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Certified domination number in corona product of graphs

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Abstract

A set S of vertices in $G=(V,E)$ is called a dominating set of G if every vertex not in S has at least one neighbour in S. A dominating set S of a graph G is said to be a certified dominating set of G if every vertex in S has either zero or at least two neighbours in $V\backslash S$. The certified domination number, $\gamma_{cer}(G)$ of G is defined as the minimum cardinality of certified dominating set of G. In this paper, we study the certified domination number of Corona product of some standard graphs.

Keywords

Dominating set, Certified Dominating set, Certified Domination Number, Corona product.

AMS Subject Classification

05C05, 0505C.

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Contents

1. Introduction

In this paper, graph $G=(V,E)$ we mean a simple, finite, connected, undirected graph with neither loops nor multiple edges. The order $|V(G)|$ is denoted by n. For graph theoretic terminology we refer to West [7]. The open neighborhood of any vertex v in G is $N(v)=\{x: xv\in E(G)\}$ and closed neighborhood of a vertex v in G is $N[v]=N(v)\cup\{v\}$. The degree of a vertex in the graph G is denoted by $\deg(v)$ and the maximum degree (minimumdegree) in the graph G is denoted by $\Delta(G)(\delta(G))$. For a set $S\subseteq V(G)$ the open (closed) neighborhood $N(S)(N[S])$ in G is defined as $N(S)=\bigcup_{v\in S} N(v)\,(N[S]=U_{v\in S}N[v]$. We write K_n, P_n, and C_n for a complete graph, a path graph, a cycle graph of order n, respectively. The complement of a graph G, denoted $by\bar{G}$, is a graph with the vertex set $V(G)$ such that for every two vertices v and $w, vw\in E(\bar{G})$ if and only if $vw\notin E(\bar{G})$.

The corona of two disjoint graphs G_1 and G_2 is defined to be the graph $G=G_1\circ G_2$ formed from one copy of G_1 and $|V(G1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the ith copy of G_2. In particular, the corona $G^\circ K_1$ is the graph constructed from a copy of H, where for each vertex $v\in V(G)$, a new vertex v' and a pendant vv' are added. This is also denoted by G^+.

The concept of certified domination in graphs was introduced by Dettlaff, Lemanska, Topp, Ziemann and Zylinski[3] and further studied in[2]. It has many application in real life situations. This motivated we to study the certified domination number in corona and Cartesian product of graphs.

In [3], authors studied certified dominaiton number in graphs which is defined as follows:

Definition 1.1. *Let $G=(V,E)$ be any graph of order n. A subset $S\subseteq V(G)$ is called a Certified dominating set of G if S is a dominating set of G and every vertex in S has either zero or at least two neighbours in $V\backslash S$. The certified domination number defined by $\gamma_{cer}(G)$ is the minimum cardinality of certified dominating set in G.*

2. Preliminaries

Theorem 2.1 ([2])**.** *For any graph Go of order $n \geq 2$, every certified dominating set of G contains its extreme vertices.*

Theorem 2.2 ([2])**.** *For any graph G of order n, $1 \leq \gamma_{cer}(G) \leq n$.*

Theorem 2.3 ([2])**.** *For any graph G of order $n \geq 3, \gamma_{cer}(G) = 1$ if and only if G has a vertex of degree $n-1$.*

Theorem 2.4 ([3])**.** *For any Path graph P_n of order $n \geq 1$,*

$$\gamma_{cer}(G) = \begin{cases} 1 & \text{if } n = 1 \text{ or } 3 \\ 2 & \text{if } n = 2 \\ 4 & \text{if } n = 4 \end{cases}$$

$$\gamma_{cer}(G) = \lceil \frac{n}{3} \rceil \text{ if } n \geq 5$$

3. Main Results

Theorem 3.1. *Let G be a connected graph with $\delta(G) \geq 2$. If S is a minimum certified dominating set of G, then $V - S$ is a dominating set of G.*

Proof. Let G be a connected graph with $\delta(G) \geq 2$ and let S be a minimum certified dominating set of G. To prove $V - S$ is a dominating set of G. Suppose $V - S$ is not a dominating set of G. Then there exists a vertex $v \in S$ such that vis not dominated by any vertex in $V - S$. This shows that $S - \{v\}$ is a certified dominating set of G, which is a contradiction to the minimality of S. Hence, $V - S$ is a dominating set of G. □

Remark 3.2. *Let G be a connected graph with $\delta(G) \geq 2$. If S is a minimumcertified dominating set of G, then $V - S$ is a need not be a certified dominating set of G.*

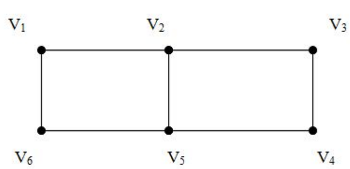

Figure 1

For the graph given in Figure 1, the set $S = \{y_2, v_5\}$ is a minimum certified dominating set of G. Also, here the set $V - S = \{v_1, v_3, v_4, v_6\}$ is a dominating set of G but no vertex in $V - S$ has greater than one neighbour in $V - (V - S)$. That is, in S. Therefore, $V - S$ is not a certified dominating set of G.

Theorem 3.3. *If G is a connected graph with $\delta(G) \geq 2$ and If S is a minimum certified dominating set of G, then $|V - S| \geq |S|$.*

Proof. Let G be a connected graph with $\delta(G) \geq 2$ and let S be a minimum certified dominating set of G. To prove $V - S$ is a dominating set of G. Then by Theorem 3.1, $V - S$ is a dominating set of G. Therefore $|V - S| \neq 0$ and $N[V - S] = V(G)$. This implies that S has a neighbour in $V - S$.*But* since $\delta(G) \geq 2$ that S has at least two neighbours in $V - S$. Hence, $V - S >| S$. □

Theorem 3.4. *If G is a connected graph with $\delta(G) \geq 2$, then $\gamma_{cer}(G) \leq n/2$.*

Proof. Let G be a connected graph with $\delta(G) \geq 2$. let S be a minimum certified dominating set of G. Suppose $\gamma_{cer}(G) > n/2$. By Theorem 3.1, $V - S$ is a dominating set of G. Also $|V - S| = |V| - |S| < n - n/2$. This implies that $|V - S| < n/2$. Therefore by Theorem 3.3, $S |< n/2$, which is a contradiction. Hence $\gamma_{\text{cer}}(G) < n/2$. □

Theorem 3.5. *Let G be a connected graph of even order $n \geq 2$. If $\delta(G) \geq 2$, then $\gamma_{cer}(G) = n/2$ if and only if $G \approx C_4$.*

Proof. Let G be a connected graph of even order $n \geq 2$ with $\delta(G) \geq 2$. Suppose $G \approx C_4$. Then by Theorem 2.5, $\gamma_{\text{cer}}(G) = 2 = n/2$.

Conversely, assume $\gamma_{cer}(G) = n/2$. Then by Theorem 3.3. $V - S \backslash > n/2$. Since n is even that $|V - S| = n/2$. This shows that every vertex in S has exactly two neighbours in $V - S$. Let $\{S_1, S_2 \ \ldots_1 S_n\}$ be the set of stars that covers all the vertices in G. Let $S_i = \{u_i, y_i\}$. Suppose $n \geq 3$. If there is a i such that $\deg(u_i) \geq 2$ and $\deg(v_i) \geq 2$. Thus we can find a certified dominating set of G with cardinality less than $n/2$, which is a contradiction. Therefore for every i, either u_i or y_i must be of degree equal to 1 . Since $\delta(G) \geq 2$, which is not possible. Therefore $n < 2$. Hence G must be isomorphic to either K_2 or $K_{2\text{m}}$ or the cycle C_4. If K_2 or $K_2{}^+$, then $\delta(G) = 1$ which is a contradiction. Hence, $G \approx C_4$. □

4. Corona of Graphs

For every $v \in V(G)$ denote by H_v the copy of H whose vertices are adioint to the vertex v. Denote $v + H_v$ be the subgraph of the corona $G \circ H$ corresponding to the join $< \{v\} + H_v >$.

Theorem 4.1. *Let G be a connected graph of order n and H be any graph of order $m \geq 2$. Then, $S \subseteq V(G \circ H)$ is a certified dominating set of $G \circ H$ if and only if $S \cap V(v + H_v)$ is a certified dominating set of $v + H_v$ for every $v \in V(G)$.*

Proof. Let G be a connected graph of order n and H be any graph of order $m \geq 2$. First assume that $S \subseteq V(G \circ H)$ be a certified dominating set of $G \circ H$. Let $v \in V(G)$. Let $S_v = S \cap V(v + H_v)$. We show that S_v is a certified dominating set of $v + H_v$. If $v \in S_v$, then $V(H_v) \subseteq N_{H_v + v}[v] \subseteq N_{v + H_v}[v]$. Then S_v is a certified dominating set in $G \circ H$. Suppose $v \notin S_v$. Then clearly $v \in N_{v+H_v}[S_v]$. Let $u \in V(H_v) - S_v$. Then there exists $w \in S$ such that $u \in N_{GoH}[w]$. Also, $ux \notin E(GoH)$ for all $x \in V(G) - \{v\}$. This means that S_v is a dominating set of $v + H_v$. Now we prove that S_v is a certified dominating set of $v + H_v$. Since $n \geq 2$, it is clear that $|N_{v+H_v}[y] \cap V(v + H_v) - S_v| \geq 2$ for every $y \in S_v$. Therefore, S_v is a certified dominating set of $v + H_v$.

Conversely, assume $S_v = S \cap V(v + H_v)$ is a certified dominating set of $v + H_v$ for every $v \in V(G)$. To prove that $S \subseteq V(G \circ H)$ is a certified dominating set of $G \circ H$. Let $u \in V(G \circ H) - S$. Since $N_{v+H_v}[x] = N_{GoH}[x]$ for every $x \in S_v$, that the certified dominating set S_v in $v + H_v$ implies the existence of $x \in S_v$ such that $u \in N_{v+H_v}[x]$. Hence, S is a certified dominating set of $G \circ H$. □

Corollary 4.2. *Let G be a connected graph of order n and H be any graph of order $m \geq 2$. Then $\gamma_{cer}(G \circ H) = n$.*

Theorem 4.3. *Let G be a connected graph of order n and H be any trivial graph. Then $\gamma_{cer}(G \circ H) = 2n$.*

Proof. Let G be a connected graph of order n and H be any trivial graph. Since $H \approx K_1$, that $G \circ H = G^+$. Also, that G^+ is a connected graph of order $2n$. By definition, every vertex in G^+ is either a support vertex or a pendant vertex. Let S be a minimum certified dominating set of G^+. By Theorem 2.1, every support vertices must in S. Also, every support vertex is adjacent to exactly one pendant vertex implies every pendent vertices are in S. Then $S = V(G^+)$. Hence, we conclude $\gamma_{cer}(G \circ H) = 2n$. □

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