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Prime and odd prime graphs

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Abstract
An odd prime labeling of a graph G is a labeling of the vertices with distinct positive integers from the set $\{1,3,5,\ldots,2n-1\}$, where n is the number of vertices such that the labels of vertices are relatively prime to the labels of all adjacent vertices. Prime and an odd prime graphs for one point union of graphs, graphs with identification of some graphs and some graphs have been described in this paper.

Keywords
Prime labeling, relatively prime, star, path, one point union of graphs, odd prime.

AMS Subject Classification
05C78.

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1. Introduction

Consider connected graphs are finite, simple and undirected. For more details the reader is referred to [1]. The symbols $|V(G)|$ and $|E(G)|$ will denote the number of vertices and edges of G respectively. In 1980, Entringer originated the prime labeling, it was first studied by Tout, Dabboucy and Howalla [8] in 1982. Defined this labeling as follows. A graph has a prime labeling if its vertices of G can be labeled distinctly with first n positive integers such that each pair of adjacent vertices are relatively prime. Such a prime labeling is said to be prime graph. An odd prime labeling is a variation of prime labeling. In 2018, Prajapati et.al.[3] have introduced the concept of odd prime graphs. They proved that certain class of graphs such as path, complete graph, complete bipartite graphs under certain conditions, wheel, helm, fan, friendship, Petersen graph $P(n,2)$ are odd prime graphs. They [4] also showed that graphs obtained by duplicating each vertex by an edge and each edge by a vertex in path, star, cycle and wheel are odd prime graphs. Further, they [5] investigated various snake graphs are odd prime. Youssef et. al., [9] studied some families of odd prime graphs and some necessary conditions for a graph to be odd prime.

A bijection g from vertex set $V(G)$ to $\{1,3,5,\ldots,2n-1\}$ of a graph G is called an odd prime labeling of G if for each edge $e=uv\in E(G)$, such that $\gcd(g(u),g(v))=1$. A graph which admits that labeling is said to be an odd prime graph. We will show that few families of one point union of graphs, graphs with identification of some graphs and some graphs are prime and an odd prime Here we are listing some preliminary definitions. The following definitions are taken from [2], [6], [7]. The flower graph Fl_n is the graph obtained from a helm H_n by joining each pendent vertex to the apex of helm. The star graph is a complete bipartite graph $K_{1,m}$, where m represents the number of vertices and S_m has $m-1$ edges. A fan graph f_m is defined to be the join of the complete graph K_1 and path P_n. i.e, $f_m=P_n\odot K_1$. A friendship graph F_n is a one point union of n cycles of C_3. The sunlet graph is a graph of $2n$ vertices is obtained by attaching n pendent edges to the cycle C_n and it is denoted by S_n. The Lilly graph $L(m,n),(m,n\geq 2)$ can be constructed by two star graphs $2K_{1,m},(m\geq 2)$ joining two path graphs $2P_n,(n\geq 2)$ with sharing a common vertex. i.e., $L(m,n)=2K_{1,m}\odot 2P_n$. An udukkai graph $A(m,n),(m,n\geq 2)$ can be constructed by two fan graphs $2f_m,(m\geq 2)$ joining two path graphs $2P_n,(n\geq 2)$ with sharing a common vertex. i.e., $A(m,n)=2f_m\odot 2P_n$. The butterfly graph $BF(m,n)$ is a graph obtained from two cycles C_n of the same order, sharing a common vertex with an arbitrary number m of pendent edges attached at the common vertex. An Octopus graph $O(m,n),(m,n\geq 2)$ can be constructed by a fan graph $f_m,(m\geq 2)$ joining a star graph $K_{1,n}$ with sharing a common vertex. i.e., $O(m,n) = f_m \odot K_{1,n}$. The drums graph $D(m,n), (m \geq 3, n \geq 2)$ can be constructed by two cycle graphs $2C_m$, $m \geq 3$ joining two path graphs $2P_n, n \geq 2$ with sharing common vertex. i.e., $D(m,n) = 2C_m + 2P_n$. The planter graph $R(m,n), (m \geq 2, n \geq 3)$ can be constructed by joining a fan graph $f_m, (m \geq 2)$ and a cycle graph $C_n, (n \geq 3)$ with sharing a common vertex $R(m,n) = f_m \odot C_n$.

A triangular snake is the graph obtained from a path $x_1, x_2, \ldots, x_p$ by joining x_r and x_{r+1} to a new vertex y_r for $r = 1, 2, \ldots, p-1$.. The $k-$ polygonal book, denoted $B_{k,n}$, is formed by n copies of a $k-$ polygonal sharing a single edge. Each k - polygonal is referred to as a page of the book graph. A circular ladder graph CL_n is a 3-regular simple graph consists of two concentric n-cycle in which each of the n corresponding vertices joining by an edge. Barycentric subdivision is the graph obtained by insering a vertex of degree two into every edge of original graph. Consider barycentric subdivision of cycle and join each newly inserted vertices of incident edges by an edge. Denote the new graph by $C_n\,(C_n)$ as it look like C_n inscribed in C_n. A graph G in which a vertex is distinguished from other vertices is called a rooted graph and the vertex is called the root of G. Let G be rooted graph. The graph $G^{(n)}$ obtained by identifying the roots of n copies of G is called a one vertex union of the n copies of G. The bistar $B_{m,n}$ is the graph obtained by joining the apex vertices of two copies of star $K_{1,m}$ and $K_{1,n}$ by an eadge.

2. Prime Graphs

In this section, we investigate prime graphs of few graphs.

Theorem 2.1. *If $G(l,m)$ has prime graph, then there exists a graph from the class $G \odot (P_n + 2K_1)$ admits prime graph, when $l+n+1$ is prime.*

Proof. Let $G = (l,m)$ be prime graph with l vertices and m edges. Define a one to one and onto function. $g_1 : V_1 \rightarrow \{1, 2, \ldots, l\}$ with the property that given any two adjacent vertices are prime labels. Consider the graph $(P_n + 2K_1)$ with vertex set $\{x, y, z_s : 1 \leq s \leq n\}$ and edge set $\{xz_s, yz_s : 1 \leq s \leq n\} \cup \{z_s z_{s+1} : 1 \leq s \leq n-1\}$ We superimpose one of the vertex say x of $(P_n + 2K_1)$ on selected vertex v_1 in G with $g_1(v_1) = 1$ in G. Now we define a new graph $G^* = G \odot (P_n + 2K_1)$ with vertex set $V^* = V_1 \cup \{x, y, z_s : 1 \leq s \leq n\}$ and edge set $E^* = E_1 \cup \{xz_s, yz_s : 1 \leq s \leq n\} \cup \{z_s z_{s+1} : 1 \leq s \leq n-1\}$. Also $|V(G^*)| = l + n + 1$.

Define *a* bijective function $h_1 : V^* \rightarrow \{1, 2, 3, \ldots, l, l+1, \ldots, l+n+1\}$ by $h_1(v) = g_1(v)$ for all $v \in V(G), h_1(x) = h_1(v_1) = 1, h_1(z_s) = l + s$ for $l \leq s \leq n, h_1(y) = l+n+1$ which is prime. We have to prove that G^* is prime graph. Earlier, G is prime graph, it is enough to prove that for any two adjacent vertices $uv \in E^*$, which is not in G, the numbers $h_1(u)$ and $h_1(v)$ are relatively prime. Hence $G \odot (P_n + 2K_1)$ admits prime graph for $l+n+1$ is prime. ◻

Theorem 2.2. *If G has prime graph, then there exists a graph from the class $G \odot F_p^t$ (where F_p^t the one point union of t copies of flower pot graph) admits prime graph.*

Proof. Let $G(m,n)$ be prime graph with m vertices and n edges. Define a bijective function $g_2 : V_2 \rightarrow \{1, 2, 3, \ldots, m\}$ with the property that given any two adjacent vertices are relatively prime labels. Let F_p^t be the one point union of flower pot graph with vertex set $V_2\left(F_p^t\right) = \left\{d_r^t, e_S^t : 1 \leq r \leq k, 1 \leq s \leq l\right\}$ and $E_2\left(F_p^t\right) = \{d_1 e_s^t : 1 \leq s \leq l\} \cup \left\{d_r^t d_{r+1}^t : 1 \leq r \leq k-1\right\} \cup \left\{d_1 d_k^t\right\}$. We layover one of the vertex, say d_1 of F_p^t on selected vertex v_1 in G with $g_2(v_1) = 1$. Now we define a graph $G^* = G \cup F_p^t$ with vertex set $V^* = V_2 \cup \{d_r^t, e_s^t : 1 \leq r \leq k, 1 \leq s \leq l\}$ and $E^* = E_2 \cup \{d_1 e_s^t : 1 \leq s \leq l\} \cup \left\{d_r^t d_{r+1}^t : 1 \leq r \leq k-1\right\} \cup \left\{d_1 d_k^t\right\}$. Also $|V(G^*)| = m + t(k+l) - 1$. Define a bijective function $h_2 : V^* \rightarrow \{1, 2, 3, \ldots, m, m+1, \ldots, m + t(k+l) - 1\}$ by $h_2(v) = g_2(v)$ for all $v \in V(G), h_2(v_1) = g_2(v_1) = h_2(d_1) = 1$ $h_2(d_r^t) = m + (t-1)(k+l-1) + r$ for $2 \leq r \leq k$ and $h_2(e_s^t) = m + (t-1)(k+l-1) + k + s$ for $1 \leq s \leq l$. We have to prove that G^* is prime graph. Already, G is prime graph, it is sufficient to prove that for any two adjacent vertices $uv \in E^*$, which is not in G, the numbers $h_2(u)$ and $h_2(v)$ are relatively prime. Hence $G \odot F_p^t$ admits prime graph, where F_p^t is the one point union of t copies of flower pot graph. ◻

Theorem 2.3. *If $G(l,m)$ has prime graph, then there exists a graph $G \odot L(S_n)$ admits prime graph, where $L(S_n)$ is the line graph of sunlet graph for $n \geq 3$.*

Proof. Let $G(l,m)$ be a prime graph with 1 odd vertices and m edges. Define an one-to-one and onto function $g_3 : V_3 \rightarrow \{1, 2, \ldots, l\}$ with the property that given any two adjacent vertices are prime labels. Consider the line graph of sunlet graph $L(S_n)$ with vertex set $\{r_t, s_t : 1 \leq t \leq n\}$ and edge set $\{r_t r_{t+1}, r_t s_t, s_t r_{t+1} : 1 \leq t \leq n-1\} \cup \{r_1 r_n\} \cup \{r_1 s_n\} \cup \{r_n s_n\}$. We layover one of the vertex say r_1 of $L(S_n)$ on selected vertex q_1 in G with $g_3(q_1) = 1$. Now we define a new graph $G^* = G \odot L(S_n)$ with vertex set $V^* = V_3 \cup \{r_t, s_t : 1 \leq t \leq n\}$ and edge set $E^* = E_3 \cup \{r_t r_{t+1}, r_t s_t, s_t r_{t+1} : 1 \leq t \leq n-1\} \cup \{r_1 r_n\} \cup \{r_1 s_n\} \cup \{r_n s_n\}$. Also $|V(G^*)| = l + 2n$. Define a bijective function $h_3 : V^* \rightarrow \{1, 2, 3, \ldots, l, l+2n\}$ by $h_3(v) = g_3(v)$ for all $v \in V(G), h_3(q_1) = h_3(r_1) = 1$. Consider the following cases

Case 1: l is odd. $h_3(s_1) = l + 1$ which is even. For each $2 \leq t \leq n, h_3(r_t) = l + 2(t-1)$ and $h_3(s_t) = l + 2(t-1)$

Case 2: $l$is even and either $l+1$ or $l+2n-1$ is prime. $h_3(r_2) = l+1, h_3(s_1) = l+2n-1, h_3(s_2) = l+2$, for each $2 \leq t \leq n, h_3(r_t) = l + 2t - 3$, for each $3 \leq t \leq n$ $h_3(s_t) = l + 2t - 2$. We have to prove that G^* is prime graph. Earlier, G is a prime graph, it is sufficient to prove that for any two adjacent vertices $uv \in E^*$, which is not in G, the numbers $h_3(u)$ and $h_3(v)$ are relatively prime. Hence $G \odot L(s_n)$ admits prime graph when l is odd and either $l+1$ or $l+2n-1$ is prime. ◻

Theorem 2.4. *If $G(l,m)$ has prime graph, then there exists a graph from the class $G \odot G_n$ admits a prime graph.*

Proof. Let $G(l,m)$ be prime graph wih l odd vertices and m edges. Define an one to one and onto function $g_4 : V_4 \rightarrow \{1,2,\ldots,l\}$ with the property that given any two adjacent vertices are prime labels. Consider the gear graph G_n with vertex set $\{x, x_s : 1 \leq s \leq 2n\}$ and edge set $\{xx_s : s \text{ is odd}\} \cup \{x_s x_{s+1} : 1 \leq s \leq 2n-\ 1\} \cup \{x_1 x_{2n}\}$. We superimpose one of the vertex say x of G_n on selected vertex v_1 in G with $g(v_1) = 1$. Now we define a new graph $G^* = G \odot G_n$ with vertex set $V^* = V_4 \cup \{x, x_s : 1 \leq s \leq n\}$ and edge set $E^* = E_4 \cup \{xx_s : sisodd\} \cup \{x_s x_{s+1} : 1 \leq\ s \leq 2n-1\} \cup \{x_1 x_{2n}\}$. Also $|V(G^*)| = l + 2n$.

Define a bijective function $h_4 : V^* \rightarrow \{1,2,3,\ldots,l,l+1,\ldots,l+2n\ \}$ by $h_4(v) = g_4(v)$ for all $v \in V(G), h_4(x) = g_4(v_1) = 1, h_4(x_s) = l+s$ for $1 \leq s \leq 2n$. Consider the following cases

Case 1: l is even Suppose $h_4(x_1) = l+1$ is odd Consider the following subcases.

Subcase 1a. $l+1$ is prime. Suppose that $h_4(x_1) = l+1$ is prime. Then $h_4(x_{2n}) = l+2n$ is even label, which is not a multiple of $l+1$.

Case 2: l is odd. Suppose $h_4(x_1) = l+1$ is even. Then $h_4(x_{2n}) = l+2n$ must be prime number. We have to prove that G^* is prime graph. Already, G is prime graph, it is enough to prove that for any two adjacent vertices $uv \in E^*$, which is not in G, the numbers $h_4(u)$ and $h_4(v)$ are relatively prime. Here $G \odot G_n$ is prime graph with $l+1$ and $l+2n$ are prime number.

A snake graph $C^m_{k,1}$ is the fusion of $mk-$ cycles, C_k such that for $2 \leq j \leq m$, a shared vertex is called the vertebrae, denoted r_j, results from the fusion where a minimal path of length 1 joins r_{j-1} and r_j. □

Theorem 2.5. *If $G(l,m)$ has prime graph, then there exists a graph from the class $G \odot C^m_{k,1}$ admits prime.*

Proof. Let $G(l,m)$ be prime graph with l odd vertices and m edges. Define a bijective function $g_5 : V_5(G) \rightarrow \{1,2,3,\ldots,l\}$ with the property that given any two adjacent vertices are relatively prime labels. Let $C^m_{k,1}$ be a fusion of identical cycles with minimum distance 1 joins r_{q-1} and r_{qk} in general r_q, r_{s-1} and r_s with vertex set $V_5\left(C^m_{k,1}\right) = \{r_s : 1 \leq s \leq mk-(m-1)\}$ and edge set $E_5\left(C^m_{k,1}\right) = \{r_s r_{s+1} : 1 \leq s \leq mk-1-(m-1)\} \cup \{r_1 r_k\} \cup \left\{r_{(s-1)k} r_{sk-(s-1)} : 2 \leq s \leq m\right\}$. We fusion one of the vertex say r_1 of $C^m_{k,1}$ on selected vertex q_1 of G. Now we define a new graph $G^* = G \odot C^m_{k,1}$ with vertex set $V^* = V_5 \cup \{r_s : 1 \leq s \leq mk-(m-1)\}$ and edge set $E^* = E_5 \cup \{r_s r_{s+1} : 1 \leq s \leq mk-1-(m-1)\} \cup \{r_1 r_k\} \cup \left\{r_{(s-1)k} r_{sk-(s-1)}\ : 2 \leq s \leq m\right\}$. Also $|V(G)| = l + mk - (m-1)$. Define a bijective function $h_5 :\ V^* \rightarrow \{1,2,3\ldots,l,l+1,\ldots,l+m(k-1)\}$ by $h_5(v) = g_5(v)$ for all $v \in V(G)$ $h_5(q_1) = h_5(r_1) = 1$ $h_5(r_s) = l+s-1$ for $(m-1)k+1 \leq s \leq mk-(m-1)$ We have to prove that G^* is prime graph. Already, G is prime graph, it is sufficient to prove that for any two adjacent vertices $uv \in E^*$, which is not in G the numbers $h_5(u)$ and $h_5(v)$ are relatively prime. Hence $G \odot C^m_{k,1}$ admits prime graph. □

Theorem 2.6. *The One vertex union of q copies of snake graph $\left(C^m_{k,1}\right)^q$ is prime.*

Proof. Let $G = \left(C^m_{k,1}\right)^q$ be the one point union of q copies of snake graph $C^m_{k,1}$. Then vertex set $V_6(G) = \left\{r^q_d : 1 \leq d \leq q(mk\ -(m-1))\right\}$ and edge *set* $E_6(G) = \left\{r^q_d r^q_{d+1} ; 1 \leq d \leq q(mk-1\ -(m-1))\right\} \cup \left\{r_1 r^q_k\right\} \cup \left\{r^q_{dk-k} r^q_{dk-(d-1)} : 2 \leq d \leq mq\right\}$. Also $|V(G)| = q[mk-(m-1)]$. Define a bijective function $h_6 : V_6(G) \rightarrow \{1,2,3,\ldots,q(mk-(m-1))\} by\ h_6\left(h^q_1\right) = 1, h_6\left(r^q_e\right) = e$ for $q = 1, e \in \{2,3,\ldots,m\}, q \geq 2, h_6\left(r^q_e\right) = (q-1)mk - ((q-1)m-1) + (e-1)$ for $e \in \{2,3,\ldots,k,k+1,2k,2k+1,\ldots mk\}$ or $2 \leq e \leq mk$ It is easily verified that for any two adjacent vertices $uv \in E(G)$, the numbers $h_6(u)$ and $h_6(v)$ are relatively prime. Hence $G = \left(C^m_{k,1}\right)^q$ admits prime graph. □

Theorem 2.7. *If G has prime graph, then there exists a graph from the class $G \cup (B_n \cup B_n \ldots B_n)$ admits prime graph.*

Proof. Let $G(l,m)$ be prime graph with bijective function $g_7 : V_7(G) \rightarrow \{1,2,3,\ldots,l\}$ satisfying the property of prime graph. Consider the graph $H = (B_n \cup B_n \ldots \cup B_n)$ be the union of k copies of Brush graph. Then vertex set $V_7(H) = \left\{r^k_t, s^k_t : 1 \leq t \leq n\right\}$ and $E(H) = \left\{r^k_t, s^k_t : 1 \leq t \leq n\right\} \cup \left\{r^k_t r^k_{t+1} : 1 \leq t \leq n-1\right\}$. Let $G^* = G \cup H$ with $V(G^*) = V_7(G) \cup V_7(H)$ and $E(G^*) = E_7(G) \cup E_7(H)$. Clearly, $|V(G^*)| = l + 2kn$. Define a bijective function $h_7 : V(G^*) \rightarrow \{1,2,3,\ldots,l,l+1,\ldots,l+2kn\}$ by $g_7(v) = h_7(v)$ for all $v \in V(G)$. Consider the following cases.

Case 1: l is even. $h_7\left(r^k_t\right) = l + 2(k-1)n + 2t - 1$ for $1 \leq t \leq n$ $h_7\left(s^k_t\right) = l + 2(k-1)n + 2t$ for $1 \leq t \leq n$.

Case 2: l is odd. $h_7\left(r^k_t\right) = l + 2(k-1)n + 2t$ for $1 \leq t \leq n$ $h_7\left(s^k_t\right) = l + 2(k-1)n + 2t - 1$ for $1 \leq t \leq n$ In order to show that G^* is prime graph. Clearly, G is prime graph, it is enough to show that for any two adjacent vertices $uv \in E^*$, which is not in G, the numbers $h_7(u)$ and $h_7(v)$ are relatively prime. Hence $G^* = G \cup (B_n \cup B_n \ldots \cup B_n)$ admits prime graph for all n. □

Theorem 2.8. *The barycentric cycle attached by pendant edge at each vertex of a graph is prime.*

Proof. Let G be the brycentric cycle $C_n(C_n)$ attached by pendent edge at each vertex of graph is prime. Then $V_8(G) = \{u_w, u'_w, v_w, v'_w : 1 \leq w \leq n\}$ and $E_8(G) = \{u_w v_w, u_w u'_w, v_w v'_w : 1 \leq w \leq n\} \cup \{u_w u_{w+1}, v_w u_{w+1} : 1 \leq w \leq n-\ 1\} \cup \{u_1 u_n\} \cup \{u_1 v_n\}$.

Here, $|V(G)| = 3n$ and $|E(G)| = 4n$. Define a bijective function $h_8 : V(G) \rightarrow \{1,2,\ldots,3n\}$ by $h_8(u_w) = 2w-1$ for $1 \leq w \leq n, h_8(v_w) = 2w$ for $1 \leq w \leq n$ The remaining even labels $\{2n+2, 2n+4, \ldots, 3n(n \text{ even })\}$ are labeled by $u'_W s$. If $h_8(u_w)$ and $h_8(u'_w)$ are not relatively prime, then interchange $h_8(u'_w)$ and $h_8\left(u'_{w-1}\right)$. Similarlhy, the remaining odd labels $\{2n+1, 2n+3, \ldots, 3n(n \text{ odd })\}$ are labeled by $u'_W s$. If $h_8(v_w)$ and $h_8(v'_w)$ are not relatively prime, then interchange $h_8(v'_w)$ and $h_8\left(v'_{w-1}\right)$. It is easily verified that for any two adjacent vertices $uv \in E(G)$, the numbers $h_8(u)$ and $h_8(v)$ are relatively prime. Hence the barycentric cycle $C_n\,(C_n)$ attached by pendent edge at each vertex of graph is prime. □

Theorem 2.9. *If $G(l,m)$ has prime graph, then there exists graph from the class $G\theta B_{2n}$ admits prime graph.*

Proof. Let $G(l,m)$ be prime graph with l vertices and m edges. Define a bijective function $g_9 : V_9 \to \{1,2,\ldots,l\}$ with the property that given any two adjacent vertices are relatively prime labels. Consider the rectangular book graph B_{2n} with vertex set $V_9\,(B_{2n}) = \{u,v,u_t,v_t : 1 \le t \le n\}$ and edge set $E_9\,(B_{2n}) = \{uv,uu_t,vv_t,u_tv_t : 1 \le t \le n\}$. We layover one of the edge say uv of B_{2n} on selected edge xy in G with $g_9(x) = 1$ and $g_9(y) = 2$. Now we define a new graph $G^* = G\theta B_{2n}$ with vertex set $V^* = V_9(G) \cup V_9\,(B_{2n})$ and $E^* = E_9(G) \cup E_9\,(B_{2n})$. Also, $|\,V\,(G^*\,| = l+2n$. Define a bijective function $h_9 : V^* \to \{1,2,3,\ldots,l,l+1,\ldots,l+2n\}$ by $h_9(v) = g_9(v)$ for also $v \in V(G), h_9(u) = 1, h_9(v) = 2$, for each $1 \le t \le n, h_9\,(u_t) = l+2t$ and $h_9\,(v_t) = l+2t-1$ We have to prove that G^* is prime graph, it is sufficient to prove that for any two adjacent vertices $pq \in E^*$, which is not in G, the numbers $h_9(p)$ and $h_9(q)$ are relatively prime. Hence $G\theta B_{2n}$ admits prime graph. Delete the edge $\{u_tv_t : 1 \le t \le n\}$ from the graph $G\theta B_{2n}$. Thus we have the following corollary □

Corollary 2.10. *If $G(l,m)$ has prime graph, then there exists a graph from the class $G\theta B_{n,n}$ admits prime graph.*

Theorem 2.11. *If G has prime graph, then there exists a graph from the class $G \odot C^k_{n_e}$ (where $C^k_{n_e}$ is the one point union of k copies of cycleC_{n_e}) admits prime.*

Proof. Let $G(l,m)$ be prime graph with l vertices and m edges. Define a bijective function $g_{10} : V_{10} \to \{1,2,\ldots,l\}$ with the property that given any two adjacent vertices are prime labels. Consider the one point union of k copies of cycle C_{n_e}, that is $H = C^k_{n_e}$.

Then vertex set $V_{10}(H) = \left\{x_0, x_{ef} : 1 \le e \le k, 1 \le f \le n_e\right\}$ and edge set $E_{10}(H) = \left\{x_0x_{ef}, x_0x_{e(n_{(e-1)})}, : 1 \le e \le k, 1 \le f \le n_e\right\} \cup \left\{x_{ef}x_{e(f+1)} : 1 \le e \le k, 1 \le f \le n_e\right\}$. We layover one of the vertex say x_0 of $H = C^k_{ne}$ on selected vertex v_1 in G with $g_{10}\,(v_1) = 1$. Now we define a new graph $G^* = G \odot C^k_{ne}$ with vertex Si;et $V^* = V_{10}(G) \cup V_{10}(H)$ and edge set $E^* = E_{10}(G)$ $\cup E_{10}$ (H). Also $|V\,(G^*)| = l + \sum_{e=1}^{k} n_e - k$. Define a bijective function $h_{10} : V^* \to \left\{1,2,3,\ldots,l,l+1,\ldots,l+\sum_{e=1}^{k} n_e - k\right\}$ by $g_{10}(v) = h_{10}(v)$ for all $v \in V(G), h_{10}\,(v_1) = h_{10}\,(x_0) = 1, h_{10}\left(x_{ef}\right) = l + \sum_{c=1}^{e-1} n_c + f + 1$ for $1 \le e \le k, 1 \le f \le n_k$. We have to show that G^* is prime graph. Earlier, G is prime graph $_{2n}$ it is enough to prove that for any two adjacent vertices uv in G^*, which is not in G, the numbers $h_{10}(u)$ and $h_{10}(v)$ are relatively prime. Clearly, for any two adjacent vertices $uv \in E^*$, the numbers $h_{10}(u)$ and $h_{10}(v)$ are relatively prime. Thus $G \odot C^k_{ne}$ admits prime graph. From $G \odot C^k_{ne}$, we delete the edges $\left\{x_0x_{en_e}, x_0x_{e(n_{e-1})}, 1 \le e \le k\right\}$, thus we have the following corollary. □

Corollary 2.12. *If G is prime graph, then there exists a graph from the class $G \odot P^k_{n_e}$ (where $P^k_{n_e}$ is the one point union of k copies of path P_{n_e}) admits prime.*

Theorem 2.13. *If G has prime graph, then there exists a graph from the class $G \odot kP_{n_r}$ (where kP_{n_r} is disjoint union of k copies of path P_{n_r}) admits prime.*

Proof. Let $G(l,m)$ be prime graph with l vertices and m edges. Define a bijective function $g_{11} : V_{11} \to \{1,2,\ldots,l\}$ with the property that given any two adjacent vertices are prime labels. Consider disjoint union of k copies of path P_{n_r}, that is $H = kP_{n_r}$. Then vertex set $V_{11}(H) = \{x_{rs} : 1 \le r \le k, 1 \le s \le n_r\}$ and edge set $E_{11}(H) = \left\{x_{rs}x_{r(s+1)} : 1 \le r \le k, 1 \le s \le n_r\right\}$. We layover one of the vertex x_{11} of $H = kP_{n_i}$ on selected vertex v_1 in G with $g_{11}\,(v_1) = 1$. Now we define a new graph $G^* = G \odot kP_{n_r}$ with vertex set $V^* = V_{11}(G) \cup V_{11}(H)$ and edge set $E^* = E_{11}(G) \cup E_{11}(H)$. Also $|V_{11}\,(G^*)| = l + \sum_{r=1}^{k} n_r$. Define a bijective function $h_{11} : V^* \to \{1,2,3,\ldots,l,l+1,\ldots,l+\sum_{r=1}^{k} n_r\}$ by $g_{11}(v) = h_{11}(v)$ for all $v \in V_{11}(G), h_{11}\,(v_1) = h_{11}\,(x_{11}) = 1, h_{11}\,(x_{rs}) = l + \sum_{c=1}^{r-1} n_c + s$ for $1 \le r \le k, 1 \le s \le n_k$. We have to show that G^* is prime graph. Earlier, G is prime graph, it is enough to prove that for any two adjacent vertices uv in G^*, which is not in G, the numbers $h_{11}(u)$ and $h_{11}(v)$ are relatively prime. Clearly, for any two adjacent vertices $uv \in E^*$, the numbers $h_{11}(u)$ and $h_{11}(v)$ are relatively prime. Thus $G \odot kP_{n_r}$ admits prime graph. □

3. Odd Prime Graphs

In this section, we discuss about odd prime graphs.

Theorem 3.1. *If G has odd prime graph, then there exists a graph from the class $G \odot K_{1,b}$ admits odd prime.*

Proof. Let $G = (V_{12}, E_{12})$ be odd prime graph with l vertices and m edges Define a bijective function $g_{12} : V_{12} \to \{1,3,5,\ldots,2l-1\}$ with the property that given any two adjacent vertices are relatively prime labels. Consider the star $K_{1,b}$ with vertex set $\{t, t_e : 1 \le e \le b\}$ and edge set $\{tt_e : 1 \le e \le b\}$. We identify one of the vertex say t of $K_{1,b}$ on selected vertex v_1 in G with $g_{12}\,(v_1) = 1$. Now we define a new graph $G^* = G \odot K_{1,b}$ with vertex set $V^* = V_{12} \cup \{t, t_e : 1 \le e \le b\}$ and edge set $E^* = E_{12} \cup \{tt_e : 1 \le e \le b\}$. Define the bijective function $h_{12} : V^* \to \{1,3,5,\ldots,2l-1,2l+1,\ldots,2l+2b\}$ by $g_{12}(v) = h_{12}(v)$ for all $v \in V(G), h_{12}(t) = 1, h_{12}\,(t_e) = 2l+2e-1$. for $1 \le e \le b$. We have to show that G^* is odd prime graph.

Already, G is odd prime graph, it is sufficient to prove that for any two adjacent vertices uv in G^* which are not in G, the numbers $h_{12}(u)$ and $h_{12}(v)$ are relatively prime. For any edge $tt_e \in E^*(1 \le e \le b), \gcd\left(h_{12}(t), h_{12}\,(t_e)\right) = \gcd(1, 2l+2e-1) = 1$. For any two adjacent vertices $uv \in E^*$, the numbers $h_{12}(u)$ and $h_{12}(v)$ are relatively prime. Thus $G \odot K_{1,b}$ admits odd prime graph. □

Corollary 3.2. *Octopus graph $O_n = f_n \odot K_{1,n}$ is odd prime.*

Corollary 3.3. *Coconut tree $CT\,(m,n) = P_m \odot K_{1,n}$ is odd prime.*

Theorem 3.4. *If G has odd prime graph, then there exists a graph from the class $G \odot P_w$ admits odd prime graph.*

Proof. Let $G = (V_{13}, E_{13})$ be odd prime graph with l vertices and m edges. Define a bijective function $g_{13} : V_{13} \to \{1,3,5,\ldots,2l-1\}$ with the property that adjacent vertices have relatively prime labels. Consider the path P_w with vertex set $\{t_a : 1 \leq a \leq w\}$ and edge set $\{t_a t_{a+1} : 1 \leq a \leq w-1\}$. We superimpose one of the vertex say t_1 of P_w on selected vertex u_1 in G.

Define a new graph $G^* = G \odot P_w$ with vertex set $V^* = V_{13} \cup V(P_w)$ and edge set $E^* = E_{13} \cup E(P_w)$. Now define the bijective function $h_{13} : V^* \to \{1,3,5,\ldots,2l-1,2l+1,\ldots,2l+2w-2\}$ by $g_{13}(v) = h_{13}(v)$ for all $v \in V(G)$, $h_{13}(v) = h_{13}(t_1) = 1$ and $h_{13}(t_a) = 2l+2a-2$ for $2 \leq a \leq w$. We have to show that G^* is odd prime graph. Already, G is odd prime graph, it is sufficient to prove that for any two adjacent vertices uv in G^*, which is not in G, the numbers $h_{13}(u)$ and $h_{13}(v)$ are relatively prime. For any edge $t_a t_{a+1} \in E^* (2 \leq a \leq w-1)$ $\gcd(h_{13}(t_a), h_{13}(t_{a+1})) = \gcd(2l+2a-2, 2l+2a) = 1$ Since they are consecutive odd numbers, $\gcd(h_{13}(t_1), h_{13}(t_2)) = \gcd(1, 2l+1) = 1$. For any two adjacent vertices $uv \in E^*$, the numbers $h_{13}(u)$ and $h_{13}(v)$ are relatively prime. Thus $G \odot P_w$ admits odd prime graph. □

Corollary 3.5. *Dragon or balloon graph is odd prime.*

Theorem 3.6. *If $G(l,m)$ has odd prime graph, then there exists a graph from the class $G \odot C_n$ that admits odd prime graph.*

Proof. Let $G(l,m)$ has odd prime graph. Define a bijective function $g_{14} : V_{14} \to \{1,3,5,\ldots,2l-1\}$ with the property that relatively prime labels. Consider the cycle C_d with vertex set $\{t_f : 1 \leq f \leq d\}$ and edge set $\{t_f t_{f+1} : 1 \leq f \leq d-1\} \cup \{t_1 t_d\}$. We superimpose one of the vertex t_1 of C_d on selected vertex v in G with $h_{14}(v) = 1$. Now we define a new graph $G^* = G \odot C_d$ with vertex set $V^* = V_{14} \cup \{t_f : 2 \leq f \leq d\}$ and edge set $E^* = E_{14} \cup \{t_1 t_2, t_1 t_d, t_f t_{f+1} : 2 \leq f \leq d-1\}$. Define the bijective function $h_{14} : V^* \to \{1,3,\ldots,2l-1,2l+1,\ldots,2l+2d-2\}$ by $g_{14}(v) = h_{14}(v)$ for all $v \in V(G), h_{14}(v_1) = h_{14}(t_1) = 1$ and $h_{14}(t_f) = 2l+2f-2$ for $2 \leq f \leq d$. We have to show that G^* is odd prime graph. Already G is odd prime graph, it is enough to prove that for any two adjacent vertices uv in G^*, which is not in G, the numbers $h_{14}(u)$ and $h_{14}(v)$ are relatively prime. For any edge $t_f t_{f+1} \in E^*, (2 \leq f \leq d-1)$ $\gcd(h_{14}(t_f), h_{14}(t_{f+1})) = \gcd(2l+2f-2, 2l+2f) = 1$. Since they are consecutive odd numbers.
$\gcd(h_{14}(t_1), h_{14}(t_2)) = \gcd(1, 2l+1) = 1$,
$\gcd(h_{14}(t_1), h_{14}(t_d)) = \gcd(1, 2l+2d-3) = 1$. For any two adjacent vertices $uv \in E^*$, the numbers $h_{14}(u)$ and $h_{14}(v)$ are relatively prime. Thus $G \odot C_d$ admits odd prime graph. □

Corollary 3.7. *$(n,m)-$ kite graph is odd prime.*

Corollary 3.8. *The Planter R_n graph is odd prime.*

Theorem 3.9. *If $G(l,m)$ has odd prime, then there exists a graph from the class $G \odot f_c$ that admits odd prime.*

Proof. Let $G(l,m)$ be odd prime graph with l vertices and m edges. Define a bijective function $g_{15} : V_{15} \to \{1,3,5,\ldots,2l-1\}$ with the property that given any two adjacent vertices are relatively prime labels. Consider the fan graph f_c with vertex set $\{z, z_e : 1 \leq e \leq c\}$ and edge set $\{zz_e : 1 \leq e \leq c\} \cup \{z_e z_{e+1} : 1 \leq e \leq c-1\}$. We superimpose one of the vertex say, z of f_c on selected vertex u_1 in G with $g_{15}(u_1) = 1$. Now we define new graph $G^* = G \odot f_c$ with vertex set $V^* = V_{15} \cup \{zz_e : 1 \leq e \leq c\}$ and edge set $E^* = E_{15} \cup \{zz_e : 1 \leq e \leq c\} \cup \{z_e z_{e+1} : 1 \leq e \leq c-1\}$

Define a bijective function $h_{15} : V \to \{1,2,\ldots,2l-1,2l+1,\ldots,2l+2c\}$ by $h_{15}(v) = g_{15}(v)$ for all $v \in V(G), h_{15}(u_1) = h_{15}(z) = 1, h_{15}(u_e) = 2l+2e$ for $1 \leq e \leq c$. We have to show that G^* is odd prime graph. Already, G is odd prime graph, it is sufficient to prove that for any two adjacent vertices uv in G^*, which is not in G, the numbers $h_{15}(u)$ and $h_{15}(v)$ are relatively prime. For any two adjacent vertices $uv \in E^*$, the numbers $h_{15}(u)$ and $h_{15}(v)$ are relatively prime. Thus, $G \odot f_C$ admits odd prime graph. □

Corollary 3.10. *Umbrella graph is odd prime.*

Theorem 3.11. *If $G(l,m)$ has odd prime graph, then there exists a graph from the class $G \odot F_a$ that admits odd prime.*

Proof. Let $G(l,m)$ be odd prime graph with l vertices and m edges. Define a bijective function $g_{16} : V_{16} \to \{1,3,5,\ldots,2l-1\}$ with the property that given any two adjacent vertices have relatively prime laels. Consider the friendship graph F_a with vertex set $\{z, z_b : 1 \leq b \leq a\}$ and edge set $\{zz_b : 1 \leq b \leq a\} \cup \{z_b z_{b+1} : b \text{ is odd, that is, } b = 1,3,5,\ldots\}$. We superimpose one of the vertex say z of F_a on selected vertex u_1 in G with $g_{16}(u_1) = 1$. Now, we define a new graph $G^* = G \odot F_a$ with vertex set $V^* = V_{16} \cup \{z, z_b : 1 \leq b \leq 2a\}$ and edge set $E^* = E_{16} \cup \{zz_b : 1 \leq b \leq 2a\} \cup \{z_{2b-1} z_{2b} : b \text{ is odd, i.e., } b = 1,3,5,\ldots\}$. Define a bijective function $h_{16} : V^* \to \{1,3,5,\ldots,2l-1,2l+1,\ldots,2l+4a-1\}$ by $h_{16}(u) = g_{16}(u)$ for all $v \in V(G), h_{16}(u_1) = h_{16}(z) = 1, h_{16}(z_b) = 2l+2b-1$ for $1 \leq b \leq 2a$. We have to show that G^* is odd prime graph. Already, G is odd prime graph, it is sufficient to prove that for any two adjacent vertices uv in G^*, which is not in G, the numbers $h_{16}(u)$ and $h_{16}(v)$ are relatively prime. Clearly, for any two adjacent vertices $uv \in E^*$, the numbers h_{16} (u) and $h_{16}(v)$ are relatively prime. Thus $G \odot F_a$ admits odd prime graph. □

Theorem 3.12. *If $G(l,m)$ has odd prime graph, then there exists a graph from the class $G \odot T_b$ that admits odd prime.*

Proof. Let $G(l,m)$ be odd prime with bijective function $g_{17} : V(G) \to \{1,3,5,\ldots,2l-1\}$ with the property that given any two adjacent vertices are relatively prime labels. Consider the triangular graph T_b with vertex set $\{r, r_w, s_w : 1 \leq w \leq b\}$ and edge set $\{rr_w, rs_w, r_w s_w : 1 \leq w \leq b\}$. We superimpose one of the vertex say, r of T_b on selected vertex v_1 in G with $g_{17}(v_1) = 1$. Now, we define new graph $G^* = G \odot T_b$ with vertex set $V^* = V_{17}(G) \cup V_{17}(T_b)$ and edge set $E^* =$

$E_{17}(G)\cup E_{17}(T_b)$. Define a bijective function $h_{17}: V^*\rightarrow\{1,3,5,\ldots,2l-1,2l+1,\ldots,2l+4b-1\}$ by $g_{17}(v)=h_{17}(v)$ for all $v\in V(G), h_{17}(v_1)=h_{17}(r)=1, h_{17}(r_w)=2l+4w-3$ for $1\leq w\leq b, h_{17}(s_w)=2l+4w-1$ for $1\leq w\leq b$. We have to show that G^* is an odd prime graph. Already, G is odd prime graph, it is enough to prove that for any two adjacent vertices uv in G^*, which is not in G, the numbers $h_{17}(u)$ and $h_{17}(v)$ are relatively prime. Clearly for any two adjacent vertices $uv\in E^*$, the numbers $h_{17}(u)$ and $h_{17}(v)$ are relatively prime. Thus $G\odot T_b$ admits odd prime graph. □

Corollary 3.13. *$B_{2n}\odot T_n$ is odd prime graph.*

Theorem 3.14. *The one vertex union of $C_a, K_{1,b}$ and P_c is odd prime graph.*

Proof. Let G be the one vertex union of $C_a, K_{1,b}$ and P_c. Then $V_{18}(G)=\{u_r: 1\leq r\leq a\}\cup\{v,v_s: 1\leq s\leq b\}\cup\{w_t: 1\leq t\leq c\ \}$ and $E_{18}(G)=\{u_ru_{r+1}: 1\leq r\leq a-1\}\cup\{u_1u_a\}\cup\{v_1v_s: 1\leq s\leq\ b\}\cup\{w_1w_2\}\cup\{w_tw_{t+1}: 2\leq t\leq c-1\}$ with $u_1=v_1=w_1$. Also, $|V(G)|=l+\ b+c-1$. We superimpose two of the vertices say v of $K_{1,b}$ and w_1 of P_c on selected vertex u_1 in C_a. Define a bijective function $g_{18}: V_{18}\rightarrow\{1,3,5,\ldots,2a-1,2a+1,\ldots,2a+2b-1,2a+2b+1,\ldots,2a+2b-2c-3\}$ by $g_{18}(u_1)=g_{18}(v)=g_{18}(w_1)=1$ for each $2\leq r\leq l, g_{18}(u_r)=2r-1$, for each $1\leq s\leq b, g_{18}(v_s)=2a+2s-1$, for each $2\leq t\leq c, g_{18}(w_t)=2a+2b+2t-3$ Clearly, for any edge $uv\in E(G)$, the numbers $g_{18}(u)$ and $g_{18}(v)$ are relatively prime. Hence the one vertex union of $C_l, K_{1,b}$ and P_c admits odd prime graphs. □

Corollary 3.15. *The one vertex union of k copies of shell graph is odd prime.*

Corollary 3.16. *The one vertex union of $C_l, K_{1,m}, P_n, f_n, F_n, T_n$ is an odd prime.*

Theorem 3.17 ([9])**.** *If G is odd prime of order l, then $G\cup P_n$ is odd prime.*

Corollary 3.18. *Corollary. [9] $\cup P_{n_i}$ is an odd prime.*

Corollary 3.19. *Corollary. If G is odd prime of order l, then $G\cup P_{n_i}$ is odd prime.*

Theorem 3.20. *The one vertex union of k copies of fan grapn $f_p,(p\geq 2)$ is odd prime.*

Proof. Let $G=f_p^k(p\geq 2)$ be the one vertex union of k copies of fan graph. Then vertex set $V_{19}\left(f_p^k\right)=\left\{u,u_q^k: 1\leq q\leq p\right\}$ and edge set $E_{19}\left(f_p^k\right)=\left\{uu_q^k: 1\leq q\leq\ p\right\}\cup\left\{u_q^ku_{q+1}^k: 1\leq q-1\right\}$. Note that $\left|V\left(f_p^k\right)\right|=kp+1$ and $\left|E\left(f_p^k\right)\right|=k(2p-1)$. Define a bijjective function $g_{19}: V_{19}\left(f_p^k\right)\rightarrow\{1,3,5,\ldots,2kp+1\}$ by $g_{19}(u)=1$, $g_{19}\left(u_q^k\right)=2(k-1)p+2q+1$ for $1\leq q\leq p$. It is easily verified that for any edge $uv\in E\left(f_p^k\right)$, the numbers $g_{19}(u)$ and $g_{19}(v)$ are relatively prime. Hence f_p^k the one vertex union of k copies of fan graph is odd prime. □

Theorem 3.21. *The one vertex union of t copies of octopus graph $O(m,n)=f_m\odot K_{1,n}(\ m,n\geq 2)$, that is,$O^t(\ m,n)$ admits odd prime.*

Proof. Let $H=O^t(\ m,n)$ be the one vertex union of t copies of octopus graph $O(m,\ n)=f_m\odot\quad K_{1,n}$. Then vertex set $V_{20}(O^t(\ m,n))=\{u,u_r^t,v_s^t: 1\leq r\leq m, 1\leq s\leq\ n\}$ and edge set $E_{20}(O^t(\ m,n))=\{uu_r^t: 1\leq r\leq m\}\cup\left\{u_r^tu_{r+1}^t: 1\leq r\leq m-1\right\}\cup\{uv_s^t: 1\leq s\leq n\}$. Note that, $|V(O^t(\ m,n))|=k(m+n)+1$. Define a bijective function $g_{20}: V_{20}(O^t(\ m,n))\rightarrow\{1,3,5,\ldots,2k(m+n)+1\}$ by $g_{20}(u)=1, g_{20}\ (u_r^t)=2(k-1)(m+n)+2r+1$ for $1\leq r\leq m, g_{20}(v_s^t)=2(k-1)(m+n)+2m+2s+1$ for $1\leq s\leq n$. Clearly, for any edge $uv\in E(H)$, the numbers $g_{20}(u)$ and $g_{20}(v)$ are relatively prime. Hence the one vertex union of t copies of octopus graph $H=O^t(\ m,n)$ is odd prime. □

Theorem 3.22. *The one vertex union of f copies of drum graph $D(m,n)=2C_m\odot 2P_n,(\ m\geq 3,n\geq 2)$, that is $D^f(m,n)$ admits odd prime.*

Proof. Let $H=D^f(m,n),\quad(m\geq 3,n\geq 2)$ be the one vertex union of f copies of drum graph $D(m,n)$. Then vertex set $V_{21}\left(D^f(m,n)\right)=\left\{u,u_d^f,v_e^f: 1\leq d\leq 2m-\ 2, 1\leq e\leq 2n-2\right\}$ and edge set $E_{21}\left(D^f(m,n)\right)=\left\{uu_d^f: d=1,m-1,m,2m-2\right\}\cup\left\{u_d^fu_{d+1}^f: 1\leq d\leq m-2, m\leq d\leq 2m-3\right\}\cup\left\{uv_d^f: d=1,n\right\}\cup\left\{v_d^fv_{d+1}^f: 1\leq d\leq n-2, n\leq d\leq 2n-3\right\}$.

Here, $\left|V\left(D^f(m,n)\right)\right|=k(2(\ m-1)+2\ (n-1))+1$. Define a bijective function $g_{21}: V_{21}\left(D^f(m,n)\right)\rightarrow\{1,3,5,\ldots,2k(2(\ m-1)+2(n-1))+1\}$ by $g_{21}(u)=1, g_{21}\left(u_d^f\right)=2(k-1)(2(\ m-1)+2(n-1))+2d+1$ for $1\leq d\leq 2m-2, g_{21}\left(v_e^f\right)=2(k-\ 1)\ (2(\ m-1)+2(n-1))+4(m-1)+2e+1$ for $1\leq e\leq 2n-2$. The numbers 1 is relatively prime to every natural number and any two consecutive odd integers are relatively prime. Hence, g_{21} is odd prime labeling. Thus $H=D^f(m,n)$ is odd prime. □

Theorem 3.23. *The one vertex union of w copies of udukkai graph $A(m,n)=2\ f_m\odot 2P_n(\ m,n\geq 2)$, that is,$A^w(m,n)$ admits odd prime.*

Proof. $H=A^w(m,n),(m,n\geq 2)$ be the one vertex union of w copies of Udukkai graph $A(m,n)=2f_m\odot 2P_n$. Then vertex set $V_{22}(A^w(m,n))=\{u,u_s^w,v_t^w: 1\leq\ s\leq 2m, 1\leq t\leq 2n\}$ and edge set $E_{22}(A^w(m,n))=\{uu_s^w: 1\ \leq s\leq 2\ m\}\ U\ \{u_s^wu_{s+1}^w: 1\leq s\leq m-1, m+1\leq s\leq 2m-1\}\cup\{uv_t^w: t=1,\ \ n\}\cup\{v_t^w v_{t+1}^w: 1\leq t\leq n-2, n\leq t\leq 2n-3\}$.

Note that, $|V(A^w(m,n))|=k(2\ m+\ 2(n-1))+1$. Define a bijective function $g_{22}: V_{22}(A^w(m,n))\rightarrow\{1,3,5,\ldots,2k(2m+2(n-1))+1\}$ by $g_{22}(u)=1, g_{22}(u_s^w)=2(k-\ 1)\ (2m+2(n-1))+2s+1$ for $1\leq s\leq 2m, g_{22}(v_t^w)=2(k-1)(2m+2(n-1))+4m+2t+1$ for $1\leq t\leq 2n-3$. As any integer is relatively prime to 1 and any two consecutive odd integers are relatively prime. Hence, g_{22} is odd prime labeling. Thus $H = A^w(m,n)$ is odd prime. □

Delete the edges u_i^k, u_{i+1}^k from the above theorem, we get the following corollary.

Corollary 3.24. *The one vertex union of k copies of Lilly graph $L(m,n) = 2K_{1,m} \odot 2\, P_n(\, m,n \geq 2)$, that is,$L^k(m,n)$ admits odd prime.*

Theorem 3.25. *The one vertex union of f copies of Umbrella graph $U(x,y),(x,y \geq 2)$ is odd prime graph.*

Proof. Let $H = U^f(x,y)(x,y \geq 2)$ be the one vertex union of f copies of Umbrella graph $U(x,y)$. Then $V_{23}(H) = \left\{u, u_d^f, v_e^f : 1 \leq d \leq x, 1 \leq e \leq y\right\}$ and $E_{23}(H) = \left\{uu_d^f : 1 \leq d \leq x\right\} \cup \left\{u_d^f u_{d+1}^f : 1 \leq d \leq x-1\right\} \cup \left\{uv_1^f\right\} \cup \left\{v_e^f v_{e+1}^f : 1 \leq\ e \leq y-1\right\}$. Note that $|V(H)| = k(x+y)+1$. Define a bijective function $g_{23} : V(H) \to \{1,3,5,\ldots, f(x+y-1)+1\}$ by $g_{23}(u) = 1, g_{23}\left(u_d^f\right) = (f-1)(2x+2y-1)+2d+1$ for $1 \leq d \leq x, g_{23}\left(v_d^f\right) = (f-1)(2x+2y-\ 1)\ +2x+2e+1$ for $1 \leq e \leq y$. Clearly, for any edge $uv \in E(H)$, the numbers $g_{23}(u)$ and $g_{23}(v)$ are relatively prime. Hence the one vertex union of f copies of Umbrella graph is odd prime. □

From the one vertex union of f copies of Umbrella graph $U(x,y)$, that is, $U^f(x,y)$, we delete the edge $u_d^f u_{d+1}^f (1 \leq d \leq x)$, thus we have the following corollary

Corollary 3.26. *The one vertex union of k copies of coconut tree $CT(m,n)$, that is, $CT^k(m,n)$ is odd prime.*

Theorem 3.27. *The one vertex union of f copies of Butterfly graph $B_{m,n}$ is odd prime.*

Proof. Let $B'_{m,n}$ be the one vertex union of f copies of Butterfly graph. Then vertex set $V_{24}\left(B_{m,n}^f\right) = \left\{u, u_d^f, v_e^f : 1 \leq d \leq 2m-2, 1 \leq e \leq n\right\}$ and $E_{24}\left(B_{m,n}^f\right) = \left\{uu_d^f : d = 1, m-1, m, 2m-2\right\} \cup \left\{u_d^f u_{d+1}^f : 1 \leq d \leq m-2\right\} \cup \left\{u_d^f u_{d+1}^f : 1 \leq d \leq 2m-3\right\} \cup \left\{uv_e^f : 1 \leq e \leq n\right\}$. Also $\left|V\left(B_{m,n}^f\right)\right| = f(2\,m+n-2)\ +1$. Define a bijective function $g_{24} : V\left(B_{m,n}^f\right) \to \{1,3,5\ldots, 2k(2m+n-2)-1\}$ by $g_{24}(u) = 1, g_{24}\left(u_d^f\right) = 2(f-1)(2\,m+n-2)+2d+1$, for $1 \leq d \leq 2m-2$ $g_{24}\left(v_e^f\right) = 2(f-1)(2\,m+n-2)+2(2\,m-2)+2e-1$ for $1 \leq e \leq n$. Clearly, for any two adjacent vertices $uv \in E_{24}\left(B_{m,n}^f\right)$, the numbers $g_{24}(u)$ and $g_{24}(v)$ are relatively prime. Hence $B_{m,n}^f$ the one vertex union of f copies of Butterfly graphs is odd prime. □

Theorem 3.28. *The one vertex union of c copies of Butterfly graph $B_{m,n,l}(\, m,n \geq 3)$ with shell orders m and n is odd prime.*

Proof. Let ${B_{m,n,l}}^c\,(m_2 n \geq 3)$ be the one vertex union of c copies of Butterfly graph with shell orders m and n. Then vertex set $V_{25}\left({B_{m,n,l}}^c\right) = \left\{u, u_r^c, v_S^c, w_t^c : 1 \leq r \leq\ m, 1 \leq s \leq n, 1 \leq t \leq l\right\}$ and $E_{25}\left(B_{m,n,l}^c\right) = \{uu_r^c, uv_s^c, u\quad w_t^c : 1 \leq r \leq m, 1 \leq s \leq n, 1 \leq t \leq l\} \cup \left\{u_r^c u_{r+1}^c : 1 \leq r \leq m-1\right\} \cup \left\{v_s^c v_{s+1}^c : 1 \leq s \leq n-1\right\}$.

Also $\left|V\left(B_{m,n,l}^c\right)\right| = c(\,m+n+l)+1$. Define a bijective function $g_{25} : V\left(B_{m,n,l}^c\right) \to \{1,3,5,\ldots, +c(\,m+n+l)+1\}$ by $g_{25}(u) = 1, g_{25}\left(u_r^c\right) = (c-1)(2\,m+2n+2c+1)+2r+1$, for $1 \leq r \leq m, g_{25}\left(v_r^c\right) = (c-1)(2\,m+\ 2n+2c+1)+2\,m+2r-1$, for $1 \leq r \leq n, g_{25}\left(w_r^c\right) = (c-1)(2\,m+2n+2c+\ 1)+2\,m+2n+2r-1$, for $1 \leq r \leq l$. For any edge $uv \in {B_{m,n,l}}^c$, the numbers $g_{25}(u)$ and $g_{25}(v)$ are relatively prime. Hence ${B_{m,n,l}}^c$ the one vertex union of c copies of Butterfly graphs $B_{m,n,l}(\, m,n \geq 3)$ with shell orders m and n is odd prime. □

Theorem 3.29. *The one vertex union of s copies of triangular snake $T_r(r \geq 2)$ is prime graph.*

Proof. Let T_r^s be the one vertex union of s copies of triangular snake T_r. Then vertex set $V'_{26}\left(T_r^s\right) = \{u, u_e^s, v_e^s : 1 \leq e \leq r\}$ and edge set $E_{26}\left(T_r^s\right) = \{u_e^s v_e^s : 1 \leq\ e \leq r-1\} \cup \left\{u_e^s v_{e+1}^s : 1 \leq\ e \leq r-1\right\} \cup \left\{u_e^s u_{e+1}^s : 1 \leq e \leq r-1\right\} \cup \{uv_1^s\} \cup \{uu_1^s\}$ Note that $\left|V\left(T_r^S\right)\right| = 4s(r-1)+1$. Define a bijective function $g_{26} : V\left(T_r^s\right) \to \{1,3,5,\ldots, 4s(r-1)+1\}$ by $g_{26}(u) = 1$, when $s = 1$ $g_{26}\left(u_e^1\right) = 4e-1$ for $1 \leq e \leq r-1$ $g_{26}\left(v_e^1\right) = 4e+1$ for $1 \leq e \leq r-1$ when $s \geq 2$ (two or more copies) $g_{26}\left(u_e^s\right) = 4(s-1)(r-1)+4e+1$ for $1 \leq e \leq r-1$ $g_{26}\left(v_e^s\right) = 4(s-1)(r-1)+4e-1$ for $1 \leq e \leq r-1$. For any edge $uv \in E\left(T_r^S\right)$, the numbers $g_{26}(u)$ and $g_{26}(v)$ are relatively prime. The above defined labeling gives odd prime for T_r^s. Thus T_r^s is odd prime graph. □

Theorem 3.30. *The one vertex union of c copies of planter graph $R(m,n) = f_m \odot\, C_n(\, m \geq 2, n \geq 3)$, that is,$R^c(m,n)$ admits odd prime.*

Proof. $H = R^c(\, m,n)(m \geq 2, n \geq 3)$ be the one vertex union of c copies of planter graph $R(m,n) = f_m \odot\quad K_{1,n}$. The one vertex union of c copies of planter graph $R(m,n)$. The vertex set $V_{27}\left(R^c(\, m,n)\right) = \left\{u, u_a^c, v_b^c : 1 \leq a \leq m, 1 \leq b \leq n\right\}$ and edge set $E_{27}\left(R^c(\, m,n)\right) = \{uu_a^c : 1 \leq a \leq m\} \cup \left\{u_a^c u_{a+1}^c : 1 \leq a \leq m\ \ -1\right\} \cup \left\{uv_1^c\right\} \cup \left\{v_b^c v_{b+1}^c : 1 \leq b \leq n-2\right\} \cup \left\{uv_{n-1}^c\right\}$. Note that, $\left|V\left(R^c(\, m,n)\right)\right| = k(m+\,(n-1))+1$. Define a bijective function $g_{27} : V_{27}\left(R^c(\, m,n)\right) \to \{1,3,5,\ldots, 2k(m+\,(n-1))+1\}$ by $g_{27}(u) = 1, g_{27}\left(u_a^c\right) = 2(k-1)(m+(n-1))+2a+1$ for $1 \leq a \leq m, g_{27}\left(v_b^c\right) = 2(k-1)(m+(n-1))+2m+2b+1$ for $1 \leq b \leq n-\ 1$.It is easily verified that for any two adjacent vertices $e = uv \in R^c(\, m,n)$, such that $\gcd(g(u), g(v)) = 1$. Thus $R^k(\, m,n)$ is odd prime graph. □

Theorem 3.31. *The one vertex union of t copies of Vanessa graph $V(m,n) = 2f_m \odot K_{1,n}, (\, m,n \geq 2)$, that is $V^t(m,n)$ admits odd prime.*

Proof. Let $H = V^t(m,n), (m,n \geq 2)$ be the one vertex union of t copies of Vanessa graph $V(m,n) = 2f_m \odot K_{1,n}$. Then vertex set $V_{28}(V^t(m,n)) = \{b, b_r^t, c_s^t : 1 \leq r \leq 2m, 1 \leq s \leq n\}$ and edge set $E_{28}(V^t(m,n)) = \{b \quad b_r^t : 1 \leq r \leq 2m\} \cup \{b_r^t b_{r+1}^t : 1 \leq r \leq m-1, m+1 \leq r \leq 2m-1\} \cup \{bc_s^t : 1 \leq s \leq n\}$. Note that $|V^t(m,n)| = t\ (2\,m+n)+1$ Define a bijective function $g_{28} : V_{28}((V^t(m,\ n) \to \{1,3,5,\ldots,2t(2m+n)+1\}$ by $g_{28}(b) = 1, g_{28}(b_r^t) = 2(t-1)(2\,m+n)+2r+1$ for $1 \leq r \leq 2m, g_{28}(c_s^t) = 2(t-1)(2\,m+n)+4\,m+2\,s+1$ for $1 \leq s \leq n$ It is easily verified that for any two adjacent vertices $e = bc \in V^t(m,n)$ such that $\gcd(g(b), g(c)) = 1$. Thus $H = V^t(m,n)$ is odd prime graph. □

Theorem 3.32. *The one vertex union of H - graph is odd prime.*

Proof. Let H_p^r be the one vertex union of r copies of H - graph H_p. Then vertex set $V_{29}(H_p^r) = \{u_c^r : 1 \leq c \leq p\} \cup \left\{v_c^r : 1 \leq c \leq \left[\frac{p}{2}\right]-1\right\} \cup \left\{v_{\left[\frac{p}{2}\right]}\right\} \cup \left\{v_c^r : \left[\frac{p}{2}\right]+1 \leq c \leq p\right\}$ and edge set $E_{29}(H_p^r) = \{u_c^r u_{c+1}^r : 1 \leq c \leq p-1\} \cup \left\{v_c^r v_{c+1}^r : 1 \leq c \leq \left[\frac{p}{2}\right]-1\right\} \cup \left\{v_c^r v_{c+1}^r : \left[\frac{p}{2}\right]+1 \leq c \leq p-1\right\} \cup \left\{v^r_{\left[\frac{p}{2}\right]-1} v_{\left[\frac{p}{2}\right]}\right\} \cup \left\{v_{\left[\frac{p}{2}\right]} v_{\left[\frac{p}{2}\right]+1}\right\} \cup \left\{u^r_{\frac{p+1}{2}} v_{\left[\frac{p}{2}\right]} \text{ for } p \text{ is odd }\right\}$ or $\left\{u^r_{\frac{p}{2}+1} v_{\left[\frac{p}{2}\right]} \text{ for } p \text{ is even }\right\}$. Also, $\left|V(H_p^r)\right| = r(2p-1)+1$. Define a bijective function $g_{29} : V_{29} \to \{1,3,5,\ldots,r(4p-\ 2)\ +1\}$ by $g_{29}\left(v_{\left[\frac{p}{2}\right]}\right) = 1, g_{29}(u_c^r) = (r-1)(4p-2)+2c+1$ for $1 \leq c \leq p$, $g_{29}(v_c^r) = (r-1)(4p-2)+2p+2c+1$ for $1 \leq c \leq \left[\frac{p}{2}\right]-1, g_{29}(v_c^r) = (r-\ 1)\ (4p-2)+2p+2c+1$ for $\left[\frac{p}{2}\right]+1 \leq c \leq p$. Clearly, for any edge uv $\in E\left(H_p^r\right)$, the numbers g_{29} (u) and $g_{29}(v)$ are relatively prime. □

Theorem 3.33. *The triangular book T_t is an odd prime graph.*

Proof. Let T_t be a triangular book. Then $V_{30}(T_t) = \{r, r_w, s_w : 1 \leq w \leq t\}$ and $E_{30}(T_t) = \{rr_w, rs_w, r_w s_w : 1 \leq w \leq t\}$. Also, $|V(T_t)| = 2t+1$. Define a bijective function $g_{30} : V(T_t) \to \{1,3,5,\ldots,4t+1\}$ by $g_{30}(r) = 1, g_{30}(r_w) = 4w-1$ for $1 \leq w \leq t, g_{30}(s_w) = 4w+1$ for $1 \leq w \leq t$. As any number is relatively prime to 1 and two consecutive odd integers are relatively prime. Thus, T_t admits odd prime. □

Theorem 3.34. *The rectangular book B_{2q} is odd prime graph*

Proof. Let B_{2q} be a rectangular book. Then $V_{31}(B_{2q}) = \{r, s, r_v, s_v : 1 \leq v \leq q\}$ and $E_{31}(B_{2q}) = \{rs, rr_v, ss_v, r_v s_v : 1 \leq v \leq q\}$. Also, $\left|V(B_{2q})\right| = 2q+2$. Define a bijective function $g_{31} : V_{31}(B_{2q}) \to \{1,3,5,\ldots,4q+3\}$ by $g_{31}(r) = 1, g_{31}(s) = 3, g_{31}(r_v) = 4v+1$ for $1 \leq v \leq q, v \neq 3q \cdot g_{31}(r_v) = 4v+3$ for $v = 3$q, $g_{31}(s_v) = 4v+3$ for $1 \leq v \leq q, v \neq 3q \cdot g_{31}(s_v) = 4v+1$ for $v = 3q$ As any number is relatively prime to 1 , $\gcd(3, 4v+1) = 1$ for $v \neq 3q, \gcd(3, 4v+1) = 1$ for $v = 3q$ and any two consecutive odd integers are relatively prime. □

Theorem 3.35. *The circular ladder graph $CL_w, w \geq 3$ is odd prime graph.*

Proof. The circular ladder graph $CL_w, w \geq 3$ has $2n$ vertices and $3n$ edges, whose vertex set $V_{32}(CL_w) = \{a_c, b_c : 1 \leq c \leq w\}$ and edge set $E_{32}(CL_w) = \{a_1 a_w, a_c a_{c+1} : 1 \leq c \leq w-1\} \cup \{b_1 b_w, b_c b_{c+1} : 1 \leq c \leq w-1\} \cup \{a_c b_c \ \ 1 \leq c \leq w\}$. Define odd prime labeling $g_{32} : V_{32}(CL_w) \to \{1,3,5,\ldots,4w-1\}$ by for each $1 \leq c \leq w, g_{32}(a_c) = 4c-3, g_{32}(b_c) = 4c-1$. Clearly, for any edge $uv \in E(CL_w)$, the numbers $g_{32}(a)$ and $g_{32}(b)$ are relatively prime. Hence The circular ladder graph $CL_w, w \geq 3$ is odd prime graph.
Delete the edges $b_1 b_w, b_c b_{c+1} : 1 \leq c \leq w$, from the above theorem, we get the following corollary, □

Corollary 3.36. *The sunlet graph $S_t, t \geq 3$ is odd prime.*

Theorem 3.37. *The line graph of a sunlet graph $L(S_t)$ permits odd prime for $t \geq 3$.*

Proof. Let $L(S_t)$ be the line graph of a sunlet graph with $2t$ vertices and $3t$ edges, whose vertex set $V_{33}(\ L(S_t)) = \{c_e, d_e : 1 \leq e \leq t\}$ and edge set $E_{33}(\ L(S_t)) = \{c_e c_{e+1} : 1 \leq e \leq t-1\} \cup \{c_1 c_t\} \cup \{c_e d_e : 1 \leq e \leq t\} \cup \{c_{e+1} d_e : 1 \leq e \leq t-1\} \cup \{c_1 d_t\}$. Define odd prime labeling $g_{33} : V_{33}(\ L(S_t)) \to \{1,3,5,\ldots,4t-1\}$ by for each $1 \leq e \leq t, g_{33}(c_e) = 4e-3, g_{33}(d_e) = 4e-1$. An easy verification shows that g_{33} is the desired odd prime labeling of $L(S_t)$. Thus $L(S_t)$ is odd prime graph for $t \geq 3$. □

Theorem 3.38. *The pentagonal book $B_{5,d},\ d \geq 1$ is odd prime graph.*

Proof. Let $B_{5,d}$ be a pentagonal book. Then vertex $V_{34}(B_{5,n}) = \{u_1, u_2, v_c, w_c, x_c : 1 \leq c \leq d\}$ and edge set $E_{34}(B_{5,d}) = \{u_1 u_2\} \cup \{u_2 v_c, v_c w_c, w_c x_c, u_1 x_c : 1 \leq c \leq d\}$. Note that $\left|V(B_{5,d})\right| = 3d+2$ and $\left|E(B_{5,d})\right| = 4d+1$. Define odd prime labeling $g_{34} : V_{34}(B_{5,d}) \to \{1,3,5,\ldots,6d+3\}$ by $g_{34}(u_1) = 1, g_{34}(u_2) = 3$, m for each $1 \leq c \leq d, g_{34}(v_c) = 6c-1, g_{34}(w_c) = 6c+1, g_{34}(x_c) = 6c+3$. Clearly, for any edge $uv \in E(B_{5,d})$, the numbers $g_{34}(u)$ and $g_{34}(v)$ are relatively prime. Hence, $B_{5,d}$ is odd prime graph for all $d \geq 1$. □

Theorem 3.39. *The barycentric subdivision of cycle C_n is odd prime.*

Proof. Let $\{v_1, v_2, v_3, \ldots, v_n\}$ be the vertices of cycle C_n and $\{v_1',\ v_2', v_3', \ldots, v_n'\}$ be the newly inserted vertices to obtain barycentric subdivision of cycle C_n.

Then vertex set $V_{35}(C_n(C_n)) = \{v_r, v_r' : 1 \leq r \leq n\}$ and edge set $E_{35}(C_n(C_n)) = \{v_r v_{r+1}, v_r' v_{r+1} : 1 \leq r \leq n\} \cup \{v_1 v_n\} \cup \{v_1 v_n'\} \cup \{v_r v_r' :\ 1 \leq r \leq n\}$. Here, $|V(C_n(C_n))| = 2n$ and $|E(C_n(C_n))| = 3n$. Define odd prime labeling $g_{35} : V_{35}(C_n(C_n)) \to \{1,3,5,\ldots,4n-1\}$ by for each $1 \leq r \leq n, g_{35}(v_r) = 4r-3, g_{35}(v_r') = 4r-1$. It is easily verify that for any edge uv $\in E(C_n(C_n))$ the numbers g_{35} (u) and g_{35} (v) are relatively prime. Hence, $C_n(C_n)$ is odd prime graph. □

Theorem 3.40. *The union of k-copies of triangular snake $T_r \cup T_r \cup \ldots \cup T_r, r \geq 2$ is odd prime graph.*

Proof. Let $H = T_r \cup T_r \cup \ldots \cup T_r, r \geq 2$ be the union of k copies of triangular snake. It has $k(2r-1)$ vertices and $k(3r-3)$ edges. Then vertex set

$$V_{36}(H) = \left\{c_a^k, d_b^k : 1 \leq a \leq r, 1 \leq b \leq r-1\right\}$$

and edge set

$$E_{36}(H) = \left\{c_a^k c_{a+1}^k, c_a^k d_a^k, c_{a+1}^k d_a^k : 1 \leq a \leq r-1\right\}.$$

Define odd prime labeling $g_{36} : V_{36}(H) \to \{1, 3, 5, \ldots, k(2(2r-1)) - 1\}$ by $g_{36}(c_a) = 2(k-1)(2r-1) + 4a - 3$ for $1 \leq a \leq r, g_{36}(d_a) = 2(k-1)(2r-1) + 4a - 1$ for $1 \leq a \leq r-1$. An easy check proves that g_{36} is the required odd prime labeling of H. Thus $H = T_r \cup T_r \cup \ldots \cup T_r$ is odd prime graph for $r \geq 2$. □

Theorem 3.41. *The bistar $B_{m,n}$ is odd prime graph.*

Proof. Let $B_{m,n}$ be the bistar with vertex set

$$\{a, b, a_r, b_s : 1 \leq r \leq m, 1 \leq s \leq n\}$$

where a_r, b_s are pendent vertices and edge set

$$\{ab, ab_r, bb_j : 1 \leq r \leq m, 1 \leq j \leq n\}.$$

Also, $|V(B_{m,n})| = m+n+2$ and $|E(B_{m,n})| = m+n+1$. Define the odd prime labeling $g_{37} : V_{37}(B_{m,n}) \to \{1, 3, 5, \ldots, 2(m+n+2) - 1\}$ as follows $g_{37}(a) = 1, g_{37}(b) = p$, where p is the largest prime number $(m+1)+1 \leq p \leq 2(m+n+2)-1$. Now label the vertices $a_r, (1 \leq r \leq m)$ consecutively from the set $\{3, 5, \ldots, 2(m+1)-1\}$ and label the remaining vertices $b_s, (1 \leq s \leq n)$ consecutively from the set $\{2(m+1)+1, 2(m+1)+3, \ldots, 2(m+n+2)-1\} / \{p\}$. The labeling pattern defined above covers all the possibilities in each edge $ab \in E(B_{m,n})$, the numbers $g_{37}(a)$ and $g_{37}(b)$ are relatively prime. Hence the bistar $B_{m,n}$ is odd prime graph. □

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