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# Bipolar interval valued signed neutrosophic graphs

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**Abstract**

In this article, we combine the concept of bipolar neutrosophic set with graph theory, we introduced the bipolar interval valued neutrosophic graphs, strong bipolar interval valued neutrosophic graphs, degree of bipolar interval valued neutrosophic graphs and introduce the bipolar interval valued signed neutrosophic graphs and investigate some of their properties with proofs and examples.

**Keywords**

Bipolar, signed, balanced, Interval Valued Neutrosophic.

**AMS Subject Classification**

03E72, 05C72, 05C78, 05C99.

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## Contents



## 1. Introduction

Neutosophic sets proposed by smarandache [11,12] is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. They are a generalization of the theory of fuzzy sets, intuitionistic fuzzy set, interval valued fuzzy set and interval valued intuitionistic fuzzy sets [8]. The neutrosophic sets are characterized by a truth membership function $(T)$ an indeterminacy membership function $(I)$ and a falsity membership function $(F)$ independently, which are within the real standard or nonstandard unit interval $]^{-}0,1^{+}[$. In order to practice NS in real life applications conveniently, Wang et al.[5] introduced the concept of a single valued neutrosophic sets (SVNS), a subclass of the neutrosophic sets. The same author introduced the concept of interval valued neutrosophic sets, which is more precise and flexible than single valued neutrosophic sets. The IVNS is a generalization of single valued neutrosophic sets, in which three membership functions are independent and their value belong to the unit interval $[0,1]$ some more work on single valued neutrosophic sets, interval valued neutrosophic sets and their application may be found on [1,2,4-7,13-17,19-21]

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving combinational problems in different areas such as geometry, algebra, number theory, topology, and optimization and computer science [9,10]. Most important thing which is to be noted that, when we have uncertainty regarding either the set of vertices or edges or both, the model becomes a fuzzy graph. The extension of fuzzy [3] graph theory have been developed by several researches including intuitionistic fuzzy graphs considered the vertex sets and edge sets as intuitionistic fuzzy sets. Interval value fuzzy graphs considered the vertex sets and edge sets as interval valued intuitionistic fuzzy sets. Bipolar fuzzy graph considered the vertex set and edge sets as bipolar fuzzy sets. M-polar fuzzy graph considered the vertex sets and edge sets as m-polar fuzzy sets. But, when the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graph and their extensions are failed. For this purpose, samarandache have defined four main categories of neutrosophic graphs, two based on literal indeterminancy, which called them; I edge neutrosophic graph and I-vertex neutrosophic graph, these concept ate studied deeply and has gained popularity among the researches due to its applications via real world problems the two other graph based on $(T,I,F)$ components and called them; the $(T,I,F)$-edge neutrosophic graph and the $(T,I,F)$-vertex neutrosophic graph, these concept are not developed at all. Later on, Broumi et al. [18] introduced a third neutrosophic graph model this model allow the attachment of truth-membership ($T$), indeterminacy-membership ($I$) and falsity-membership degrees ($F$) both to vertices and edges and investigated some of their properties. The third neutrosophic graph model is called single valued neutrosophic graph (SVNG) the single valued neutrosophic graph model is called single valued graph and intuitionistic fuzzy graph. Also the same authors introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph in the literature the study of interval valued neutrosophic graph is still blank we shall focus on the study of interval valued neutrosophic graphs.Then, Sudhakar et al.[22-25] introduced the concept of interval valued signed neutrosophic graph and self-centered interval valued signed neutrosophic graph.

In this paper, Bipolar interval valued neutrosophic graphs and Bipolar interval valued signed neutrosophic graphs are developed.

## 2. Preliminaries

In this section, we mainly recall some nations related to neutrosophic sets signal valued neutrosophic sets, interval valued neutrosophic sets, and bipolar interval valued neutrosophic graphs.

**Definition 2.1.** *Let $U$ be an universe of discourse; then the neutrosophic set A is an object having the from $A = \{< x : T_A(x), F_A(x) >, x \in \cup\}$, where the funtions T, I, F:U $\rightarrow$ $]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy and the degree of non-membership of the element $x \in \cup$ to the set A with the condition.*

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$$

*The functions $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]^-0,1^+[$ since it is difficult to apply NSS to practical problem. Wang etal introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.*

**Definition 2.2.** *Let $X$ be a space of points (objects) with generic elements in $X$ denoted by $X$. A single valued Neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$ . For each point $x$ in $X$. $T_A(x), I_A(x), F_A(x) \in [0,1]$. A SVNS can be written as $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$.*

**Definition 2.3.** *Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be single valued neutrosophic sets on a set $X$. If $A = (T_A, I_A, F_A)$ is a single valued neutrosophic relation on a set $X$, then $A = (T_A, I_A, F_A)$ is called a, single valued neutrosophic relation on $B = (T_B, I_B, F_B)$ If*

$$\begin{aligned} T_B(x,y) &\leq \min(T_A(x), T_A(y)) \\ I_B(x,y) &\geq \max(I_A(x), I_A(y)) \\ F_B(x,y) &\geq \max(F_A(x), F_A(y)) \quad \textit{for all } x,y \in X \end{aligned}$$

*A single valued neutrosophic relation A on X is called symmetric if $T_A(x,y) = T_A(y,x), I_A(x,y) = I_A(y,x), F_A(x,y) = F_A(y,x)$ and $T_B(x,y) = T_B(y,x), I_B(x,y) = I_B(y,x), F_B(x,y) = F_B(y,x)$ and for all $x,y \in X$.*

**Definition 2.4.** *A single valued neutrosophic graph with underlying set $V$ is defined to be a pair of $G = (A,B)$ where,*

1. *The functions $T_A : V \rightarrow [0,1]$, $I_A : V \rightarrow [0,1]$ and $F_A : V \rightarrow [0,1]$ denote the degree of truth membership degree of indeterminacy-membership and falsity-membership of the element $v_i \in V$ respectively, and $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$ for all $v_i \in V(1,2,\ldots,n)$.*
2. *The functions $T_B : E \subseteq V \times V \rightarrow [0,1]$, $I_B : E \subseteq V \times V \rightarrow [0,1]$ and $F_B : E \subseteq V \times V \rightarrow [0,1]$ are defined by*

$$\begin{aligned} T_B(\{v_i,v_j\}) &\leq \min[T_A(v_i), T_A(v_j)] \\ I_B(\{v_i,v_j\}) &\geq \max[I_A(v_i), I_A(v_j)] \\ F_B(\{v_i,v_j\}) &\geq \max[F_A(v_i), F_A(v_j)] \end{aligned}$$

*denotes the degree of truth membership, indeterminacy-membership and falsity-membership of the edge $(v_i,v_j) \in E$ respectively, where $0 < T_B(\{v_i,v_j\}) + I_B(\{v_i,v_j\}) + F_B(\{v_i,v_j\}) \leq 3$ for all $(v_i,v_j) \in E$ $(i,j = 1,2,\ldots,n)$.*

**Definition 2.5.** *A bipolar neutrosophic set A in X is defined as an object of the form $A = \{< x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x) >: x \in X\}$ where $T^P, I^P, F^P : X \rightarrow [0,1]$ and $T^N, I^N, F^N : X \rightarrow [-1,0]$. The positive membership degree $T^P(x), I^P(x), F^P(x)$ denotes the truth membership, indeterminate membership and false membership of an element $\in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^N(x), I^N(x), F^N(x)$ denotes the truth membership indeterminate membership and false membership of an element $\in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set A.*

**Example 2.6.** *Let $X = \{x_1, x_2, x_3\}$*

$$\begin{aligned} A = \quad & < x_1, 0.5, 0.3, 0.1, -0.6, -0.4, -0.05 > \\ & < x_2, 0.3, 0.2, 0.7, -0.02, -0.3, -0.02 > \\ & < x_3, 0.8, 0.05, 0.4, -0.6, -0.6, -0.03 > \end{aligned}$$

*Is a bipolar neutrosophic subset of $X$.*

**Definition 2.7.** *Let*

$$A_1 = \{< x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x) >\}$$

*and $A_2 = \{< x, T_2^P(x), I_2^P(x), F_2^P(x), T_2^N(x), I_2^N(x), F_2^N(x) >\}$ be two bipolar neutrosophic sets then $A_1 \subseteq A_2$ if and only if $T_1^P(x) \leq T_2^P(x), I_1^P(x) \leq I_2^P(x), F_1^P(x) \geq F_2^P(x)$ and $T_1^N(x) \geq T_2^N(x), I_1^N(x) \geq I_2^N(x), F_1^N(x) \leq F_2^N(x)$ for all $x \in X$.*

**Definition 2.8.** *Let*

$$A_1 = \{< x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x) >\}$$

*and* $A_2 = \{< x, T_2^P(x), I_2^P(x), F_2^P(x), T_2^N(x), I_2^N(x), F_2^N(x) >\}$ *be two bipolar neutrosophic sets then* $A_1 = A_2$ *if and only if* $T_1^P(x) = T_2^P(x), I_1^P(x) = I_2^P(x), F_1^P(x) = F_2^P(x)$ *and* $T_1^N(x) = T_2^N(x), I_1^N(x) = I_2^N(x), F_1^N(x) = F_2^N(x)$ *for all* $x \in X$.

**Definition 2.9.** *Let*

$$A_1 = \{< x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x) >\}$$

*and* $A_2 = \{< x, T_2^P(x), I_2^P(x), F_2^P(x), T_2^N(x), I_2^N(x), F_2^N(x) >\}$ *be two bipolar neutrosophic sets then their union is defined as*

$$\begin{aligned}(A_1 \cup A_2)(x) &= \max(T_1^P(x), T_2^P(x)), \frac{I_1^P(x) + I_2^P(x)}{2}, \\ &\min(T_1^P(x), T_2^P(x)) \\ &= \max(T_1^N(x), T_2^N(x)), \frac{I_1^N(x) + I_2^N(x)}{2}, \\ &\min(T_1^N(x), T_2^N(x))\end{aligned}$$

*for all* $x \in X$.

**Definition 2.10.** *Let*

$$A_1 = \{< x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x) >\}$$

*and* $A_2 = \{< x, T_2^P(x), I_2^P(x), F_2^P(x), T_2^N(x), I_2^N(x), F_2^N(x) >\}$ *be two bipolar neutrosophic sets then their intersection is defined as*

$$\begin{aligned}(A_1 \cap A_2)(x) &= \min(T_1^P(x), T_2^P(x)), \frac{I_1^P(x) + I_2^P(x)}{2}, \\ &\max(T_1^P(x), T_2^P(x)) \\ &= \max(T_1^N(x), T_2^N(x)), \frac{I_1^N(x) + I_2^N(x)}{2}, \\ &\min(T_1^N(x), T_2^N(x))\end{aligned}$$

*for all* $x \in X$.

**Definition 2.11.** *Let*

$$A_1 = \{< x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x) >: x \in X\}$$

*be a bipolar neutrosophic set in X then the complement of A is denoted by* $A^c$ *and is defined by* $T_A^P c(x) = \{1^P\} - T_A^P(x)$, $I_A^P c(x) = \{1^P\} - I_A^P(x), F_A^P c(x) = \{1^P\} - F_A^P(x)$. *and* $T_A^N c(x) = \{1^N\} - T_A^N(x)$, $I_A^N c(x) = \{1^N\} - I_A^N(x), F_A^N c(x) = \{1^N\} - F_A^N(x)$.

**Definition 2.12.** *Let* $G = (A, B)$ *be a single valued neutrosophic graph. Then the degree p of any vertex V is sum of degree of truth-membership. Sum of degree of indeterminacy-membership and sum of degree of falsity-membership of all those edges which are incident on vertex V denoted by* $d(v) = (d_T(v), d_I(v), d_F(v))$ *Where*

$$\begin{aligned}d_T(v) &= \sum_{u \neq v} T_{B(u,v)} \text{ denotes degree of truth-membership vertex} \\ d_I(v) &= \sum_{u \neq v} I_{B(u,v)} \text{ denotes degree of Interminary -membership vertex} \\ d_F(v) &= \sum_{u \neq v} F_{B(u,v)} \text{ denotes degree of Falsity -membership vertex}\end{aligned}$$

.

**Definition 2.13.** *Let X be a space of points (objects) with generic elements in X denoted by X. An interval valued neutrosophic set (IVNSA) A in X is characterized by truth-membership function* $T_A(x)$, *indeterminacy-membersip function* $I_A(x)$ *and falsity-membership function* $F_A(x)$. *For each point X in X we have that* $T_A(x) = [T_{AL}(x)T_{AU}(x)], I_A(x) = [I_{AL}(x)I_{AU}(x)], F_A(x) = [F_{AL}(x)F_{AU}(x)] \leq [0,1]$ *and* $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

**Definition 2.14.** *Let X and Y be two non-empty crisp sets. An interval valued neutrosophic relation* $R(x,y)$ *is a subset of product space* $x \times y$, *and is characterized by the truth membership function* $T_R(x,y)$, *the indeterminacy membership function* $I_R(x,y)$, *and the falsity membership function* $F_R(x,y)$, *where* $T_R(x,y), I_R(x,y), F_R(x,y) \leq [0,1]$.

## 3. Bipolar Interval Valued Neutrosophic Graphs

**Definition 3.1.** *A BIVNG can be defined as* $G = (A, B)$, *here*

$$A = < x, [T_{AL}^P T_{AU}^P][I_{AL}^P I_{AU}^P][F_{AL}^P F_{AU}^P][T_{AL}^N T_{AU}^N][I_{AL}^N I_{AU}^N][F_{AL}^N F_{AU}^N] >.$$

*is an BIVN set on vertices V and*

$$B = < x, [T_{BL}^P T_{BU}^P][I_{BL}^P I_{BU}^P][F_{BL}^P F_{BU}^P][T_{BL}^N T_{BU}^N][I_{BL}^N I_{BU}^N][F_{BL}^N F_{BU}^N] >.$$

*is an BIVN set on edges, proves the following condition*

*1.* $Y = \{v_1, v_2, \ldots, v_n\}$ *then* $T_{AL}^P : v \to [0,1], T_{AU}^P : v \to [0,1] I_{AL}^P : v \to [0,1] I_{AU}^P : v \to [0,1] F_{AL}^P : v \to [0,1] F_{AU}^P : v \to [0,1]$, $T_{AL}^N : v \to [-1,0] T_{AU}^N : v \to [-1,0] I_{AL}^N : v \to [-1,0] I_{AU}^N : v \to [-1,0] F_{AL}^N : v \to [-1,0] F_{AU}^N : v \to [-1,0]$.

*Denotes the degree of truth-membership, indeterminacy -membership and falsity-membership respectively*

$$-0 \leq T_A^P(v_i) + I_A^P(v_i) + F_A^P(v_i) \leq 3 \; \forall v_i \in v \, (i = 1, \ldots, n)$$

$$-3 \leq T_A^P(v_i) + I_A^P(v_i) + F_A^P(v_i) \leq 0$$

*2. If* $T_{BL}^P : V \times V \to [0,1] T_{BU}^P : V \times V \to [0,1] I_{BL}^P : V \times V \to [0,1] I_{BU}^P : V \times V \to [0,1] F_{BL}^P : V \times V \to [0,1] F_{BU}^P : V \times V \to$

$[0,1]$ *then*

$$\begin{aligned}
\Rightarrow \quad T^P_{BL}(\{v_i,v_j\}) &\leq \min[T^P_{AL}(v_i),T^P_{AL}(v_j)] \\
T^P_{BU}(\{v_i,v_j\}) &\leq \min[T^P_{AU}(v_i),T^P_{AU}(v_j)] \\
I^P_{BL}(\{v_i,v_j\}) &\geq \max[I^P_{AL}(v_i),I^P_{AL}(v_j)] \\
I^P_{BU}(\{v_i,v_j\}) &\geq \max[I^P_{AU}(v_i),I^P_{AU}(v_j)] \quad \textit{and} \\
F^P_{BL}(\{v_i,v_j\}) &\geq \max[F^P_{AL}(v_i),F^P_{AL}(v_j)] \\
F^P_{BU}(\{v_i,v_j\}) &\geq \max[F^P_{AU}(v_i),F^P_{AU}(v_j)]
\end{aligned}$$

*If* $T^N_{BL}: V\times V \to [-1,0] T^N_{BU}: V\times V \to [-1,0] I^N_{BL}: V\times V \to [-1,0] I^N_{BU}: V\times V \to [-1,0] F^N_{BL}: V\times V \to [-1,0] F^N_{BU}: V\times V \to [-1,0]$, *then*

$$\begin{aligned}
T^N_{BL}(\{v_i,v_j\}) &\leq \max[T^N_{AL}(v_i),T^N_{AL}(v_j)] \\
T^N_{BU}(\{v_i,v_j\}) &\leq \max[T^N_{AU}(v_i),T^N_{AU}(v_j)] \\
I^N_{BL}(\{v_i,v_j\}) &\geq \min[I^N_{AL}(v_i),I^N_{AL}(v_j)] \\
I^N_{BU}(\{v_i,v_j\}) &\geq \min[I^N_{AU}(v_i),I^N_{AU}(v_j)] \quad \textit{and} \\
F^N_{BL}(\{v_i,v_j\}) &\geq \min[F^N_{AL}(v_i),F^N_{AL}(v_j)] \\
F^N_{BU}(\{v_i,v_j\}) &\geq \min[F^N_{AU}(v_i),F^N_{AU}(v_j)] \ \forall v_i,v_j \in E
\end{aligned}$$

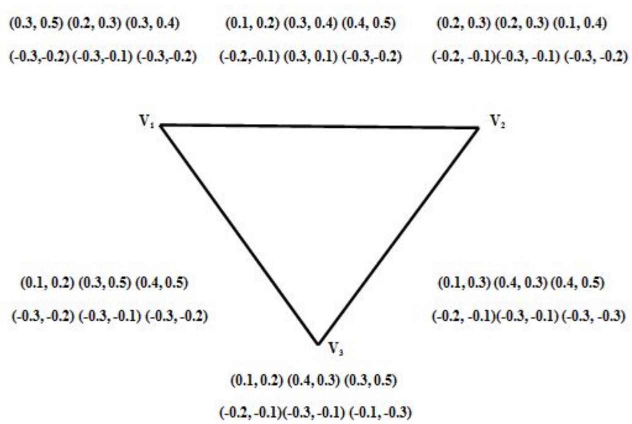


**Figure 1. BIVNG**

*The following adjacency matrix* $M^P_G$ *is representing the BIVNG diagram G*

$$\begin{bmatrix}
<[0.3\,0.5][0.2\,0.3][0.3\,0.4]><[0.1\,0.2][0.3\,0.4][0.4\,0.5]> \\
<[0.1\,0.2][0.3\,0.5][0.4\,0.6] \\
<[0.1\,0.2][0.3\,0.4][0.4\,0.5]><[0.2\,0.3][0.2\,0.3][0.1\,0.4]> \\
<[0.1\,0.3][0.4\,0.5][0.4\,0.5] \\
<[0.1\,0.2][0.3\,0.5][0.4\,0.6]><[0.1\,0.3][0.4\,0.5][0.4\,0.5]> \\
<[0.1\,0.3][0.2\,0.4][0.3\,0.5]
\end{bmatrix}$$

*And* $M^N_G$ *is*

$$\begin{bmatrix}
<(-0.3,-0.2)(-0.3,-0.1)(-0.3,-0.2)> \\
<(-0.2,-0.1)(+0.3,+0.1)(-0.3,0.2)> \\
<(-0.3,-0.2)(-0.3,-0.1)(-0.3,-0.2)> \\
<(-0.2,-0.1)(-0.3,-0.1)(-0.3,-0.2)> \\
<(-0.2,-0.1)(-0.3,-0.1)(-0.3,-0.2)> \\
<(-0.2,-0.1)(-0.3,-0.1)(-0.3,-0.3)> \\
<(-0.3,-0.2)(-0.3,-0.1)(-0.3,-0.2)> \\
<(-0.2,-0.1)(-0.3,-0.1)(-0.2,-0.1)> \\
<(-0.3,-0.1)(-0.3,-0.1)(-0.3,-0.3)>
\end{bmatrix}$$

**Definition 3.2.** *If $G=(A,B)$ be an BIVNG. Then the degree of the vertex V is summation of degree of truth membership. Summation of degree of the indeterminacy membership and summation of degree of the falsity membership of all those edges which are incident on vertex V denoted by*

$$\begin{aligned}
d^N(v) &= [d^N_{TL}(v),d^N_{TU}(v)][d^N_{IL}(v),d^N_{IU}(v)][d^N_{FL}(v),d^N_{FU}(v)] \\
d^P(v) &= [d^P_{TL}(v),d^P_{TU}(v)][d^P_{IL}(v),d^P_{IU}(v)][d^P_{FL}(v),d^P_{FU}(v)]
\end{aligned}$$

$$d^P_{TL}(v)=\sum_{u\neq v} T^P_{BL}(u,v)$$

$\Rightarrow$ *degree of lower truth positive membership vertex*

$$d^P_{TU}(v)=\sum_{u\neq v} T^P_{BU}(u,v)$$

$\Rightarrow$ *degree of upper truth positive membership vertex*

$$d^P_{IL}(v)=\sum_{u\neq v} I^P_{BL}(u,v)$$

$\Rightarrow$ *degree of lower indeterminacy positive membership vertex*

$$d^P_{IU}(v)=\sum_{u\neq v} I^P_{BU}(u,v)$$

$\Rightarrow$ *degree of upper indeterminacy positive membership vertex*

$$d^P_{FL}(v)=\sum_{u\neq v} F^P_{BL}(u,v)$$

$\Rightarrow$ *degree of lower falsity positive membership vertex*

$$d^P_{FU}(v)=\sum_{u\neq v} F^P_{BU}(u,v)$$

$\Rightarrow$ *degree of upper falsity positive membership vertex*

*Similarly we can define for negative*

$$d^N_{TL}(v)=\sum_{u\neq v} T^N_{BL}(u,v)$$

$\Rightarrow$ *degree of lower truth negative membership vertex*

$$d^N_{TU}(v)=\sum_{u\neq v} T^N_{BU}(u,v)$$

$\Rightarrow$ *degree of upper truth negative membership vertex*

$$d^N_{IL}(v)=\sum_{u\neq v} I^N_{BL}(u,v)$$

$\Rightarrow$ *degree of lower indeterminacy negative membership vertex*

$$d^N_{IU}(v)=\sum_{u\neq v} I^N_{BU}(u,v)$$

$\Rightarrow$ *degree of upper indeterminacy negative membership vertex*

$$d^N_{FL}(v)=\sum_{u\neq v} F^N_{BL}(u,v)$$

$\Rightarrow$ *degree of lower falsity negative membership vertex*

$$d^N_{FU}(v)=\sum_{u\neq v} F^N_{BU}(u,v)$$

$\Rightarrow$ *degree of upper falsity negative membership vertex*

**Definition 3.3.** *If $G = (A,B)$ is called strong BIVNG. Then*

$$\begin{aligned} T^P_{BL}(\{v_i,v_j\}) &= \min[T^P_{AL}(v_i),T^P_{AL}(v_j)] \\ T^P_{BU}(\{v_i,v_j\}) &= \min[T^P_{AU}(v_i),T^P_{AU}(v_j)] \\ I^P_{BL}(\{v_i,v_j\}) &= \max[I^P_{AL}(v_i),I^P_{AL}(v_j)] \\ I^P_{BU}(\{v_i,v_j\}) &= \max[I^P_{AU}(v_i),I^P_{AU}(v_j)] \\ F^P_{BL}(\{v_i,v_j\}) &= \max[F^P_{AL}(v_i),F^P_{AL}(v_j)] \\ F^P_{BU}(\{v_i,v_j\}) &= \max[F^P_{AU}(v_i),F^P_{AU}(v_j)] \end{aligned}$$

*Similarly*

$$\begin{aligned} T^N_{BL}(\{v_i,v_j\}) &= \max[T^N_{AL}(v_i),T^N_{AL}(v_j)] \\ T^N_{BU}(\{v_i,v_j\}) &= \max[T^N_{AU}(v_i),T^N_{AU}(v_j)] \\ I^N_{BL}(\{v_i,v_j\}) &= \min[I^N_{AL}(v_i),I^N_{AL}(v_j)] \\ I^N_{BU}(\{v_i,v_j\}) &= \min[I^N_{AU}(v_i),I^N_{AU}(v_j)] \\ F^N_{BL}(\{v_i,v_j\}) &= \min[F^N_{AL}(v_i),F^N_{AL}(v_j)] \\ F^N_{BU}(\{v_i,v_j\}) &= \min[F^N_{AU}(v_i),F^N_{AU}(v_j)] \end{aligned}$$

## 4. Bipolar Interval Valued Signed Neutrosophic Graphs

**Definition 4.1.** *A BIVNG $G^S$ is said to be signed BIVNG.*

*If $\sigma : E(G^s) \to \{3,-3\}$ this function associate from $E(G)^s$ of $G^s$ such that each edges signed to $\{+,-\}$ or all the edges and the nodes are signed to $\{+,-\}$.*

*Assign $E(G^s) \to \{3,-3\}$ based on its truth, indeterminacy, falsity membership values. If Truth =*

$$\begin{cases} > \text{Indeterminacy \& Falsity values; Positive } (+) \\ < \text{Indeterminacy \& Falsity values; Negative } (-) \\ = \text{Indeterminacy \& Falsity values; Unsigned} \end{cases}$$

*Bipolar interval valued signed Neutrosophic graph is said to be negative signed if odd numbers of edges of Bipolar interval valued signed Neutrosophic are negative.*

**Lemma 4.2.** *A BIVSNG is a Bipolar interval valued positive signed Neutrosophic graph if all the even length cycles are having all the negative signed nodes.*

*Proof.* In the following diagram, if all the edges contain negative sign is always positive. Hence it is always a positive signed graph.

In case all the edges contains negative sign is always positive.

So this graph is always a positive signed graph.

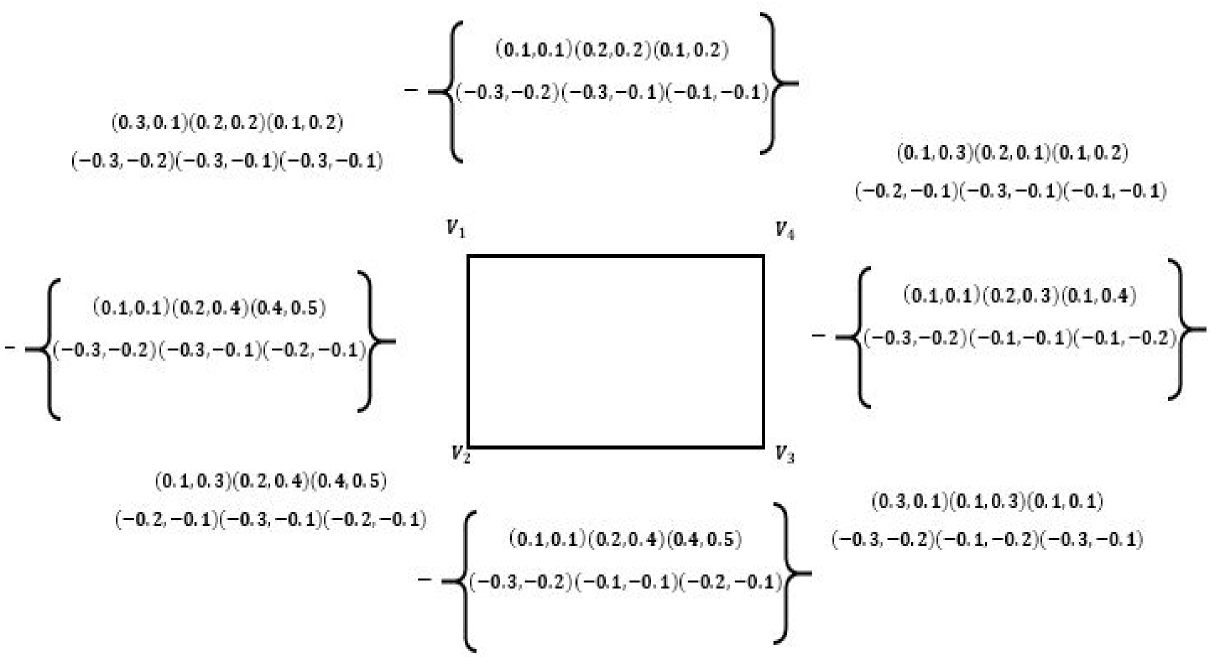


**Figure 2. BIV Positive signed N.G**

□

**Corollary 4.3.** *If a graph with odd length cycle is having all negative signed nodes is always negative signed graph.*

**Definition 4.4.** *A Bipolar interval valued signed neutrosophic graph is balanced. Then the graph have even number of negative signed edges or all positive signed edges. The Bipolar interval valued signed neutrosophic graph is completely balanced if,*

$$\sum_{i=1}^{n} T_i^P \& \sum_{i=1}^{n} T_i^N = \sum_{i=1}^{n} I_i^P \& \sum_{i=1}^{n} I_i^N = \sum_{i=1}^{n} F_i^P \& \sum_{i=1}^{n} F_i^N$$

*for all edges of G.*

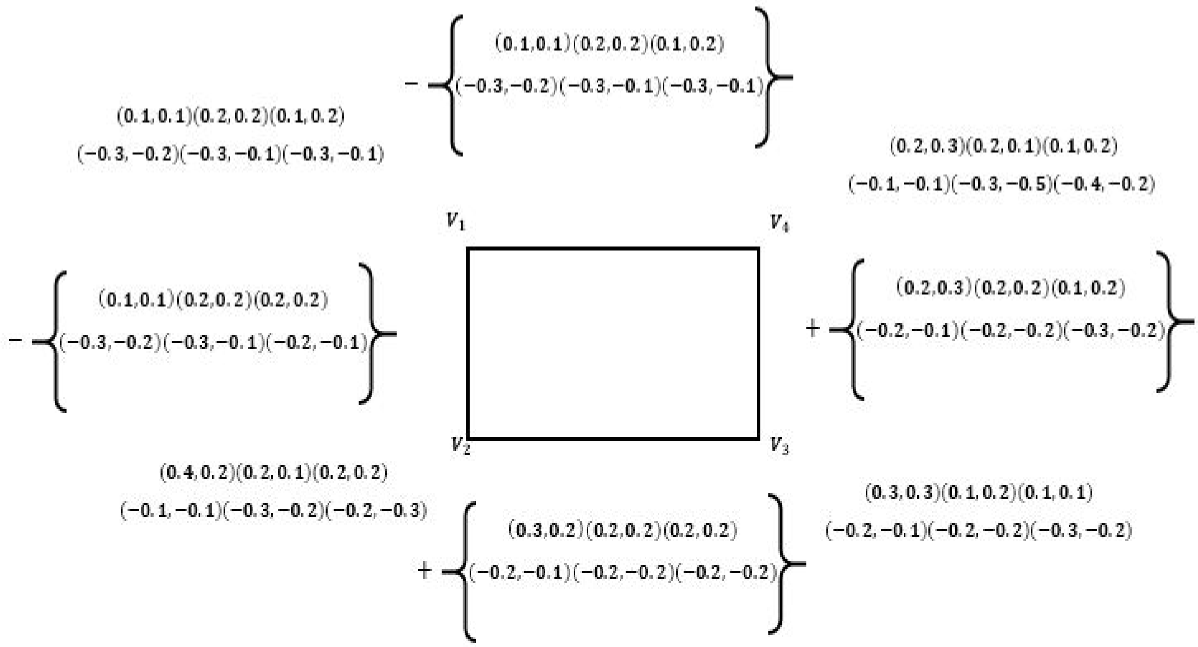


**Figure 3. Balanced BIVSNG**

**Definition 4.5.** *The complement of a Bipolar interval valued signed neutrosophic graph $G = (A,B)$ on $G^*$ is a bipolar interval valued signed neutrosophic graph $\bar{G}$., where*

$$\bar{T}_A^P(v_i) = T_A^P(v_i) \qquad \bar{T}_A^N(v_i) = T_A^N(v_i)$$
$$\bar{I}_A^P(v_i) = I_A^P(v_i) \qquad \bar{I}_A^N(v_i) = I_A^N(v_i)$$
$$\bar{F}_A^P(v_i) = F_A^P(v_i) \qquad \bar{F}_A^N(v_i) = F_A^N(v_i)$$

**Example 4.6.** *An example of complement of BIVSNG.*

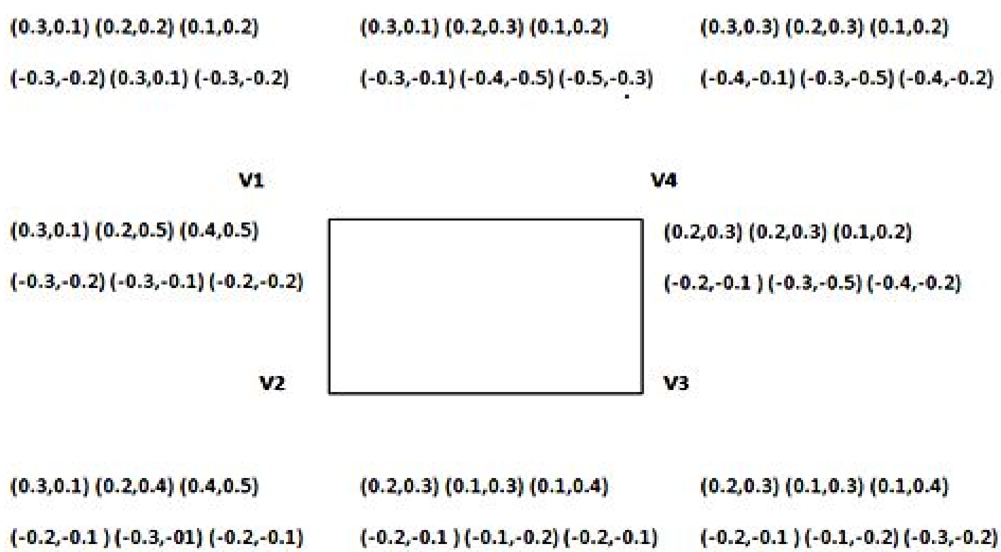


***Figure 4.***

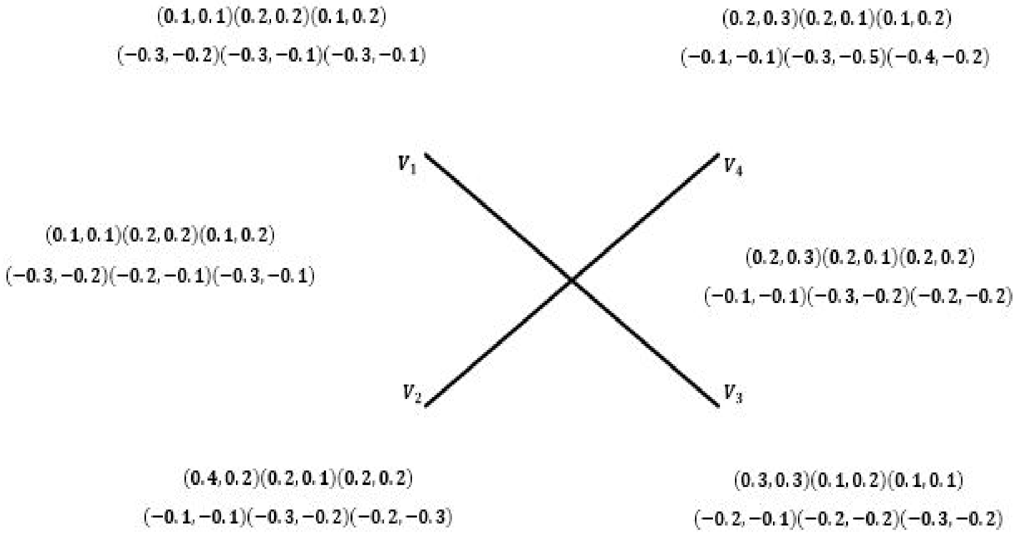


***Figure 5.***

**Proposition 4.7.** *An odd length Bipolar interval valued signed neutrosophic cycle is balanced iff it contains at least one positive edge or odd number of positive edges*

**Proposition 4.8.** *If G is a strong BIVNG then* $\bar{\bar{G}} = G$.

*Proof.* By the definition,

$$\begin{aligned}
T^P_{BL}(u,v) &= \min(T^P_{AL}(u)T^P_{AL}(v))\\
T^P_{BU}(u,v) &= \min(T^P_{AU}(u)T^P_{AU}(v))\\
T^N_{BL}(u,v) &= \max(T^N_{AL}(u)T^N_{AL}(v))\\
T^N_{BU}(u,v) &= \max(T^N_{AU}(u)T^N_{AU}(v))\\
I^P_{BL}(u,v) &= \max(I^P_{AL}(u)I^P_{AL}(v))\\
I^P_{BU}(u,v) &= \max(I^P_{AU}(u)I^P_{AU}(v))\\
I^N_{BL}(u,v) &= \min(I^N_{AL}(u)I^N_{AL}(v))\\
I^N_{BU}(u,v) &= \min(I^N_{AU}(u)I^N_{AU}(v))\\
F^P_{BL}(u,v) &= \max(F^P_{AL}(u)F^P_{AL}(v))\\
F^P_{BU}(u,v) &= \max(F^P_{AU}(u)F^P_{AU}(v))\\
F^N_{BL}(u,v) &= \min(F^N_{AL}(u)F^N_{AL}(v))\\
F^N_{BU}(u,v) &= \min(F^N_{AU}(u)F^N_{AU}(v))
\end{aligned}$$

Let

$\overline{\overline{T}}^P_{BL}(u,v)$

$= \min(T^P_{AL}(u), T^P_{AL}(v)) - \overline{T}^P_{BL}(u,v)$

$= \min(T^P_{AL}(u), T^P_{AL}(v)) - [\min(T^P_{AL}(u), T^P_{AL}(v)) - T^P_{BL}(u,v)]$

$= \min(T^P_{AL}(u), T^P_{AL}(v)) - \min(T^P_{AL}(u), T^P_{AL}(v)) + T^P_{BL}(u,v)$

$= T^P_{BL}(u,v)$

$\overline{\overline{T}}^P_{BU}(u,v)$

$= \min(T^P_{AU}(u), T^P_{AU}(v)) - \overline{T}^P_{BU}(u,v)$

$= \min(T^P_{AU}(u), T^P_{AU}(v)) - [\min(T^P_{AU}(u), T^P_{AU}(v)) - T^P_{BU}(u,v)]$

$= \min(T^P_{AU}(u), T^P_{AU}(v)) - \min(T^P_{AU}(u), T^P_{AU}(v)) + T^P_{BU}(u,v)$

$= T^P_{BU}(u,v)$

$\overline{\overline{T}}^N_{BL}(u,v)$

$= \max(T^N_{AL}(u), T^N_{AL}(v)) - \overline{T}^N_{BL}(u,v)$

$= \max(T^N_{AL}(u), T^N_{AL}(v)) - [\max(T^N_{AL}(u), T^N_{AL}(v)) - T^N_{BL}(u,v)]$

$= \max(T^N_{AL}(u), T^N_{AL}(v)) - \max(T^N_{AL}(u), T^N_{AL}(v)) + T^N_{BL}(u,v)$

$= T^N_{BL}(u,v)$

$\overline{\overline{T}}^N_{BU}(u,v)$

$= \max(T^N_{AU}(u), T^N_{AU}(v)) - \overline{T}^N_{BU}(u,v)$

$= \max(T^N_{AU}(u), T^N_{AU}(v)) - [\max(T^N_{AU}(u), T^N_{AU}(v)) - T^N_{BU}(u,v)]$

$= \max(T^N_{AU}(u), T^N_{AU}(v)) - \max(T^N_{AU}(u), T^N_{AU}(v)) + T^N_{BU}(u,v)$

$= T^N_{BU}(u,v)$

$\overline{\overline{I}}^P_{BL}(u,v)$

$= \max(I^P_{AL}(u), I^P_{AL}(v)) - \overline{I}^P_{BL}(u,v)$

$= \max(I^P_{AL}(u), I^P_{AL}(v)) - [\max(I^P_{AL}(u), I^P_{AL}(v)) - I^P_{BL}(u,v)]$

$= \max(I^P_{AL}(u), I^P_{AL}(v)) - \max(I^P_{AL}(u), I^P_{AL}(v)) + I^P_{BL}(u,v)$

$= I^P_{BL}(u,v)$

$\overline{\overline{I}}^P_{BU}(u,v)$

$= \max(I^P_{AU}(u), I^P_{AU}(v)) - \overline{I}^P_{BU}(u,v)$

$= \max(I^P_{AU}(u), I^P_{AU}(v)) - [\max(I^P_{AU}(u), I^P_{AU}(v)) - I^P_{BU}(u,v)]$

$= \max(I^P_{AU}(u), I^P_{AU}(v)) - \max(I^P_{AU}(u), I^P_{AU}(v)) + I^P_{BU}(u,v)$

$= I^P_{BU}(u,v)$

$\overline{\overline{I}}^N_{BL}(u,v)$
$= \min(I^N_{AL}(u), I^N_{AL}(v)) - \overline{I}^N_{BL}(u,v)$
$= \min(I^N_{AL}(u), I^N_{AL}(v)) - [\min(I^N_{AL}(u), I^N_{AL}(v)) - I^N_{BL}(u,v)]$
$= \min(I^N_{AL}(u), I^N_{AL}(v)) - \min(I^N_{AL}(u), I^N_{AL}(v)) + I^N_{BL}(u,v)$
$= I^N_{BL}(u,v)$

$\overline{\overline{I}}^N_{BU}(u,v)$
$= \min(I^N_{AU}(u), I^N_{AU}(v)) - \overline{I}^N_{BU}(u,v)$
$= \min(I^N_{AU}(u), I^N_{AU}(v)) - [\min(I^N_{AU}(u), I^N_{AU}(v)) - I^N_{BU}(u,v)]$
$= \min(I^N_{AU}(u), I^N_{AU}(v)) - \min(I^N_{AU}(u), I^N_{AU}(v)) + I^N_{BU}(u,v)$
$= I^N_{BU}(u,v)$

$\overline{\overline{F}}^P_{BL}(u,v)$
$= \max(F^P_{AL}(u), F^P_{AL}(v)) - \overline{F}^P_{AL}(u,v)$
$= \max(F^P_{AL}(u), F^P_{AL}(v)) - [\max(F^P_{AL}(u), F^P_{AL}(v)) - F^P_{AL}(u,v)]$
$= \max(F^P_{AL}(u), F^P_{AL}(v)) - \max(F^P_{AL}(u), F^P_{AL}(v)) + F^P_{AL}(u,v)$
$= F^P_{AL}(u,v)$

$\overline{\overline{F}}^P_{BU}(u,v)$
$= \max(F^P_{AU}(u), F^P_{AU}(v)) - \overline{F}^P_{AU}(u,v)$
$= \max(F^P_{AU}(u), F^P_{AU}(v)) - [\max(F^P_{AU}(u), F^P_{AU}(v)) - F^P_{AU}(u,v)]$
$= \max(F^P_{AU}(u), F^P_{AU}(v)) - \max(F^P_{AU}(u), F^P_{AU}(v)) + F^P_{AU}(u,v)$
$= F^P_{AU}(u,v)$

$\overline{\overline{F}}^N_{BL}(u,v)$
$= \min(F^N_{AL}(u), F^N_{AL}(v)) - \overline{F}^N_{AL}(u,v)$
$= \min(F^N_{AL}(u), F^N_{AL}(v)) - [\min(F^N_{AL}(u), F^N_{AL}(v)) - F^N_{AL}(u,v)]$
$= \min(F^N_{AL}(u), F^N_{AL}(v)) - \min(F^N_{AL}(u), F^N_{AL}(v)) + F^N_{AL}(u,v)$
$= F^N_{AL}(u,v)$

$\overline{\overline{F}}^N_{BU}(u,v)$
$= \min(F^N_{AU}(u), F^N_{AU}(v)) - \overline{F}^N_{AU}(u,v)$
$= \min(F^N_{AU}(u), F^N_{AU}(v)) - [\min(F^N_{AU}(u), F^N_{AU}(v)) - F^N_{AU}(u,v)]$
$= \min(F^N_{AU}(u), F^N_{AU}(v)) - \min(F^N_{AU}(u), F^N_{AU}(v)) + F^N_{AU}(u,v)$
$= F^N_{AU}(u,v)$

$\therefore$ The theorem proved. □

**Proposition 4.9.** *If G is a complete BIVNG then the complement of complete BIVNG has no edges.*

*Proof.*

$$T^P_{BL}(u,v) = \min(T^P_{AL}(u)T^P_{AL}(v))$$
$$T^P_{BU}(u,v) = \min(T^P_{AU}(u)T^P_{AU}(v))$$
$$T^N_{BL}(u,v) = \max(T^N_{AL}(u)T^N_{AL}(v))$$
$$T^N_{BU}(u,v) = \max(T^N_{AU}(u)T^N_{AU}(v))$$
$$I^P_{BL}(u,v) = \max(I^P_{AL}(u)I^P_{AL}(v))$$
$$I^P_{BU}(u,v) = \max(I^P_{AU}(u)I^P_{AU}(v))$$
$$I^N_{BL}(u,v) = \min(I^N_{AL}(u)I^N_{AL}(v))$$
$$I^N_{BU}(u,v) = \min(I^N_{AU}(u)I^N_{AU}(v))$$
$$F^P_{BL}(u,v) = \max(F^P_{AL}(u)F^P_{AL}(v))$$
$$F^P_{BU}(u,v) = \max(F^P_{AU}(u)F^P_{AU}(v))$$
$$F^N_{BL}(u,v) = \min(F^N_{AL}(u)F^N_{AL}(v))$$
$$F^N_{BU}(u,v) = \min(F^N_{AU}(u)F^N_{AU}(v))$$

Now $\overline{G}$

$$\begin{aligned}
\overline{T}^P_{BL} &= \min(T^P_{AL}(u), T^P_{AL}(v)) - T^P_{BL}(u,v) \quad \forall u,v \in V \\
&= \min(T^P_{AL}(u), T^P_{AL}(v)) - \min(T^P_{AL}(u), T^P_{AL}(v)) \\
&= 0 \quad \forall u,v \in V \\
\overline{T}^P_{BU} &= \min(T^P_{AU}(u), T^P_{AU}(v)) - T^P_{BU}(u,v) \quad \forall u,v \in V \\
&= \min(T^P_{AU}(u), T^P_{AU}(v)) - \min(T^P_{AU}(u), T^P_{AU}(v)) \\
&= 0 \quad \forall u,v \in V \\
\overline{T}^N_{BL} &= \max(T^N_{AL}(u), T^N_{AL}(v)) - T^N_{BL}(u,v) \quad \forall u,v \in V \\
&= \max(T^N_{AL}(u), T^N_{AL}(v)) - \max(T^N_{AL}(u), T^N_{AL}(v)) \\
&= 0 \\
\overline{T}^N_{BU} &= \max(T^N_{AU}(u), T^N_{AU}(v)) - T^N_{BU}(u,v) \quad \forall u,v \in V \\
&= \max(T^N_{AU}(u), T^N_{AU}(v)) - \max(T^N_{AU}(u), T^N_{AU}(v)) \\
&= 0 \\
\overline{I}^P_{BL} &= \max(I^P_{AL}(u), I^P_{AL}(v)) - I^P_{BL}(u,v) \\
&= \max(I^P_{AL}(u), I^P_{AL}(v)) - \max(I^P_{AL}(u), I^P_{AL}(v)) \\
&= 0 \\
\overline{I}^P_{BU} &= \max(I^P_{AU}(u), I^P_{AU}(v)) - I^P_{BU}(u,v) \\
&= \max(I^P_{AU}(u), I^P_{AU}(v)) - \max(I^P_{AU}(u), I^P_{AU}(v)) \\
&= 0
\end{aligned}$$

and

$$\begin{aligned}
\overline{I}^N_{BL} &= \min(I^N_{AL}(u), I^N_{AL}(v)) - I^N_{BL}(u,v) \\
&= \min(I^N_{AL}(u), I^N_{AL}(v)) - \min(I^N_{AL}(u), I^N_{AL}(v)) \\
&= 0 \\
\overline{I}^N_{BU} &= \min(I^N_{AU}(u), I^N_{AU}(v)) - I^N_{BU}(u,v) \\
&= \min(I^N_{AU}(u), I^N_{AU}(v)) - \min(I^N_{AU}(u), I^N_{AU}(v)) \\
&= 0
\end{aligned}$$

Similarly,

$$\begin{aligned}
\overline{F}^P_{BL}(u,v) &= \max(F^P_{AL}(u), F^P_{AL}(v)) - F^P_{BL}(u,v) \\
&= \max(F^P_{AL}(u), F^P_{AL}(v)) - \max(F^P_{AL}(u), F^P_{AL}(v)) \\
&= 0 \\
\overline{F}^P_{BU}(u,v) &= \max(F^P_{AU}(u), F^P_{AU}(v)) - F^P_{BU}(u,v) \\
&= \max(F^P_{AU}(u), F^P_{AU}(v)) - \max(F^P_{AU}(u), F^P_{AU}(v)) \\
&= 0
\end{aligned}$$

and

$$\begin{aligned}\overline{F}^N_{BL}(u,v) &= \min(F^N_{AL}(u),F^N_{AL}(v)) - F^N_{BL}(u,v)\\ &= \min(F^N_{AL}(u),F^N_{AL}(v)) - \min(F^N_{AL}(u),F^N_{AL}(v))\\ &= 0\\ \overline{F}^N_{BU}(u,v) &= \min(F^N_{AU}(u),F^N_{AU}(v)) - F^N_{BU}(u,v)\\ &= \min(F^N_{AU}(u),F^N_{AU}(v)) - \min(F^N_{AU}(u),F^N_{AU}(v))\\ &= 0\end{aligned}$$

$\overline{T}^P_{BL},\overline{T}^P_{BU},\overline{T}^N_{BL},\overline{T}^N_{BU},\overline{I}^P_{BL},\overline{I}^P_{BU},\overline{I}^N_{BL},\overline{I}^N_{BU},\overline{F}^P_{BL},\overline{F}^P_{BU},\overline{F}^N_{BL},\overline{F}^N_{BU} = (0,0,0,0,0,0,0,0,0,0,0,0)$. Hence the edge set of $\overline{G}$ is empty if $G$ is complete Bipolar interval valued neutrosophic graph. □

## 5. Conclusion

In this paper, we have defined for the Bipolar interval valued signed neutrosophic graphs, Strong bipolar interval valued neutrosophic graphs and Degree of bipolar interval valued neutrosophic graphs with examples. In future study, we plan to extend our research work to regular bipolar interval valued neutrosophic graphs and irrgular bipolar interval valued neutrosophic graphs.

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