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A study on lower Q-level subsets of l-subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring

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Abstract
In this paper, we made an attempt to study the algebraic nature of Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring and we introduce the some theorems in lower Q-level subsets of l-subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring.

Keywords
(Q,L)-fuzzy set, (Q,L)-fuzzy l-subsemiring, (Q,L)-fuzzy relation, Product of (Q,L)-fuzzy subsets, homomorphism, anti-homomorphism, Q-level subset.

AMS Subject Classification
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Contents

1. Introduction

There are many concepts of universal algebras generalizing an associative ring $(R;+;\cdot)$. Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra $(R;+;\cdot)$ is said to be a semiring if $(R;+)$ and $(R;\cdot)$ are semigroups satisfying $a\cdot(b+c)=a\cdot b+a\cdot c$ and $(b+c)\cdot a=b\cdot a+c\cdot a$ for all a,b and c in R. A semiring R is said to be additively commutative if $a+b=b+a$ for all a,b in R. A semiring R may have an identity 1, defined by $1\cdot a=a=a\cdot 1$ and a zero 0, defined by $0+a=a=a+0$ and $a\cdot 0=0=0\cdot a$ for all a in R. After the introduction of fuzzy sets by L.A.Zadeh [1], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subnearrings and ideals was introduced by S.Abou Zaid [1]. K.H.kim in [7] introduced the concept of intuitionistic Q-fuzzy semi prime ideals in semigroups. A.Solairaju and R.Nagarajan [14, 15] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. In this paper, we introduce the some theorems in Q-level subsets of an (Q,L)-fuzzy l-subsemiring of a l-semiring. We also made an attempt to study the properties of lower Q-level subsets of an (Q,L)-fuzzy l-subsemirings of l-semiring under homomorphism and anti-homomorphism.

2. Preliminaries

Definition 2.1. *Let X be a non-empty set. A fuzzy subset A of X is a function $A:X\to[0,1]$.*

Definition 2.2. *Let X be a non-empty set and $L=(L,\leq)$ be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q,L)-**fuzzy subset** A of X is a function $A:X\times Q\to L$.*

Definition 2.3. *Let R be a l-semiring and Q be a non empty set. A (Q,L)-fuzzy subset A of R is said to be a (Q,L)-**fuzzy l-subsemiring (QLFLSSR)** of R if the following conditions are satisfied:*

1. $A(x+y,q)\geq A(x,q)\wedge A(y,q)$,
2. $A(xy,q)\geq A(x,q)\wedge A(y,q)$,
3. $A(x\vee y,q)\geq A(x,q)\wedge A(y,q)$,

4. $A(x \wedge y,q) \geq A(x,q) \wedge A(y,q)$, *for all x and y in R and q in Q.*

Example 2.4. *Let* $(N,+,\cdot,\vee,\wedge)$ *be a l-semiring and* $Q = \{p\}$. *Then the* (Q,L)*-Fuzzy Set A of N is defined by*

$$A(x) = \begin{cases} 0.63 & \text{if } x \text{ is even} \\ 0.37 & \text{if } x \text{ is odd} \end{cases}$$

Clearly A is an (Q,L)*-Fuzzy l-subsemiring of l-semiring.*

Definition 2.5. *Let A and B be any two* (Q,L)*-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by* $A \times B$, *is defined as* $A \times B = \{\langle((x,y),q), A \times B((x,y),q)\rangle /$ *for all x in G and y in H and q in Q*$\}$, *where* $A \times B((x,y),q) = A(x,q) \wedge B(y,q)$.

Definition 2.6. *Let R and R′ be any two l-semirings and Q be a non empty set. Let* $f : R \to R'$ *be any function and A be a* (Q,L)*-fuzzy l-subsemiring in R,V be a* (Q,L)*-fuzzy l-subsemiring in* $f(R) = R'$, *defined by* $V(y,q) = \sup_{x=f^{-1}(y)} A(x,q)$, *for all x in R and y in R′ and q in Q. Then A is called a pre-image of V under f and is denoted by* $f^{-1}(V)$.

Definition 2.7. *Let R and R′ be any two l-semirings. Then the function* $f : R \to R'$ *is called a* **l-semiring homomorphism** *if* $f(x+y) = f(x)+f(y), f(xy) = f(x)f(y)$, *for all x and y in R.*

Definition 2.8. *Let R and R′ be any two l-semirings. Then the function* $f : R \to R'$ *is called a* **l-semiring anti- homomorphism** *if* $f(x+y) = f(y)+f(x), f(xy) = f(y)f(x)$, *for all x and y in R.*

Definition 2.9. *Let A be a* (Q,L)*-fuzzy l-subsemiring of a l-semiring R. Then* A^0 *is defined as* $A^0(x,q) = A(x,q)/A(0,q)$, *for all x in R and q in Q, where 0 is the identity element of R.*

Definition 2.10. *Let A be a* (Q,L)*-fuzzy subset of X. For* α *in L, a Q-level subset of A corresponding to* α *is the set* $A_\alpha = \{x \in X : A(x,q) \geq \alpha\}$.

3. Properties of Q-level l-subsemiring of (Q,L)-fuzzy l-subsemiring of a l-semiring

Theorem 3.1. *Let A be an* (Q,L)*-fuzzy l-subsemiring of a l-semiring R. Then for* α *in L such that* $A(0,q) \geq \alpha, A_\alpha$ *is a l-subsemiring of R.*

Proof. For all x and y in A_α,
we have, $A(x,q) \geq \alpha$ and $A(y,q) \geq \alpha$.
Now, $A(x+y,q) \geq A(x,q) \wedge A(y,q) \geq \alpha \wedge \alpha = \alpha$, which implies that $A(x+y,q) \geq \alpha$.
And, $A(xy,q) \geq A(x,q) \wedge A(y,q) \geq \alpha \wedge \alpha = \alpha$, which implies that $A(xy,q) \geq \alpha$.
Now $A(x \vee y,q) \geq A(x,q) \wedge A(y,q) \geq \alpha \wedge \alpha = \alpha$ which implies that
$A(x \vee y,q) \geq \alpha, A(x \wedge y,q) \geq A(x,q) \wedge A(y,q) \geq \alpha \wedge \alpha = \alpha$ which implies that $A(x \vee y,q) \geq \alpha$.
Therefore, $A(x+y,q) \geq \alpha, A(xy,q) \geq \alpha$ and $A(x \vee y,q) \geq \alpha, A(x \wedge y,q) \geq \alpha$.
Therefore $x+y, xy$ and $x \vee y, x \wedge y$ in A_α.
Hence A_α is a l-subsemiring of a l-semiring R. □

Theorem 3.2. *Let A be an* (Q,L)*-fuzzy l-subsemiring of a l-semiring R. Then two Q-level l-subsemiring* $A_{\alpha 1}, A_{\alpha 2}$ *and* α_1, α_2 *are in L such that* $A(0,q) \geq \alpha_1$ *and* $A(0,q) \geq \alpha_2$ *with* $\alpha_2 < \alpha_1$ *of A are equal iff there is no x in R such that* $\alpha_1 > A(x,q) > \alpha_2$.

Proof. Assume that $A_{\alpha 1} = A_{\alpha 2}$.
Suppose there exists x in R such that $\alpha_1 > A(x,q) > \alpha_2$.
Then $A_{\alpha 1} \subseteq A_{\alpha 2}$ implies x belongs to $A_{\alpha 2}$, but not in $A_{\alpha 1}$.
This is contradiction to $A_{\alpha 1} = A_{\alpha 2}$.
Therefore there is no $x \in R$ such that $\alpha_1 > A(x,q) > \alpha_2$.
Conversely if there is no $x \in R$ such that $\alpha_1 > A(x,q) > \alpha_2$.
Then $A_{\alpha 1} = A_{\alpha 2}$. □

Theorem 3.3. *Let R be a l-semiring and A be a fuzzy subset of R such that* A_α *be a Q-level l-subsemiring of R. If* α *in L is such that* $A(0,q) \geq \alpha$, *then A is an* (Q,L)*-fuzzy l-subsemiring of R.*

Proof. Let R be a l-semiring and Q be a non-empty set.
For $x,y \in R$ and $q \in Q$.
Let $A(x,q) = \alpha_1$ and $A(y,q) = \alpha_2$.

Case (i): If $\alpha_1 < \alpha_2$, then $x,y \in A_{\alpha 1}$.
As $A_{\alpha 1}$ is a Q-level l-subsemiring of R, $x+y$ and xy in $A_{\alpha 1}$.
Now $A(x+y,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A(x,q) \wedge A(y,q)$, which implies that $A(x+y,q) \geq A(x,q) \wedge A(y,q)$, for all $x,y \in R$ and $q \in Q$.
And, $A(xy,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A(x,q) \wedge A(y,q)$, implies that $A(xy,q) \geq A(x,q) \wedge A(y,q)$, for all $x,y \in R$ and $q \in Q$.
Also $A(x \vee y,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A(x,q) \wedge A(y,q)$, for all $x,y \in R$ and $q \in Q$.
And $A(x \wedge y,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A(x,q) \wedge A(y,q)$ for all $x,y \in R$ and $q \in Q$.

Case (ii): If $\alpha_1 > \alpha_2$, then $x,y \in A_{\alpha 2}$.
As $A_{\alpha 2}$ is a Q-level l-subsemiring of R, $x+y$ and xy in $A_{\alpha 2}$.
Now $A(x+y,q) \geq \alpha_2 = \alpha_2 \wedge \alpha_1 = A(y,q) \wedge A(x,q)$, which implies that $A(x+y,q) \geq A(x,q) \wedge A(y,q)$, for all $x,y \in R$ and $q \in Q$.
Also, $A(xy,q) \geq \alpha_2 = \alpha_2 \wedge \alpha_1 = A(y,q) \wedge A(x,q)$, implies that $A(xy,q) \geq A(x,q) \wedge A(y,q)$, for all $x,y \in R$ and $q \in Q$.
Also $A(x \vee y,q) \geq \alpha_2 = \alpha_1 \wedge \alpha_2 = A(x,q) \wedge A(y,q)$, for all $x,y \in R$ and $q \in Q$.
And $A(x \wedge y,q) \geq \alpha_2 = \alpha_1 \wedge \alpha_2 = A(x,q) \wedge A(y,q)$ for all $x,y \in R$ and $q \in Q$.

Case (iii): If $\alpha_1 = \alpha_2$. It is trivial.

In all the cases, A is an (Q,L)-fuzzy l-subsemiring of a l-semiring R. □

Theorem 3.4. *Let A be an (Q,L)-fuzzy l-subsemiring of a l-semiring R. The intersection of two Q-level l-subsemiring of A in R is also a Q-level l-subsemiring of A in R.*

Proof. Let α_1 and α_2 be in $L, A(0,q) \geq \alpha_1$ and $A(0,q) \geq \alpha_2$.

Case (i): If $\alpha_1 > A(x,q) > \alpha_2$, then $A_{\alpha 2} \subseteq A_{\alpha 1}$.
Therefore, $A_{\alpha 1} \cap A_{\alpha 2} = A_{\alpha 2}$, but $A_{\alpha 2}$ is a Q-level l-subsemiring of A.

Case (ii): If $\alpha_1 < A(x,q) < \alpha_2$, then $A_{\alpha 1} \subseteq A_{\alpha 2}$.
Therefore, $A_{\alpha 1} \cap A_{\alpha 2} = A_{\alpha 1,}$ but $A_{\alpha 1}$ is a Q-level l-subsemiring of A.

Case (iii): If $\alpha_1 = \alpha_2$, then $A_{\alpha 1} = A_{\alpha 2}$.

In all cases, intersection of any two Q-level l-subsemirings is also Q-level l-subsemiring of A. □

Theorem 3.5. *Let A be an (Q,L)-fuzzy l-subsemiring of a l-semiring R. If $\alpha_i \in L$, such that $A(0,q) \geq \alpha_i$ and $\{A_i\}_{i \in I}$ is a collection of Q-level l-subsemirings of A, then their intersection is also a Q-level l-subsemiring of A.*

Proof. It is trivial. □

Theorem 3.6. *Let A be an (Q,L)-fuzzy l-subsemiring of a l-semiring R. The union of any two Q-level l-subsemirings of A in R, is also a Q-level l-subsemiring of A in R.*

Proof. Let $\alpha_1, \alpha_2 \in L, A(0,q) \geq \alpha_1$ and $A(0,q) \geq \alpha_2$.

Case (i): If $\alpha_1 < A(x,q) < \alpha_2$, then $A_{\alpha 2} \subseteq A_{\alpha 1.}$
Therefore, $A_{\alpha 1} \cup A_{\alpha 2} = A_{\alpha 1}$, but $A_{\alpha 1}$ is a Q-level l-subsemiring of A.

Case (ii): If $\alpha_1 > A(x,q) > \alpha_2$, then $A_{\alpha 1} \subseteq A_{\alpha 2.}$
Therefore, $A_{\alpha 1} \cup A_{\alpha 2} = A_{\alpha 2}$, but $A_{\alpha 2}$ is a Q-level l-subsemiring of A.

Case (iii): If $\alpha_1 = \alpha_2$, then $A_{\alpha 1} = A_{\alpha 2.}$
In all cases, union of any two Q-level l-subsemiring is also a Q-level l-subsemiring of A.

□

Theorem 3.7. *Let A be an (Q,L)-fuzzy l-subsemiring of a l-semiring R. If $\alpha_i \in L, A(0,q) \geq \alpha_i$ and $\{A_i\}_{i \in I}$ is a collection of Q-level l-subsemirings of A, then their union is also a Q-level l-subsemiring of A.*

Proof. It is trivial. □

Theorem 3.8. *The homomorphic image of a Q-level l- subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring R is a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring R'.*

Proof. Let R and R' be any two semirings and $f : R \to R'$ be a homomorphism.
That is, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R.
Let $V = f(A)$, where A is an (Q,L)-fuzzy l-subsemiring of a l-semiring R.
Clearly V is an (Q,L)-fuzzy l-subsemiring of a l-semiring R'.
Let x and y in R, implies $f(x)$ and $f(y)$ in R'.
Let A is a Q-level l-subsemiring of A.
That is, $A(x,q) \geq \alpha$ and $A(y,q) \geq \alpha; A(x+y,q) \geq \alpha, A(xy,q) \geq \alpha$.
We have to prove that $f(A_\alpha)$ is a Q-level l-subsemiring of V.
Now, $V(f(x),q) \geq A(x,q) \geq \alpha$, which implies that $V(f(x),q) \geq \alpha$ and $V(f(y),q) \geq A(y,q) \geq \alpha$,
which implies that $V(f(y),q) \geq \alpha$ and $V(f(x)+f(y),q) = V(f(x+y),q) \geq A(x+y,q) \geq \alpha$, which implies that $V(f(x)+f(y),q) \geq \alpha$.
And, $V(f(x)f(y),q) = V(f(xy),q) \geq A(xy,q) \geq \alpha$, which implies that $V(f(x)f(y),q) \geq \alpha$.
Also, $V(f(x) \vee f(y),q) = V(f(x \vee y),q) \geq A(x \vee y,q) \geq \alpha$, which implies that $V(f(x) \vee f(y),q) \geq \alpha$ and $V(f(x) \wedge f(y),q) = V(f(x \wedge y),q) \geq A(x \wedge y,q) \geq \alpha$, which implies that $V(f(x) \wedge f(y),q) \geq \alpha$.
Therefore, $V(f(x)+f(y),q) \geq \alpha, V(f(x)f(y),q) \geq \alpha, V(f(x) \vee f(y),q) \geq \alpha V(f(x) \wedge f(y),q) \geq \alpha$.
Hence $f(A_\alpha)$ is a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring V of a l-semiring R'. □

Theorem 3.9. *The homomorphic pre-image of a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring R' is a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring R.*

Proof. Let R and R' be any two semirings and $f : R \to R'$ be a homomorphism.
That is, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all x and y in R.
Let $V = f(A)$, where V is an (Q,L)-fuzzy l-subsemiring of a l-semiring R'.
Clearly A is an (Q,L)-fuzzy l-subsemiring of a l-semiring R.
Let $f(x)$ and $f(y)$ in R', implies x and y in R.
Let $f(A)$ is a Q-level l-subsemiring of V.
That is, $V(f(x),q) \geq \alpha$ and $V(f(y),q) \geq \alpha; V(f(x)+f(y),q) \geq \alpha, V(f(x)f(y),q) \geq \alpha, V(f(x) \vee f(y),q) \geq \alpha, V(f(x) \wedge f(y),q) \geq \alpha$.
We have to prove that A_α is a Q-level l-subsemiring of A.
Now, $A(x,q) = V(f(x),q) \geq \alpha$,
implies that $A(x,q) \geq \alpha; A(y,q) = V(f(y),q) \geq \alpha$, implies that $A(y,q) \geq \alpha$ and $A(x+y,q) = V(f(x+y),q) = V(f(x)+f(y),q) \geq \alpha$, which implies that $A(x+y,q) \geq \alpha$.
And, $A(xy,q) = V(f(xy),q) = V(f(x)f(y),q) \geq \alpha$, Which implies that $A(xy,q) \geq \alpha$.
Also $A(x \vee y,q) = V(f(x \vee y),q) = V(f(x) \vee f(y),q) \geq \alpha$, which implies that $A(x \vee y,q) \geq \alpha, A(x \wedge y,q) = V(f(x \wedge y),q) = V(f(x) \wedge f(y),q) \geq \alpha$, which implies that $A(x \wedge y,q) \geq \alpha$.
Therefore, $V(f(x)+f(y),q) \geq \alpha, V(f(x)f(y),q) \geq \alpha, V(f(x) \vee f(y),q) \geq \alpha, V(f(x) \wedge f(y),q) \geq \alpha$.

Hence, A_α is a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring A of R. □

Theorem 3.10. *The anti-homomorphic image of a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring R is a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring R'.*

Proof. Let R and R' be any two semirings and $f: R \to R'$ be an anti-homomorphism.
That is, $f(x+y) = f(y)+f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R.
Let $V = f(A)$, where A is an (Q,L)-fuzzy l-subsemiring of R.
Clearly V is an (Q,L)-fuzzy l-subsemiring of R'.
Let x and y in R, implies $f(x)$ and $f(y)$ in R'.
Let A is a Q-level l-subsemiring of A.
That is $A(x,q) \geq \alpha$ and $A(y,q) \geq \alpha . A(y+x,q) \geq \alpha, A(yx,q) \geq \alpha, A(y \vee x,q) \geq \alpha, A(y \wedge x,q) \geq \alpha$.
We have to prove that $f(A)$ is a Q-level l-subsemiring of V.
Now, $V(f(x),q) \geq A(x,q) \geq \alpha$,
which implies that $V(f(x),q) \geq \alpha; V(f(y),q) \geq A(y,q) \geq \alpha$, which implies that $V(f(y),q) \geq \alpha$.
Now, $V(f(x)+f(y),q) = V(f(x)+f(y),q) = V(f(y+x),q) \geq A(y+x,q) \geq \alpha$, which implies that, $V(f(x)+f(y),q) \geq \alpha$.
And, $V(f(x)f(y),q) = V(f(yx),q) \geq A(yx,q) \geq \alpha$, which implies that $V(f(x)f(y),q) \geq \alpha$.
Also, $V(f(x) \vee f(y),q) = V(f(y \vee x),q) \geq A(y \vee x,q) \geq \alpha$, which implies that $V(f(x) \vee f(y),q) \geq \alpha$.
And, $V(f(x) \wedge f(y),q) = V(f(y \wedge x),q) \geq A(y \wedge x,q) \geq \alpha$, which implies that $V(f(x) \wedge f(y),q) \geq \alpha$.
Therefore, $V(f(x)+f(y),q) \geq \alpha, V(f(x)f(y),q) \geq \alpha$ and $V(f(x) \vee f(y),q) \geq \alpha, V(f(x) \wedge f(y),q) \geq \alpha$.
Hence $f(A_\alpha)$ is a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring V of R'. □

Theorem 3.11. *The anti-homomorphic pre-image of a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring R' is a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring of a l-semiring R.*

Proof. Let R and R' be any two l-semirings and $f: R \to R'$ be an anti-homomorphism.
That is, $f(x+y) = f(y)+f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R.
Let $V = f(A)$, where V is an (Q,L)-fuzzy l-subsemiring of a l-semiring R'.
Clearly A is an (Q,L)-fuzzy l-subsemiring of a l-semiring R.
Let $f(x)$ and $f(y)$ in R', implies x and y in R.
Let $f(A_\alpha)$ is a Q-level l-subsemiring of V.
That is, $V(f(x),q) \geq \alpha$ and $V(f(y),q) \geq \alpha; V(f(y)+f(x),q) \geq \alpha, V(f(y)f(x),q) \geq \alpha, V(f(y) \vee f(x),q) \geq \alpha, V(f(y) \wedge f(x),q) \geq \alpha$.
We have to prove that A_α is a Q-level subsemiring of A.
Now, $A(x,q) = V(f(x),q) \geq \alpha$, which implies that $A(x,q) \geq \alpha; A(y,q) = V(f(y),q) \geq \alpha$, which implies that $A(y,q) \geq \alpha$.
Now, $A(x+y,q) = V(f(x+y),q) = V(f(y)+f(x),q) \geq \alpha$, which implies that $A(x+y,q) \geq \alpha$.
And, $A(xy,q) = V(f(xy),q) = V(f(y)f(x),q) \geq \alpha$, which implies that $A(xy,q) \geq \alpha$.
Also $A(x \vee y,q) = V(f(x \vee y),q) = V(f(y) \vee f(x),q) \geq \alpha$, which implies that $A(x \vee y,q) \geq \alpha$ and $A(x \wedge y,q) = V(f(x \wedge y),q) = V(f(y) \wedge f(x),q) \geq \alpha$, which implies that $A(x \wedge y,q) \geq \alpha$.
Therefore, $V(f(x)+f(y),q) \geq \alpha, V(f(x)f(y),q) \geq \alpha$ and $V(f(x) \vee f(y),q) \geq \alpha, V(f(x) \wedge f(y),q) \geq \alpha$.
Hence A_α is a Q-level l-subsemiring of an (Q,L)-fuzzy l-subsemiring A of R. □

Theorem 3.12. *If A is an (Q,L)-fuzzy l-subsemiring of a l-semiring R then $H = \{x | x \in R : A(x,q) = 1\}$ is either empty or is a l-subsemiring of R.*

Proof. If no element satisfies this condition, then H is empty.
If x and y in H, then $A((x+y),q) \geq A(x,q) \wedge A(y,q) = 1 \wedge 1 = 1$.
Therefore, $A((x+y),q) = 1$.
And, $A(xy,q) \geq A(x,q) \wedge A(y,q) = 1 \wedge 1 = 1$.
Therefore, $A(xy,q) = 1$.
Also $A((x \vee y),q) \geq A(x,q) \wedge A(y,q) = 1 \wedge 1 = 1$.
Therefore,
$A((x \vee y),q) = 1$ and $A((x \wedge y),q) \geq A(x,q) \wedge A(y,q) = 1 \wedge 1 = 1$.
Therefore, $A((x \wedge y),q) = 1$.
We get $x+y, xy, x \vee y, x \wedge y$ in H.
Therefore, H is a l-subsemiring of R.
Hence H is either empty or is a l-subsemiring of R. □

Theorem 3.13. *If A be an (Q,L)-fuzzy l-subsemiring of a l-semiring R, then if $A((x+y),q) = 0$, then either $A(x,q) = 0$ or $A(y,q) = 0$, for all x and y in R and q in Q.*

Proof. Let x and y in R and q in Q.
By the definition $A((x+y),q) \geq A(x,q) \wedge A(y,q)$, which implies that $0 \geq A(x,q) \wedge A(y,q)$.
Therefore, either $A(x,q) = 0$ or $A(y,q) = 0$. □

Theorem 3.14. *Let A be a (Q,L)-fuzzy l-subsemiring of a l-semiring R. Then A^0 is a (Q,L)-fuzzy l-subsemiring of a l-semiring R.*

Proof. For any x in R and q in Q, we have

$$\begin{aligned} A^0(x+y,q) =& A(x+y,q)/A(0,q) \\ \geq& [1/A(0,q)]\{A(x,q) \wedge A(y,q)\} \\ =& [A(x,q)/A(0,q)] \wedge [A(y,q)/A(0,q)] \\ =& A^0(x,q) \wedge A^0(y,q). \end{aligned}$$

That is $A^0(x+y,q) \geq A^0(x,q) \wedge A^0(y,q)$ for all x and y in R and q in Q.
$A^0(xy,q) = A(xy,q)/A(0,q) \geq [1/A(0,q)]\{A(x,q) \wedge A(y,q)\} = [A(x,q)/A(0,q)] \wedge [A(y,q)/A(0,q)] = A^0(x,q) \wedge A^0(y,q)$.
That is $A^0(xy,q) \geq A^0(x,q) \wedge A^0(y,q)$ for all x and y in R and

q in Q.

$$\begin{aligned} A^0(x\vee y,q) =& A(x\vee y,q)/A(0,q) \\ \geq& [1/A(0,q)]\{A(x,q)\wedge A(y,q)\} \\ =& [A(x,q)/A(0,q)]\wedge[A(y,q)/A(0,q)] \\ =& A^0(x,q)\wedge A^0(y,q). \end{aligned}$$

That is $A^0(x\vee y,q)\geq A^0(x,q)\wedge A^0(y,q)$ for all x and y in R and q in Q.

$$\begin{aligned} A^0(x\wedge y,q) =& A(x\wedge y,q)/A(0,q) \\ \geq& [1/A(0,q)]\{A(x,q)\wedge A(y,q)\} \\ =& [A(x,q)/A(0,q)]\wedge[A(y,q)/A(0,q)] \\ =& A^0(x,q)\wedge A^0(y,q). \end{aligned}$$

That is $A^0(x\wedge y,q)\geq A^0(x,q)\wedge A^0(y,q)$ for all x and y in R and q in Q.
Hence A^0 is a (Q,L)-fuzzy l-subsemiring of a l-semiring R. □

Theorem 3.15. *Let A be an (Q,L)-fuzzy l-subsemiring of a l-semiring R.A^+ be a fuzzy set in R defined by $A^+(x,q)=A(x,q)+1-A(0,q)$, for all x in R and q in Q, where 0 is the identity element. Then A^+ is an (Q,L)-fuzzy l-subsemiring of a l-semiring R.*

Proof. Let x and y in R and q in Q.
We have, $A^+(x+y,q)=A(x+y,q)+1-A(0,q)\geq\{A(x,q)\wedge A(y,q)\}+1-A(0,q)=\{A(x,q)+1-A(0,q)\}\wedge\{A(y,q)+1-A(0,q)\}=A^+(x,q)\wedge A^+(y,q)$, which implies that $A^+(x+y,q)\geq A^+(x,q)\wedge A^+(y,q)$ for all x,y in R and q in Q.
$A^+(xy,q)=A(xy,q)+1-A(0,q)\geq\{A(x,q)\wedge A(y,q)\}+1-A(0,q)=\{A(x,q)+1-A(0,q)\}\wedge\{A(y,q)+1-A(0,q)\}=A^+(x,q)\wedge A^+(y,q)$.
Therefore, $A^+(xy,q)\geq A^+(x,q)\wedge A^+(y,q)$ for all x,y in R and q in Q.
Also

$$\begin{aligned} A^+(x\vee y,q) =& A(x\vee y,q)+1-A(0,q) \\ \geq& \{A(x,q)\wedge A(y,q)\}+1-A(0,q) \\ =& \{A(x,q)+1-A(0,q)\}\wedge\{A(y,q)+1-A(0,q)\} \\ =& A^+(x,q)\wedge A^+(y,q), \end{aligned}$$

which implies that $A^+(x\vee y,q)\geq A^+(x,q)\wedge A^+(y,q)$ for all x,y in R and q in Q.
$A^+(x\wedge y,q)=A(x\wedge y,q)+1-A(0,q)\geq\{A(x,q)\wedge A(y,q)\}+1-A(0,q)=\{A(x,q)+1-A(0,q)\}\wedge\{A(y,q)+1-A(0,q)\}=A^+(x,q)\wedge A^+(y,q)$, which implies that $A^+(x\wedge y,q)\geq A^+(x,q)\wedge A^+(y,q)$ for all x,y in R and q in Q.
Hence A^+ is an (Q,L)-fuzzy l-subsemiring of a l-semiring R. □

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