d_7) : d_7 \in N_E(a_7)\} = 1$.
Hence $G_7 \odot H_7$ is vertex edge neighborhood prime graph. □

4. Graph identification of m fold types of graphs

In this section, we deal with m fold types of graphs.

Theorem 4.1. *If $G_1(p_1, q_1)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_1 \odot$ m fold Petersen graph $P(n,2)$ that admits vertex edge neighborhood prime for all $n \geq 5$.*

Proof. Let $G_1(p_1, q_1)$ be vertex edge neighborhood prime graph with bijection $g_1 : V(G_1) \cup E(G_1) \rightarrow \{1, 2, ..., |V(G_1) \cup E(G_1)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_1 be m fold Petersen graph $P(n,2)$, where $n \geq 5$ with
$V(H_1) = \{u_j, v_j : 1 \leq j \leq n\} \cup \{w_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and
$E(H_1) = \{u_j u_{j+2} : 1 \leq j \leq n-2\} \cup \{u_j v_j : 1 \leq j \leq n\} \cup \{v_j v_{j+1} : 1 \leq j \leq n-1\} \cup \{v_j w_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{v_{j+1} w_{ij} : 1 \leq i \leq m, 1 \leq j \leq n-1\} \cup \{v_1 w_{in} : 1 \leq i \leq m\} \cup \{u_1 u_{n-1}\} \cup \{u_2 u_n\} \cup \{v_1 v_n\}$.
We overlay one of the vertex say v_1 of H_1 on selected vertex of s_1 in G_1 with $g_1(s_1) = 1$.
Note that $G_1^* = G_1 \odot H_1$ with $V(G_1^*) = V(G_1) \cup V(H_1)$ and $E(G_1^*) = E(G_1) \cup E(H_1)$.
$|V(G_1^*)| = p_1 + n(m+2) - 1$ and $|E(G_1^*)| = q_1 + n(2m+3)$.
Define $h_1 : V(G_1^*) \cup E(G_1^*) \rightarrow \{1, 2, ..., p_1 + q_1 + n(3m+5) - 1\}$ as follows:
$g_1(z_1) = h_1(z_1)$ for all $z_1 \in V(G_1)$ and $g_1(e_1) = h_1(e_1)$ for all $e_1 \in E(G_1)$.
$h_1(v_1) = h_1(s_1) = 1, h_1(v_1 v_n) = p_1 + q_1 + n(m+2) + 3n - 1$.
$h_1(v_1 w_{in}) = p_1 + q_1 + n(m+2) + n(2i+3) - 1$ for $1 \leq i \leq m$.
$h_1(u_j v_j) = p_1 + q_1 + n(m+2) + n + j - 1$ for $1 \leq j \leq n$.
For each $1 \leq i \leq m$ and $1 \leq j \leq n$, $h_1(w_{ij}) = p_1 + q_1 + (i+1)n + j - 1, h_1(v_j w_{ij}) = p_1 + q_1 + n(m+2) + (2i+1)n + 2j - 2, h_1(v_{j+1} w_{ij}) = p_1 + q_1 + n(m+2) + (2i+1)n + 2j - 1$.
$h_1(v_j v_{j+1}) = p_1 + q_1 + n(m+2) + 2n + j - 1$ for $1 \leq j \leq n-1$.
Consider the following four cases.
Case 1. $p_1 + q_1$ is odd
$h_1(u_j) = p_1 + q_1 + 2j - 1$ for $1 \leq j \leq n$.
$h_1(v_{j+1}) = p_1 + q_1 + 2j$ for $1 \leq j \leq n-1$.
Case 2. $p_1 + q_1$ is even
$h_1(u_n) = p_1 + q_1 + 2n - 1$.
For each $1 \leq j \leq n-1, h_1(u_j) = p_1 + q_1 + 2j, h_1(v_{j+1}) = p_1 + q_1 + 2j - 1$.
Case 3. n is odd,
$h_1(u_j u_{j+2}) = p_1 + q_1 + n(m+2) + j - 1$ for $1 \leq j \leq n-2$.
$h_1(u_1 u_{n-1}) = p_1 + q_1 + n(m+2) + n - 2, h_1(u_2 u_n) = p_1 + q_1 + n(m+2) + n - 1$.
Case 4. n is even,
$h_1(u_j u_{j+2}) = p_1 + q_1 + n(m+2) + j - 2$ for $2 \leq j \leq n-2$.
$h_1(u_1 u_{n-1}) = p_1 + q_1 + n(m+2) + n - 3, h_1(u_2 u_n) = p_1 + q_1 + n(m+2) + n - 2, h_1(u_1 u_3) = p_1 + q_1 + n(m+2) + n - 1$.
Clearly, G_1 is vertex edge neighborhood prime graph. We claim that H_1 is vertex edge neighborhood prime graph. Let x_1 be any vertex of H_1.
For $x_1 = u_j, v_j, w_{ij}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$ with $deg(x_1) \geq 2$. Here, $\gcd\{h_1(w_1) : w_1 \in N_V(x_1)\} = 1$ and
$\gcd\{h_1(e_1) : e_1 \in N_E(x_1)\} = 1$.
Hence $G_1^* = G_1 \odot H_1$ is vertex edge neighborhood prime graph. □

Theorem 4.2. *If $G_2(p_2, q_2)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_2 \odot$ m fold prism $C_n \times K_2$ that admits vertex edge neighborhood prime for all n.*

Proof. Let $G_2(p_2, q_2)$ be vertex edge neighborhood prime graph with bijection $g_2 : V(G_2) \cup E(G_2) \rightarrow \{1, 2, ..., |V(G_2) \cup E(G_2)|\}$ satisfying the property of vertex edge neighborhood prime graph.
Consider H_2 be m fold prism graph $C_n \times K_2$ with
$V(H_2) = \{u_j, v_j : 1 \leq j \leq n\} \cup \{w_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and
$E(H_2) = \{u_j u_{j+1}, v_j v_{j+1} : 1 \leq j \leq n-1\} \cup \{v_1 v_n\} \cup \{u_{j+1} w_{ij} : 1 \leq i \leq m, 1 \leq j \leq n-1\} \cup \{u_1 u_n\} \cup \{u_j v_j : 1 \leq j \leq n\} \cup \{u_j w_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{v_1 w_{in} : 1 \leq i \leq m\}$.
We overlay one of the vertex say v_1 of H_2 on selected vertex of t_1 in G_2 with $g_2(t_1) = 1$.
Also, $G_2^* = G_2 \odot H_2$ with $V(G_2^*) = V(G_2) \cup V(H_2)$ and $E(G_2^*) = E(G_2) \cup E(H_2)$.
$|V(G_2^*)| = p_2 + n(m+2) - 1$ and $|E(G_2^*)| = q_2 + (2m+3)n$.
Define $h_2 : V(G_2^*) \cup E(G_2^*) \rightarrow \{1, 2, ..., p_2 + q_2 + (3m+5)n - 1\}$ as follows:
$g_2(z_2) = h_2(z_2)$ for all $z_2 \in V(G_2)$ and $g_2(e_2) = h_2(e_2)$ for all $e_2 \in E(G_2)$.
$h_2(v_1) = h_2(t_1) = 1, h_2(u_1 u_n) = p_2 + q_2 + n(m+2) + n - 1, h_2(v_1 v_n) = p_2 + q_2 + n(m+2) + 3n - 1$.
For each $1 \leq j \leq n, h_2(u_j v_j) = p_2 + q_2 + n(m+2) + n + j - 1, h_2(v_j v_{j+1}) = p_2 + q_2 + n(m+2) + 2n + j - 1$.
For each $1 \leq i \leq m$ and $1 \leq j \leq n, h_2(w_{ij}) = p_2 + q_2 + (i+$

$1)n+j-1, h_2(v_jw_{ij}) = p_2+q_2+n(m+2)+(2i+1)n+2j-2$.
$h_2(u_ju_{j+1}) = p_2+q_2+n(m+2)+j-1$ for $1 \leq j \leq n-1$.
$h_2(u_{j+1}w_{ij}) = p_2+q_2+n(m+2)+(2i+1)n+2j-1$ for $1 \leq i \leq m$ and $1 \leq j \leq n-1$.
$h_2(u_1w_{in}) = p_2+q_2+n(m+2)+(2i+3)n-1$ for $1 \leq i \leq m$.
We consider the following cases.
Case 1. p_2+q_2 is odd
$h_2(u_j) = p_2+q_2+2j-1$ for $1 \leq j \leq n$.
$h_2(v_{j+1}) = p_2+q_2+2j$ for $1 \leq j \leq n-1$.
Case 2. p_2+q_2 is even
$h_2(u_n) = p_2+q_2+2n-1$.
For each $1 \leq j \leq n-1, h_2(u_j) = p_2+q_2+2j, h_2(v_{j+1}) = p_2+q_2+2j-1$.
Already, G_2 is vertex edge neighborhood prime graph. Now we have to prove H_2 is vertex edge neighborhood prime graph.
Let a_2 be any vertex of H_2.
For $a_2 = u_j, v_j, w_{ij}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$ with $deg(a_2) \geq 2$. Here, $\gcd\{h_2(b_2) : b_2 \in N_V(a_2)\} = 1$ and
$\gcd\{h_2(d_2) : d_2 \in N_E(a_2)\} = 1$.
Hence $G_2^* = G_2 \odot H_2$ admits vertex edge neighborhood prime graph. □

Theorem 4.3. *If $G_3(p_3,q_3)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_3 \odot m$ fold triangular snake T_n that admits vertex edge neighborhood prime for all n.*

Proof. Let $G_3(p_3,q_3)$ be vertex edge neighborhood prime graph with bijection $g_3 : V(G_3) \cup E(G_3) \to \{1,2,...,|V(G_3) \cup E(G_3)|\}$ satisfying the property of vertex edge neighborhood prime graph.
Consider H_3 be the m fold triangular snake graph T_n with
$V(H_3) = \{u_j : 1 \leq j \leq n\} \cup \{v_{ij} : 1 \leq i \leq m, 1 \leq j \leq n-1\}$
and
$E(H_3) = \{u_ju_{j+1} : 1 \leq j \leq n-1\} \cup$
$\{u_jv_{ij}, u_{j+1}v_{ij} : 1 \leq i \leq m, 1 \leq j \leq n-1\}$
We overlay one of the vertex say u_1 of H_3 on selected vertex of c_1 in G_3 with $g_3(c_1) = 1$.
Note that $G_3^* = G_3 \odot H_3$ with $V(G_3^*) = V(G_3) \cup V(H_3)$ and $E(G_3^*) = E(G_3) \cup E(H_3)$.
$|V(G_3^*)| = p_3+n+m(n-1)-1$ and $|E(G_3^*)| = q_3+(n-1)(2m+1)$.
Define $h_3 : V(G_3^*) \cup E(G_3^*) \to \{1,2,...,p_3+q_3+n(3m+2)-(3m+1)-1\}$ as follows:
$g_3(z_3) = h_3(z_3)$ for all $z_3 \in V(G_3)$ and $g_3(e_3) = h_3(e_3)$ for all $e_3 \in E(G_3)$.
$h_3(u_1) = h_3(c_1) = 1, h_3(u_{n-1}v_{1n-1}) = p_3+q_3+4n+m(n-1)-4, h_3(u_nv_{1n-1}) = p_3+q_3+4n+m(n-1)-5$.
$h_3(u_ju_{j+1}) = p_3+q_3+n+m(n-1)+3j-3$ for $1 \leq j \leq n-1$.
For each $1 \leq j \leq n-2, h_3(u_jv_{1j}) = p_3+q_3+n+m(n-1)+3j-2, h_3(u_{j+1}v_{1j}) = p_3+q_3+n+m(n-1)+3j-1$.
For each $2 \leq i \leq m$ and $1 \leq j \leq n-1, h_3(v_{ij}) = p_3+q_3+i(n-1)+j, h_3(u_jv_{ij}) = p_3+q_3+n+m(n-1)+(2i-1)(n-2)+2(i-1)+2j-1, h_3(u_{j+1}v_{ij}) = p_3+q_3+n+m(n-1)+(2i-1)(n-2)+(2i-1)+2j-1$.
We consider the following two cases.
Case 1. p_3+q_3 is odd
For each $1 \leq j \leq n-1, h_3(v_{1j}) = p_3+q_3+2j-1, h_3(u_{j+1}) = p_3+q_3+2j$.
Case 1. p_3+q_3 is even
For each $1 \leq j \leq n-1, h_3(v_{1j}) = p_3+q_3+2(n-j), h_3(u_{j+1}) = p_3+q_3+2(n-j)-1$.
For proving G_3^* is vertex edge neighborhood prime graph. In earlier, G_3 is vertex edge neighborhood prime graph. Now we have to prove H_3 is vertex edge neighborhood prime graph.
Let a_3 be any vertex of H_3.
For $a_3 = u_j$ for $1 \leq j \leq n$ and v_{ij} for $1 \leq i \leq m$ and $1 \leq j \leq n-1$ with $deg(a_3) \geq 2$. Here, $\gcd\{h_3(b_3) : b_3 \in N_V(a_3)\} = 1$ and $\gcd\{h_3(d_3) : d_3 \in N_E(a_3)\} = 1$.
Hence $G_3^* = G_3 \odot H_3$ admits vertex edge neighborhood prime graph. □

Theorem 4.4. *If G_4 has vertex edge neighborhood prime graph, then there exists a graph from the class $G_4 \odot m$ fold alternate triangular snake $A(T_n)$ that admits vertex edge neighborhood prime for all $n = 4,6,8,10,...$*

Proof. Let $G_4(p_4,q_4)$ be vertex edge neighborhood prime graph with labeling $g_4 : V(G_4) \cup E(G_4) \to \{1,2,...,|V(G_4) \cup E(G_4)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_4 be m fold alternate triangular snake $A(T_n)$, where $n = 4,6,8,10,...$ with
$V(H_4) = \{u_j : 1 \leq j \leq n\} \cup \{v_{ij} : 1 \leq i \leq m, 1 \leq j \leq (\frac{n}{2})-1\}$
and
$E(H_4) = \{u_ju_{j+1} : 1 \leq j \leq n-1\} \cup$
$\{u_{2j}v_{ij}, u_{2j+1}v_{ij} : 1 \leq i \leq m, 1 \leq j \leq \frac{n}{2}-1\}$.
We overlay one of the vertex say u_2 of H_4 on selected vertex of b_1 in G_4 with $g_4(b_1) = 1$.
Note that $G_4^* = G_4 \odot H_4$ with $V(G_4^*) = V(G_4) \cup V(H_4)$ and $E(G_4^*) = E(G_4) \cup E(H_4)$.
$|V(G_4^*)| = p_4+n+m(\frac{n}{2}-1)-1$ and $|E(G_4^*)| = q_4+(n-1)+2m(\frac{n}{2}-1)$.
Define $h_4 : V(G_4^*) \cup E(G_4^*) \to \{1,2,...,p_4+q_4+2n-2+3n(\frac{n}{2}-1)\}$ as follows:
$g_4(z_4) = h_4(z_4)$ for all $z_4 \in V(G_4)$ and $g_4(e_4) = h_4(e_4)$ for all $e_4 \in E(G_4)$.
$h_4(u_2) = h_4(b_1) = 1, h_4(u_1) = p_4+q_4+\frac{3n}{2}-1, h_4(v_{11}) = p_4+q_4+1, h_4(u_{n-1}u_n) = p_4+q_4+\frac{3n}{2}-3$.
For each $1 \leq j \leq \frac{n}{2}-1, h_4(u_{2j+2}) = p_4+q_4+3j+1, h_4(u_{2j+1}) = p_4+q_4+3j-1, h_4(u_{2j-1}u_{2j}) = p_4+q_4+n+m(\frac{n}{2}-1)+4j-3, h_4(u_{2j}u_{2j+1}) = p_4+q_4+n+m(\frac{n}{2}-1)+4j-2$,
$h_4(u_{2j}v_{1j}) = p_4+q_4+n+m(\frac{n}{2}-1)+4j, h_4(u_{2j+1}v_{1j}) = p_4+q_4+n+m(\frac{n}{2}-1)+4j-1$.
$h_4(v_{1j+1}) = p_4+q_4+3j$ for $1 \leq j \leq \frac{n}{2}-2$.
For each $2 \leq i \leq m$ and $1 \leq j \leq \frac{n}{2}-1, h_4(v_{ij}) = p_4+q_4+\frac{(i+1)n}{2}+(i-2)+j-1, h_4(u_{2j}v_{ij}) = p_4+q_4+n+m(\frac{n}{2}-1)+i(n-2)+2j-1, h_4(u_{2j+1}v_{ij}) = p_4+q_4+n+m(\frac{n}{2}-1)+i(n-2)+2j$.

Clearly, G_4 is vertex edge neighborhood prime graph. In order to show that H_4 is vertex edge neighborhood prime graph. Let a_4 be any vertex of H_4.
If $a_4 = u_1, u_n$ with $deg(a_4) = 1$, then
$\gcd\{h_4(w_4), h_4(a_4w_4) : w_4 \in N_V(a_4)\} = 1$.
For $a_4 = v_{ij}, u_k$, for $1 \leq i \leq m, 1 \leq j \leq \frac{n}{2} - 1$ and $2 \leq k \leq n-1$ with $deg(a_4) \geq 2$. Here, $\gcd\{h_4(w_4) : w_4 \in N_V(a_4)\} = 1$ and $\gcd\{h_4(d_4) : d_4 \in N_E(a_4)\} = 1$.
Hence $G_4^* = G_4 \odot H_4$ is vertex edge neighborhood prime graph. □

Theorem 4.5. *If G_5 has vertex edge neighborhood prime graph, then there exists a graph from the class $G_5 \odot m$ fold antiprism graph A_b that admits vertex edge neighborhood prime.*

Proof. Let $G_5(p_5, q_5)$ is vertex edge neighborhood prime graph with bijection $g_5 : V(G_5) \cup E(G_5) \rightarrow \{1, 2, ..., |V(G_5) \cup E(G_5)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_5 be m fold antiprism graph A_b with
$V(H_5) = \{x'_s, y'_s : 1 \leq s \leq b\} \cup \{z'_{rs} : 1 \leq r \leq m, 1 \leq s \leq b\}$ and
$E(H_5) = \left\{x'_s x'_{s+1}, x'_s y'_{s+1}, y'_s y'_{s+1} : 1 \leq s \leq b-1\right\} \cup \left\{x'_1 x'_b\right\} \cup \left\{y'_1 y'_b\right\} \cup \{y'_s z'_{rs} : 1 \leq r \leq m, 1 \leq s \leq b\} \cup \left\{y'_{s+1} z'_{rs} : 1 \leq r \leq m, 1 \leq s \leq b-1\right\} \cup \left\{y'_1 z'_{rb} : 1 \leq r \leq m\right\} \cup \left\{x'_b y'_1\right\} \cup \{x'_s y'_s : 1 \leq s \leq b\}$.
We overlay one of the vertex say y_1 of H_5 on selected vertex of r_1 in G_5 with $g_5(r_1) = 1$.
Also, $G_5^* = G_5 \odot H_5$ with $V(G_5^*) = V(G_5) \cup V(H_5)$ and $E(G_5^*) = E(G_5) \cup E(H_5)$.
$\left|V(G_5^*)\right| = p_5 + 2b + mb - 1$ and $\left|E(G_5^*)\right| = q_5 + 4b + 2mb$.
Define $h_5 : V(G_5^*) \cup E(G_5^*) \rightarrow \{1, 2, ..., p_5 + q_5 + 6b + 3mb - 1\}$ as follows:
$g_5(u_5) = h_5(u_5)$ for all $u_5 \in V(G_5)$ and $g_5(e_5) = h_5(e_5)$ for all $e_5 \in E(G_5)$.
$h_5(y_1) = h_5(r_1) = 1, h_5(x'_1 x'_b) = p_5 + q_5 + 3b - 1, h_5(y'_1 y'_b) = p_5 + q_5 + 6b - 1, h_5(x'_b y'_1) = p_5 + q_5 + 3b$.
$h_5(x'_s y'_s) = p_5 + q_5 + 3b + 2s - 1$ for $1 \leq s \leq b$.
For each $1 \leq s \leq b-1, h_5(x'_s y'_{s+1}) = p_5 + q_5 + 3b + 2s$,
$h_5(x'_s x'_{s+1}) = p_5 + q_5 + 2b + s - 1, h_5(y'_s y'_{s+1}) = p_5 + q_5 + 5b + s - 1$.
For each $1 \leq r \leq m$ and $1 \leq s \leq b, h_5(z'_{rs}) = p_5 + q_5 + (r+5)b + s - 1, h_5(y'_s z'_{rs}) = p_5 + q_5 + (2r+4)b + mb + 2s - 2$.
$h_5(y'_{s+1} z'_{rs}) = p_5 + q_5 + (2r+4)b + mb + 2s - 1$ for $1 \leq r \leq m$ and $1 \leq s \leq b-1$.
$h_5(y'_1 z'_{rb}) = p_5 + q_5 + (2r+6)b + mb - 1$ for $1 \leq r \leq m$.
Consider the following cases.
Case 1. $p_3 + q_3$ is odd
$h_5(x'_s) = p_5 + q_5 + 2s - 1$ for $1 \leq s \leq b$.
$h_5(y'_{s+1}) = p_5 + q_5 + 2s$ for $1 \leq s \leq b-1$.
Case 1. $p_3 + q_3$ is even
$h_5(x'_s) = p_5 + q_5 + 2s - 1$.
For each $1 \leq s \leq b-1, h_5(x'_s) = p_5 + q_5 + 2s, h_5(y'_{s+1}) = p_5 + q_5 + 2s - 1$.
Clearly, G_5 is vertex edge neighborhood prime graph. We need to prove H_5 is vertex edge neighborhood prime graph. Let u_5 be any vertex of H_5.
For $u_5 = x'_s, y'_s, z'_{rs}$ for $1 \leq r \leq m$ and $1 \leq s \leq b$ with $deg(u_5) \geq 2$. Here, $gcd\{h_5(w_5) : w_5 \in N_V(u_5)\} = 1$ and $\gcd\{h_5(d_5) : d_5 \in N_E(u_5)\} = 1$.
Hence $G_5^* = G_5 \odot H_5$ admits vertex edge neighborhood prime graph. □

Theorem 4.6. *If G_6 has vertex edge neighborhood prime graph, then there exists a graph from the class $G_6 \odot m$ fold cycle graph C_y that admits vertex edge neighborhood prime.*

Proof. Let $G_6(p_6, q_6)$ be vertex edge neighborhood prime graph with bijection $g_6 : V(G_6) \cup E(G_6) \rightarrow \{1, 2, ..., |V(G_6) \cup E(G_6)|\}$ satisfying the condition of vertex edge neighborhood prime graph.
Consider H_6 be m fold cycle graph C_y with
$V(H_6) = \{u''_t : 1 \leq t \leq y\} \cup \{v''_{st} : 1 \leq s \leq m, 1 \leq t \leq y\}$ and
$E(H_6) = \left\{u''_{t+1} v''_{st} : 1 \leq s \leq m, 1 \leq t \leq y-1\right\} \cup \left\{u''_1 u''_y\right\} \cup \left\{u''_1 v''_{sy}\right\} \cup \left\{u''_t v''_{st} : 1 \leq s \leq m, 1 \leq t \leq y\right\} \cup \left\{u''_t u''_{t+1} : 1 \leq t \leq y-1\right\}$.
We overlay one of the vertex say u''_1 of H_6 on selected vertex of z_1 in G_6 with $g_6(z_1) = 1$.
Note that $G_6^* = G_6 \odot H_6$ with $V(G_6^*) = V(G_6) \cup V(H_6)$ and $E(G_6^*) = E(G_6) \cup E(H_6)$
$\left|V(G_6^*)\right| = p_6 + y + my - 1$ and $\left|E(G_6^*)\right| = q_6 + y + 2my$.
Define $h_6 : V(G_6^*) \cup E(G_6^*) \rightarrow \{1, 2, ..., p_6 + q_6 + 2y + 3my - 1\}$ as follows:
$g_6(z_6) = h_6(z_6)$ for all $z_6 \in V(G_6)$ and $g_6(d_6) = h_6(d_6)$ for all $d_6 \in E(G_6)$.
$h_6(u''_1) = h_6(z_1) = 1, h_6(u''_1 u''_y) = p_6 + q_6 + 2y + my - 1$.
$h_6(u''_t) = p_6 + q_6 + t - 1$ for $2 \leq t \leq y$.
$h_6(u''_t u''_{t+1}) = p_6 + q_6 + y + my + t - 1$ for $1 \leq t \leq y-1$.
For each $1 \leq s \leq m$ and $1 \leq t \leq y, h_6(v''_{st}) = p_6 + q_6 + sy + t - 1, h_6(u''_t v''_{st}) = p_6 + q_6 + 2sy + my + 2t - 2$.
$h_6(u''_{t+1} v''_{st}) = p_6 + q_6 + 2sy + my + 2t - 1$ for $1 \leq s \leq m$ and $1 \leq t \leq y-1$.
$h_6(u''_1 v''_{sy}) = p_6 + q_6 + (2s+2)y + my - 1$ for $1 \leq s \leq m$.
Already, G_6 is vertex edge neighborhood prime graph. It's enough to prove H_6 is vertex edge neighborhood prime graph. Let a_6 be any vertex of H_6.
For $a_6 = u''_t, v''_{st}$ for $1 \leq s \leq m$ and $1 \leq t \leq y$ for $1 \leq i \leq n$ with $deg(a_6) \geq 2$. Here, $\gcd\{h_6(w_6) : w_6 \in N_V(a_6)\} = 1$ and $\gcd\{h_6(d_6) : d_6 \in N_E(a_6)\} = 1$.
Hence $G_6 \odot H_6$ admits vertex edge neighborhood prime graph. □

Theorem 4.7. *If $G_7(p_7, q_7)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_7 \odot m$ fold double triangular snake graph $D(T_y)$ that admits vertex edge neighborhood prime.*

Proof. Let $G_7(p_7, q_7)$ be vertex edge neighborhood prime graph with bijection $g_7 : V(G_7) \cup E(G_7) \rightarrow \{1, 2, ..., |V(G_7) \cup E(G_7)|\}$ satisfying the property of vertex

edge neighborhood prime graph.
Consider H_7 be the m fold double triangular snake graph $D(T_y)$ with
$V(H_7) = \{c_b : 1 \leq b \leq y\} \cup$
$\{d_{ab}, e_{ab} : 1 \leq a \leq m, 1 \leq b \leq y-1\}$ and
$E(H_7) = \{c_b c_{b+1} : 1 \leq b \leq y-1\} \cup$
$\{c_b d_{ab}, c_b e_{ab}, c_{b+1} d_{ab}, c_{b+1} e_{ab} : 1 \leq a \leq m, 1 \leq b \leq y-1\}$.
We overlay one of the vertex say c_1 of H_7 on selected vertex of a_1 in G_7 with $g_7(a_1) = 1$.
Also, $G_7^* = G_7 \odot H_7$ with $V(G_7^*) = V(G_7) \cup V(H_7)$ and $E(G_7^*) = E(G_7) \cup E(H_7)$.
Here, $|V(G_7^*)| = p_7 + y + 2m(y-1) - 1$ and $|E(G_7^*)| = q_7 + (4m+1)(y-1)$.
Define $h_7 : V(G_7^*) \cup E(G_7^*) \to$
$\{1, 2, ..., p_7 + q_7 + 6m(y-1) + 2y - 2\}$ as follows:
$g_7(z_7) = h_7(z_7)$ for all $z_7 \in V(G_7)$ and $g_7(d_7) = h_7(d_7)$ for all $d_7 \in E(G_7)$.
$h_7(c_1) = h_7(a_1) = 1$.
For each $1 \leq b \leq y-1, h_7(c_b c_{b+1}) = p_7 + q_7 + 2m(y-1) + y + 5b - 1, h_7(c_b d_{1b}) = p_7 + q_7 + 2m(y-1) + y + 5b - 5$,
$h_7(c_b e_{1b}) = p_7 + q_7 + 2m(y-1) + y + 5b - 2$,
$h_7(c_{b+1} d_{1b}) = p_7 + q_7 + 2m(y-1) + y + 5b - 4, h_7(c_{b+1} e_{1b}) = p_7 + q_7 + 2m(y-1) + y + 5b - 3$.
For each $2 \leq a \leq m$ and $1 \leq b \leq y-1, h_7(d_{ab}) = p_7 + q_7 + (a+1)y - a + b - 1, h_7(e_{ab}) = p_7 + q_7 + m(y-1) + ay - (a-1) + b - 1, h_7(c_b d_{ab}) = p_7 + q_7 + 2m(y-1) + (2a+2)y - (2a+1) + 2b - 2, h_7(c_{b+1} d_{ab}) = p_7 + q_7 + 2m(y-1) + (2a+2)y - (2a+1) + 2b - 1, h_7(c_b e_{ab}) = p_7 + q_7 + 4m(y-1) + 2a(y-1) + 2b - 1, h_7(c_{b+1} e_{ab}) = p_7 + q_7 + 4m(y-1) + 2ay - (2a-1) + 2b - 1$.
We consider the following two cases.
Case 1. $p_7 + q_7$ is odd.
For each $1 \leq b \leq y-1, h_7(c_{b+1}) = p_7 + q_7 + 2b, h_7(d_{1b}) = p_7 + q_7 + 2b - 1, h_7(e_{1b}) = p_7 + q_7 + 2y + b - 2$.
Case 2. $p_7 + q_7$ is even.
$h_7(e_{11}) = p_7 + q_7 + 1$.
For each $1 \leq b \leq y-1, h_7(c_{b+1}) = p_7 + q_7 + 2b + 1, h_7(d_{1b}) = p_7 + q_7 + 2b$.
$h_7(e_{1(b+1)}) = p_7 + q_7 + 2y + b - 1$ for $1 \leq b \leq y-2$.
We claim that G_7^* is vertex edge neighborhood prime graph. Clearly, G_7 is vertex edge neighborhood prime graph. We have to prove H_7 is vertex edge neighborhood prime graph.
Let a_7 be any vertex of H_7.
For $a_7 = c_b$, for $1 \leq b \leq y$ and d_{ab}, e_{ab} for $1 \leq a \leq m, 1 \leq b \leq y-1$ with $deg(a_7) \geq 2$. Here, $\gcd\{h_7(y_7) : y_7 \in N_V(a_7)\} = 1$ and $\gcd\{h_7(d_7) : d_7 \in N_E(a_7)\} = 1$.
Hence $G_7 \odot H_7$ is vertex edge neighborhood prime graph. □

References

[1] Balakrishnan. R and Ranganathan. K, *A Text Book of Graph Theory*, Second Edition, Springer, New York, 2012.

[2] Gallian. J. A, *A Dynamic Survey of Graph Labeling*, Electronic Journal of Combinatorics, DS6, 2015.

[3] Parul B Pandya and Shrimali. N. P, *Vertex Edge Neighborhood Prime Labeling of Some Graphs*, International Journal of Scientific Research and Review, Volume 7, Issue 10, 735-743, 2018.

[4] Patel. S. K and Shrimali. N. P , *Neighborhood Prime Labeling*, International Journal of Mathematics and Soft Computing, Volume 5, No. 2, 135 – 143, 2015.

[5] Rajesh Kumar. T. J and Mathew Varkey. T. K, *A note on Total Neighborhood Prime Labeling*, International Journal of Pure and Applied Mathematics (ISSN 1314-3395), Vol.118 , 1007-1013, 2018.

[6] Shalini Rajendra Babu and Ramya. N, *On Prime Cordial Labeling of Crown, Armed Crown, H - graph and Butterfly graph*, International Journal of Innovative Technology and Exploring Engineering (ISSN 2278-3075), Volume 9, Issue 2, 735-743, 2019.

[7] Simaringa. M and Thirunavukkarasu. S, *New Type of Results on Prime Graphs*, Jour of Adv Research in Dynamical and Control systems, Volume 12, No. 8, 2020.

[8] Simaringa. M and Vijayalakshmi. K, *Vertex Edge Neighborhood Prime Labeling of Some Graphs*, Malaya Journal of Mathematik, Volume 7, No. 4, Page 775 – 785, 2019.

[9] Simaringa. M and Vijayalakshmi. K, *Vertex Edge Neighborhood Prime Labeling For Union And One Point Union of Graphs*, International Journal of Advanced Science and Technology, Vol.29, No. 4, Page 309 – 320, 2020.

[10] Umer Ali, Muhammad bilal and Zohaib Zahid, *Some Families of Convex Polytopes Labeled by 3-Total Edge Product Cordial Labeling*, Punjab University Journal of Mathematics (ISSN 1016-2526), Vol 49(3), Page 119-132, 2017.

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