Malaya Journal of Matematik, Vol. 9, No. 1, 1095-1099, 2021

https://doi.org/10.26637/MJM0901/0190

Oblong mean prime labeling of variations of cycle, star and path

S. Suganya[1*], C. Santhanaraju[2] and V.J. Sudhakar[3]

Abstract

A graph $G=(V,E)$ with p vertices and q edges is said to be admit oblong mean prime labeling if there exists a bijection $f:V(G)\rightarrow\{2,6,12,,...p(p+1)\}$ such that the induced function $f^*_{ompl}:E(G)\rightarrow N$ given by

$$f^*_{ompl}(uv)=\frac{f(u)+f(v)}{2},\ \ \forall edges\ \ uv\in E(G)$$

the induced function $f^*_{ompl}(uv)$ is said to be an oblong mean prime labeling if the gcd of each vertex of degree atleast 2, is one . A graph which admits oblong mean prime labeling is called oblong mean prime graph. In the paper we proved that shadow graph of even cycle , splitting graph of even cycle,square graph P_n^2 & union of two paths.

Keywords

Oblong mean prime labeling, oblong mean prime graph, shadow graph, even cycle, splitting graph, union of paths and P_n^2.

AMS Subject Classification

05C78, 05C38.

[1,3] *Research Department of Mathematics, Islamiah college, Vaniyambadi-635752, Tamil Nadu, India.*

[2] *Research Department of Mathematics, Arignar Anna Government Arts College, Cheyyar-604407, Tamil Nadu, India.*

*Corresponding author: [1] suganyasureshkumar25@gmail.com

Article History: Received **08** January **2021**; Accepted **22** March **2021**

Contents

1. Introduction

All graphs in this paper are finite, simple and undirected graphs. Let (p,q) be a graph with $p=|V(G)|$ vertices and $q=|E(G)|$ edges. A graph labeling is an assignment of integers to the vertices or edges. Some basic notation and definitions are taken from [1],[2]and [3]. We worked odd mean labeling & even mean labeling of some families graphs. A detailed survey can be found in [4] and [5] and we worked cube difference labeling of some graphs. Sunoj B.S & Mathew Varkey T.K introduced the concept of oblong mean prime labeling & they proved the results for some path related graphs. In the paper we investigated the oblong mean prime labeling of shadow of even cycle, splitting graph of even cycle, union of two paths and square graph p_n^2.
we provide a brief summary of the definitions and other information are useful for the present investigation.

2. Preliminaries

Definition 2.1. *A walk in which $u_1,u_2,..u_n$ are distinct is called a path. A path on n vertices is denoted by P_n.*

Definition 2.2. *A closed path is called a cycle A cycle on n vertices is denoted by C_n.*

Definition 2.3. *For any graph G, the splitting graph $S(G)$ is obtained by adding to each vertex v_i in G a new vertex $v_i^{'}$ is adjacent to the neighbours of v_i in G.*

Definition 2.4. *Let G be a graph with p vertices and q edges. The greatest common divisor of a vector of degrees greater than or equal to 2, is the gcd of the labels of the incident edges.*

Definition 2.5. *An oblong number is the product of a number with its successor, algebraically it has the form $n(n+1)$. The oblong numbers are 2,6,12,20,..*

Definition 2.6. *Let G be a graph with p vertices and q edges. Define a bijection $f : V(G) \to \{2,6,12,20,..p(p+1)\}$ by $f(v_i) = i(i+1)\ \forall\ 1 \le i \le p$ and define a 1-1 mapping $f^*_{ompl} : E(G) \to N$ by $f^*_{ompl}(uv) = \frac{f(u)+f(v)}{2}$ the induced function f^*_{ompl} is said to be an oblong mean prime labeling, if the gcd of each vertex of degree atleast 2 is one.*

Definition 2.7. *A graph which admits oblong mean prime labeling is called mean prime graph.*

Definition 2.8 (Shadow graph)**.** *Let G be a connected graph. A graph, constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G_1 to the neighbours of the corresponding vertex v in G_2, that is for every vertex u in G_1 there exist v in G_2 such that $N(u) = N(v)$ the resulting graph is known as shadow graph it is denoted by $D_2(G)$.*

3. Main Results

Theorem 3.1. *$D_2(C_n), n \ge 4, n$ even is oblong mean prime graph.*

Proof. Let $G = D_2(C_n), n \ge 4, n$ even.
Let $V[D_2(C_n)] = \{u_i | 1 \le i \le 2n\}$. Let $E[D_2(C_n)] = [(u_iu_{i+1}) \cup (u_{n+i}u_{n+i+1}) | 1 \le i \le n-1] \cup [(u_1u_n) \cup (u_{n+1}u_{2n}) \cup (u_{2n}u_1) \cup (u_{n+1}u_n)] \cup [(u_iu_{n+i+1}) \cup (u_{n+i}u_{i+1}) | 2 \le i \le n-1] \cup [(u_{n+i}u_{i-1}) \cup (u_iu_{n+i-1}) | i = 2]$

$$|V(G)| = 2n$$

$$|E(G)| = 4n$$

Define a function $f : V \to \{2,6,12,,...2n(2n+1)\}$ by
$f(u_i) = i(i+1), 1 \le i \le n$
$f(u_{n+i}) = (n+i)(n+i+1), 1 \le i \le n$ clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$$f^*_{ompl}(u_iu_{i+1}) = (i+1)^2, 1 \le i \le n-1$$

$$f^*_{ompl}(u_{n+i}u_{n+i+1}) = (n+i+1)^2, 1 \le i \le n-1$$

$$f^*_{ompl}(u_1u_n) = \frac{n^2+n+2}{2}$$

$$f^*_{ompl}(u_{n+1}u_{2n}) = \frac{5n^2+5n+2}{2}$$

$$f^*_{ompl}(u_1u_{2n}) = 2n^2+n+1$$

$$f^*_{ompl}(u_{n+1}u_n) = (n+1)^2$$

$$f(u_iu_{n+i+1}) = \frac{i(i+1)+(n+i+1)(n+i+2)}{2}, 2 \le i \le n-1$$

$$f(u_iu_{n+i-1}) = \frac{i(i+1)+(n+i-1)(n+i)}{2}, i = 2$$

$$f(u_{n+i}u_{i+1}) = \frac{(n+i)(n+i+1)+(i+1)(i+2)}{2}, 2 \le i \le n-1$$

$$f(u_{n+i}u_{i-1}) = \frac{(n+i)(n+i+1)+(i-1)i}{2}, i = 2$$

Clearly f^*_{ompl} is an injection
$d(u_i) \ge 2,\ 1 \le i \le 2n.$

$$\begin{aligned}
gcd\ \text{ of } (u_i) =& gcd\ \text{ of } [f^*_{ompl}(u_iu_{i+1}), f^*(u_iu_{i-1}),\\
& f^*(u_iu_{n+i-1}), f^*(u_iu_{n+i+1})]\\
gcd \text{ of } (u_i) =& gcd \text{ of } \Big[(i+1)^2, i^2,\\
& \frac{i(i+1)+(n+i-1)(n+i)}{2},\\
& \frac{i(i+1)+(n+i+1)(n+i+2)}{2}\Big]\\
gcd\ \text{ of } (u_i) =& 1,\ 2 \le i \le n-1\\
gcd\ \text{ of } (u_1) =& gcd\ \text{ of } [f^*_{ompl}(u_1u_2), f^*_{ompl}(u_1u_n),\\
& f^*_{ompl}(u_1u_{n+2}), f^*_{ompl}(u_1u_{2n})]\\
gcd\ \text{ of } (u_1) =& gcd\ \text{ of } [4, \frac{n^2+n+2}{2}, \frac{n^2+5n+8}{2},\\
& 2n^2+n+1]\\
gcd\ \text{ of } (u_1) =& 1\\
gcd\ \text{ of } (u_n) =& gcd\ \text{ of } [f^*_{ompl}(u_nu_1), f^*_{ompl}(u_nu_{n+1}),\\
& f^*_{ompl}(u_nu_{n-1}), f^*_{ompl}(u_nu_{2n-1})]\\
gcd\ \text{ of } (u_n) =& gcd\ \text{ of } [\frac{n^2+n+2}{2}, (n+1)^2, n^2,\\
& \frac{n(n+1)+(2n-1)2n}{2}]\\
gcd\ \text{ of } (u_n) =& 1\\
gcd\ \text{ of } (u_{n+i}) =& gcd\ \text{ of } [f^*_{ompl}(u_{n+i}u_{n+i+1}),\\
& f^*_{ompl}(u_{n+i}u_{n+i-1}),\\
& f^*_{ompl}(u_{n+i}u_{i-1}), f^*_{ompl}(u_{n+i}u_{i+1})]\\
gcd\ \text{ of } (u_{n+i}) =& gcd\ \text{ of } \Big[(n+i+1)^2, (n+i)^2,\\
& \frac{(n+i)(n+i+1)+(i-1)i}{2},\\
& \frac{(n+i)(n+i+1)+(i+1)(i+2)}{2}\Big]\\
gcd\ \text{ of } (u_{n+i}) =& 1,\ 2 \le i \le n-1\\
gcd\ \text{ of } (u_{n+i}) =& gcd\ \text{ of } [f^*_{ompl}(u_{n+1}u_{2n}), f^*_{ompl}(u_{n+1}u_n),\\
& f^*_{ompl}(u_{n+1}u_{n+2}), f^*_{ompl}(u_{n+1}u_2)]\\
gcd\ \text{ of } (u_{n+i}) =& gcd\ \text{ of } \Big[\frac{5n^2+5n+2}{2}, (n+1)^2, (n+2)^2,\\
& \frac{n^2+3n+8}{2}\Big]\\
gcd\ \text{ of } (u_{n+i}) =& 1\\
gcd\ \text{ of } (u_{2n}) =& gcd\ \text{ of } [f^*_{ompl}(u_{2n}u_{2n-1}), f^*_{ompl}(u_{2n}u_{n+1}),\\
& f^*_{ompl}(u_{2n}u_{n-1}), f^*_{ompl}(u_{2n}u_1)]
\end{aligned}$$

$$gcd \text{ of } (u_{2n}) = gcd \text{ of } \left[4n^2, \frac{5n^2+5n+2}{2}, \frac{2n(2n+1)+n(n-1)}{2}, 2n^2+n+1\right]$$

$$gcd \text{ of } (u_{2n}) = 1$$

So gcd of each vertex of degree greater than atleast 2 is one. Hence $D_2(C_n),\ n \geq 4,\ n$ even admits oblong mean prime labeling $D_2(C_n),\ n \geq 4,\ n$ even is oblong mean prime graph. □

Example 3.2. *See Fig. 1.*

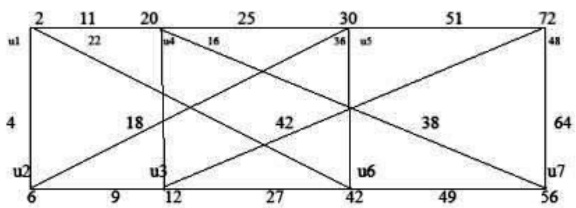

Figure 1

Theorem 3.3. *The splitting graph of even cycle $C_n,\ n \geq 4,\ n$ even is oblong mean prime graph.*

Proof. Let $C_n,\ n \geq 4\ n$ even be an even cycle with n vertices. Let $G = S(C_n),\ \ n \geq 4,\ n$ even.

$$V(G) = \{u_i | 1 \leq i \leq 2n\}$$

$E(G) = \{u_i u_{i+1} |\ 1 \leq i \leq n-1\} \cup \{u_1 u_n\} \cup \{u_i u_{n+i+1} |,\ 1 \leq i \leq n-1\} \cup \{u_{i+1} u_{n+i} |\ 1 \leq i \leq n-1\} \cup \{u_1 u_{2n}\} \cup \{u_n u_{n+1}\}$

$$|V(G)| = 2n$$

$$|E(G)| = 3n$$

Define a function $f : V \to \{2, 6, 12, ..2n(2n+1)\}$ by $f(u_i) = i(i+1),\ 1 \leq i \leq 2n$. For the vertex labeling, the induced edge labeling is $f^*_{ompl} : E(G) \to N$ by

$$f^*_{ompl}(u_i u_{i+1}) = (i+1)^2,\ 1 \leq i \leq n-1$$

$$f^*_{ompl}(u_1 u_n) = \frac{n^2+n+2}{2}$$

$$f^*_{ompl}(u_i u_{n+i+1}) = \frac{i(i+1)+(n+i+1)(n+i+2)}{2},\ 1 \leq i \leq n-1$$

$$f^*_{ompl}(u_{i+1} u_{n+i}) = \frac{(i+1)(i+2)+(n+i)(n+i+1)}{2},\ 1 \leq i \leq n-1$$

$$f^*_{ompl}(u_1 u_{2n}) = 2n^2+n+1$$

$$f^*_{ompl}(u_n u_{n+1}) = (n+1)^2$$

$$gcd \text{ of } (u_1) = gcd \text{ of } \{f^*_{ompl}(u_1 u_2), f^*_{ompl}(u_1 u_n), f^*_{ompl}(u_1 u_{n+2}), f^*_{ompl}(u_1 u_{2n})\}$$

$$gcd \text{ of } (u_1) = gcd \text{ of } \{4, \frac{n^2+n+2}{2}, \frac{n^2+5n+8}{2}, 2n^2+n+1\}$$

$$gcd \text{ of } (u_1) = 1$$

$$gcd \text{ of } (u_{i+1}) = gcd \text{ of } \left\{f^*_{ompl}(u_{i+1} u_i), f^*_{ompl}(u_{i+1} u_{n+1}), f^*_{ompl}(u_{i+1} u_{n+i+2}), f^*_{ompl}(u_{i+1} u_{i+2})\right\}$$

$$gcd \text{ of } (u_{i+1}) = gcd \text{ of } \left\{(i+1)^2, \frac{(i+1)(i+2)+(n+1)(n+2)}{2}, \frac{(i+1)(i+2)+(n+i+2)(n+i+3)}{2}, i^2+3i+3\right\}$$

$$gcd \text{ of } (u_{i+1}) = 1,\ 1 \leq i \leq n-1$$

$$gcd \text{ of } (u_n) = gcd \text{ of } \left\{f^*_{ompl}(u_n u_{n-1}), f^*_{ompl}(u_n u_1), f^*_{ompl}(u_n u_{2n-1}), f^*_{ompl}(u_n u_{n+1})\right\}$$

$$gcd \text{ of } (u_n) = gcd \text{ of } \left\{n^2, \frac{n^2+n+2}{2}, \frac{5n^2-n}{2}, (n+1)^2\right\}$$

$$gcd \text{ of } (u_n) = 1$$

$$gcd \text{ of } (u_{n+1}) = gcd \text{ of } \left\{f^*_{ompl}(u_{n+1} u_n), f^*_{ompl}(u_{n+1} u_2)\right\}$$

$$gcd \text{ of } (u_{n+1}) = gcd \text{ of } \left\{(n+1)^2, \frac{n^2+3n+4}{2}\right\}$$

$$gcd \text{ of } (u_{n+1}) = 1$$

$$gcd \text{ of } (u_{n+i+1}) = gcd \text{ of } \left\{f^*_{ompl}(u_{n+i+1} u_i), f^*_{ompl}(u_{n+i+1} u_{i+2})\right\}$$

$$gcd \text{ of } (u_{n+i+1}) = gcd \text{ of } \left\{\frac{i(i+1)+(n+i+1)(n+i+2)}{2}, \frac{(n+i+1)(n+i+2)+(i+2)(i+3)}{2}\right\}$$

$$gcd \text{ of } (u_{n+i+1}) = 1,\ 1 \leq i \leq n-1$$

$$gcd \text{ of } (u_{2n}) = gcd \text{ of } \left\{f^*_{ompl}(u_{2n} u_1), f^*_{ompl}(u_{2n} u_{n-1})\right\}$$

$$gcd \text{ of } (u_{2n}) = gcd \text{ of } \left\{2n^2+n+1, \frac{5n^2+n}{2}\right\}$$

$$gcd \text{ of } (u_{2n}) = 1$$

Therefore $S(C_n)),\ n \geq 4,\ n$ even admits oblong mean prime labeling.
Therefore $S(C_n))n \geq 4,\ n$ even is oblong mean prime graph. □

Example 3.4. *See Fig. 2.*

Theorem 3.5. *The square graph P_n^2 is oblong mean prime graph.*

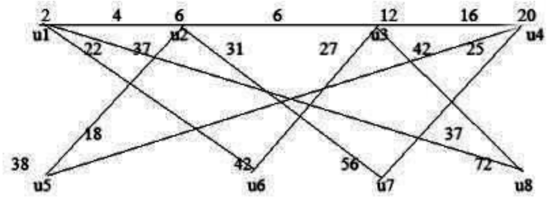

Figure 2

Proof. Let $P_n : u_1u_2...u_n$ be a path.
Let

$$G = P_n^2$$

Let

$$V(P_n^2) = \{u_i | 1 \leq i \leq n\}$$
$$E(P_n^2) = \{u_iu_{i+1} | 1 \leq i \leq n-1\} \cup \{u_iu_{i+2} | 1 \leq i \leq n-2\}$$
$$|V(P_n^2)| = n$$
$$E(P_n^2) = 2n-3$$

Define a vertex labeling $f : V \to \{2, 6, 12, 20, ...n(n+1)\}$ by $f(u_i) = i(i+1), 1 \leq i \leq n$ the induced edge labeling

$$f^*_{ompl}(u_iu_{i+1}) = i^2 + 2i + 1, \ 1 \leq i \leq n-1$$
$$f^*_{ompl}(u_iu_{i+2}) = i^2 + 3i + 3, \ 1 \leq i \leq n-2$$
$$d(u_i) \geq 2, \ 1 \leq i \leq n$$
$$gcd \ \text{ of } (u_1) = gcd \ \text{ of } \left[f^*_{ompl}(u_1u_2), f^*_{ompl}(u_1u_3)\right]$$
$$gcd \ \text{ of } (u_1) = gcd \ \text{ of } [4,7]$$
$$gcd \ \text{ of } (u_1) = 1$$
$$gcd \ \text{ of } (u_2) = gcd \ \text{ of } \left[f^*_{ompl}(u_2u_1), f^*_{ompl}(u_2u_3), f^*_{ompl}(u_2u_4)\right]$$
$$gcd \ \text{ of } (u_1) = gcd \ \text{ of } [4,9,13]$$
$$gcd \ \text{ of } (u_1) = 1$$
$$gcd \ \text{ of } (u_i) = gcd \ \text{ of } \left[f^*_{ompl}(u_iu_{i-1}), f^*_{ompl}(u_iu_{i+1}), f^*_{ompl}(u_iu_{i-2}), f^*_{ompl}(u_iu_{i+2})\right]$$
$$gcd \ \text{ of } (u_i) = gcd \ \text{ of } [i^2, (i+1)^2, i^2 - i + 1, i^2 + 3i + 3]$$
$$gcd \ \text{ of } (u_i) = 1, \ \ 3 \leq i \leq n-2$$
$$gcd \ \text{ of } (u_{n-1}) = gcd \ \text{ of } [f^*_{ompl}(u_{n-1}u_n), f^*_{ompl}(u_{n-1}u_{n-2}), f^*_{ompl}(u_{n-1}u_{n-3})]$$
$$gcd \ \text{ of } (u_{n-1}) = gcd \ \text{ of } [n^2, n^2 - 2n + 1, n^2 - 3n + 3]$$
$$gcd \ \text{ of } (u_{n-1}) = 1$$
$$gcd \ \text{ of } (u_n) = gcd \ \text{ of } [f^*_{ompl}(u_nu_{n-1}), f^*_{ompl}(u_nu_{n-2})$$
$$= gcd \ \text{ of } [n^2, n^2 - n + 1]$$
$$gcd \ \text{ of } (u_n) = 1$$

For $d(u_i) \geq 2 \ \forall i$, gcd of $(u_i) = 1, \ \forall \ i$.
So P_n^2 admits oblong mean prime labeling.
Therefore P_n^2 oblong mean prime graph. □

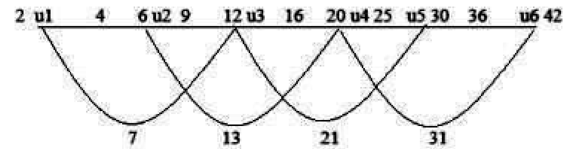

Figure 3

Example 3.6. *See Fig. 3.*

Theorem 3.7. *The graph $P_m \cup P_n$ is oblong mean prime graph.*

Proof. Let P_m be path on m vertices.
Let P_n be path on n vertices.
Let $G = P_m \cup P_n$ Let

$$V(P_m \cup P_n) = \{u_i | 1 \leq i \leq m+n\}$$
$$E(P_m \cup P_n) = \{u_iu_{i+1} | 1 \leq i \leq m-1\} \cup \{u_{m+i}u_{m+i+1} | 1 \leq i \leq m-1\}$$
$$V(P_m \cup P_n) = m+n$$
$$E(P_m \cup P_n) = m+n-2$$

Define a function $f : V \to \{2, 6, 12, ...(m+n)(m+n+1)\}$ by $f(u_i) = i(i+1), \ 1 \leq i \leq m+n$ For vertex labeling f, the induced edge labeling $f^*_{ompl} : E(G) \to N$ by

$$f^*_{ompl}(u_iu_{i+1}) = (i+1)^2, 1 \leq i \leq m-1$$
$$f^*_{ompl}(u_{m+i}u_{m+i+1}) = (m+i+1)^2, 1 \leq i \leq n-1$$
$$d(u_i) \geq 2, \ 2 \leq i \leq m-1$$
$$gcd(u_i) = gcd \ \text{ of } [f^*(u_iu_{i-1}), f^*(u_iu_{i+1})], \ 2 \leq i \leq m-1$$
$$gcd \ \text{ of } (u_i) = gcd \ \text{ of } [i^2, 2i^2 + 4i + 2]$$
$$gcd \ \text{ of } (u_i) = 1, 2 \leq i \leq m-1$$
$$d(u_{m+i+1}) \geq 2, \ 2 \leq i \leq n-2$$
$$gcd \ \text{ of } \ (u_{m+i+1}) = gcd \ \text{ of } \left[f^*_{ompl}(u_{m+i+1}u_{m+i}), f^*_{ompl}(u_{m+i+1}u_{m+i+2})\right] \ 2 \leq i \leq n-2$$
$$gcd \ \text{ of } \ (u_{m+i+1}) = gcd \ \text{ of } [(m+i+1)^2, (m+i+2)^2]$$
$$gcd \ \text{ of } \ (u_{m+i+1}) = 1, \ 2 \leq i \leq n-2$$

$d(u_i) \geq 2$, i is different from end vertices of paths. $gcd \, of(u_i) = 1 \forall$ vertices of degree atleast 2.
Therefore $P_m \cup P_n$ admits oblong mean prime labeling.
$P_m \cup P_n$ is oblong mean prime graph. □

Example 3.8. *See Fig. 4.*

4. Conclusion

In this paper, we have proved that shadow graph of even cycle, splitting graph of even cycle,square graph P_n^2 & union of two path are oblong mean prime graph.It is an interesting work. some one may extend this work in future.

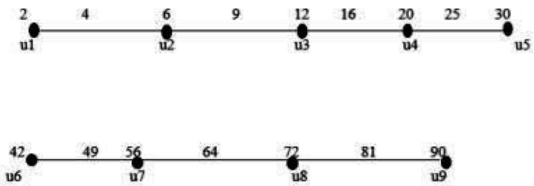

Figure 4

References

[1] F.Harary, *Graph theory*, Addison-Wesley, Reading, Mass, 1972.

[2] Joseph A. Gallian, *A Dynamic Survey of Graph Labeling the electronic journal of combinatorics*, 1996, #DS6,pp1-408.

[3] C. Santhanaraju and S.Meena, Odd mean & Even mean graphs of some graphs, *International journal of modern research and reviews*, 4(5)(2016), 1157-1159.

[4] E. Shakthivel and C. Santhanaraju A study on Odd mean & Even mean labeling of special graph families, *International journal of modern research and reviews*, 5(3)(2017), 1530-1532.

[5] V.J.Sudhakar, C. Santhanaraju and G.A.Mythili, *Cube difference labeling for some alternate triangular snake, cycle, star related graphs.* (communicated).

[6] Suroj B.S and Mathew Varkey, Oblong mean prime Labeling of some cycle graphs, *Adavances in Dynamical Systems and Applications*, 12(2)(2017), 181-186.

[7] A.Nellai Murugan and G.Esther, Some results on mean cordial graphs, *International journal of mathematics trends and technology*, 11(2)(2014).

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆