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A new closure operator via (1, 2)S_β-open sets in bi-topological spaces

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Abstract

The aim of this paper is to define a new closure operator of $\underset{(1,2)S_\beta}{\wedge}$ - set and to generate $\tau^{\overset{\wedge}{(1,2)S_\beta}}$, $\rho^{\overset{\wedge}{(1,2)S_\beta}}$ using (1,2)S_β- open sets in bitopological spaces and study some of their properties.

Keywords

(1, 2)semi-open sets, (1, 2)S_β-open sets, (1, 2)β-closed sets, (1, 2)S_β-Interior, (1, 2)S_β-Closure, $\underset{(1,2)S_\beta}{\wedge}$-sets, $\underset{(1,2)S_\beta}{\vee}$ -sets, $D^{\overset{\wedge}{(1,2)S_\beta}}$ -sets, $D^{\overset{\vee}{(1,2)S_\beta}}$-sets, $Int^{\overset{\vee}{(1,2)S_\beta}}$-sets, $C^{\overset{\wedge}{(1,2)S_\beta}}$-sets, $\tau^{\overset{\wedge}{(1,2)S_\beta}}$-sets, $\rho^{\overset{\wedge}{(1,2)S_\beta}}$-sets.

AMS Subject Classification

11B05.

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Contents

1. Introduction

In the year 1963, Kelly initiated the systematic study of bitopology which is a triple (X, τ, σ), where X is a non-empty set together with two distinct topologies τ, σ on X. Levine introduced the notion of semi-open sets and their properties in 1963. In 1983, Abd-El-monsef introduced β-open sets and β-continuity in topological spaces. In 2013, Alias B.Khalaf and Nehmat K.Ahmed introduced and defined a class of semi-open sets called S_β-open sets in topological spaces. Andrijevic introduced a class of generalized open sets in topological spaces. In 1986, Maki introduced some forms of open and closed sets known as $\wedge$-sets and $\vee$-sets The aim of this paper is to define a new closure operator of $\underset{(1,2)S_\beta}{\wedge}$ - set and to generate $\tau^{\overset{\wedge}{(1,2)S_\beta}}$, $\rho^{\overset{\wedge}{(1,2)S_\beta}}$ using (1,2)S_β- open sets in bitopological spaces and study some of their properties.

2. Preliminaries

Definition 2.1 ([6])**.** *Let A be a subset of a bitopological space $(X,\ \tau_1,\ \tau_2)$. Then A is said to be*

(i) $\tau_1\tau_2$-open if $A \in \tau_1 \cup \tau_2$,

(ii) $\tau_1\tau_2$-closed if $A^c \in \tau_1 \cup \tau_2$,

(iii) (1, 2)β-open if $A \subseteq \tau_1\tau_2 - cl(\tau_1 - int(\tau_1\tau_2 - cl(A)))$, where τ_1-Int(A) is the interior of A with respect to the topology τ_1 and $\tau_1\tau_2$-Cl(A) is the intersection of all $\tau_1\tau_2$-closed sets containing A.

(iv) (1, 2)β-Int(A) is the union of all (1, 2)β-open sets contained in A.

(v) (1, 2)β-Cl(A) is the intersection of all (1, 2)β-closed sets containing A.

Definition 2.2 ([6]). *A subset A of X is said to be*

(i) *(1, 2)semi-open if* $A \subseteq \tau_1\tau_2\text{-}Cl(\tau_1 - Int(A))$,

(ii) *(1, 2)regular-open if* $A = \tau_1\text{-}Int(\tau_1\tau_2 - Cl(A))$,

(iii) *(1, 2)β-open if* $A \subseteq \tau_1\tau_2\text{-}Cl(\tau_1\text{-}Int(\tau_1\tau_2\text{-}Cl(A)))$.

The set of all (1, 2)semi-open, (1, 2)regular-open, (1, 2)β-open are denoted as (1, 2)SO(X,τ_1,τ_2)*, (1, 2)RO*(X,τ_1,τ_2)*, (1, 2)β*$O(X,\tau_1,\tau_2)$ *or simply (1, 2)SO(X), (1, 2)RO(X), (1, 2)β O(X) respectively.*

Definition 2.3 ([4]). *A subset A of X is said to be*

(i) *(1, 2)semi-closed if* $\tau_1\tau_2\text{-}Int(\tau_1\text{-}Cl(A)) \subseteq A$.

(ii) *(1, 2)regular-closed if* $A = \tau_1\text{-}Cl(\tau_1\tau_2\text{-}Int(A))$

(iii) *(1, 2)β- closed if* $\tau_1\tau_2\text{-}Int(\tau_1 - Cl(\tau_1\tau_2\text{-}Int(A))) \subseteq A$.

The set of all (1, 2)semi-closed, (1, 2)regular-closed, (1, 2)β-closed are denoted as (1, 2)SCL(X,τ_1,τ_2)*, (1, 2)RCL* (X,τ_1,τ_2)*, (1, 2)βCL*(X,τ_1,τ_2) *or simply (1, 2)SCL(X), (1, 2)RCL(X), (1, 2)βCL(X) respectively.*

Remark 2.4 ([6]). *For any subset A of X,*

(i) $\tau_1\text{-}Int(A) \subseteq \tau_1\tau_2\text{-}Int(A)$ *and* $\tau_2\text{-}Int(A) \subseteq \tau_1\tau_2\text{-}Int(A)$.

(ii) $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_1\text{-}Cl(A)$ *and* $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_2\text{-}Cl(A)$.

(iii) $\tau_1\tau_2\text{-}Cl(A \cap B) \subseteq \tau_1\tau_2\text{-}Cl(A) \cap \tau_1\tau_2\text{-}Cl(B)$.

(iv) $\tau_1\tau_2\text{-}Int(A) \cup \tau_1\tau_2\text{-}Int(B) \subseteq \tau_1\tau_2\text{-}Int(A \cup B)$.

Theorem 2.5 ([1]). *Let* (X,τ_1,τ_2) *be a bitopological space. If* $A \in \tau_1$ *and B ∈ (1, 2)SO(X), then A ∩ B ∈ (1, 2)SO(X).*

Theorem 2.6 ([1]). *Let* $A \subset Y \subset (X,\tau_1,\tau_2)$ *and if A is* τ_i*-semi open in X, then A is* τ_i*-semi open in Y.*

Definition 2.7 ([9]). *A (1, 2)semi-open subset A of a bitopological space* (X,τ_1,τ_2) *is said to be (1, 2)*S_β*- open if for each* $x \in A$ *there exists a (1, 2)β - closed set F such that* $x \in F \subseteq A$.

Definition 2.8 ([10]). *In a bitopological space X, a subset B of X is said to be (1, 2)*S_β*-∧-set (* $\underset{(1,2)S_\beta}{\wedge}$ *-set) if* $B = B^{\overset{\wedge}{(1,2)S_\beta}}$,

where $B^{\overset{\wedge}{(1,2)S_\beta}} = \cap\{G/G \supseteq B$ *and* $G \in$ *(1, 2)*S_β*-O(X)*$\}$.

Definition 2.9 ([10]). *In a bitopological space X, a subset B of X is said to be (1, 2)*S_β*-∨-set (* $\underset{(1,2)S_\beta}{\vee}$ *-set) if* $B = B^{\overset{\vee}{(1,2)S_\beta}}$,

where $B^{\overset{\vee}{(1,2)S_\beta}} = \cup\{F/F \subseteq B$ *and* $F \in$ *(1, 2)*$S_\beta CL(X)$.

Proposition 2.10 ([10]). *Let A and B be two subsets of a bitopological space X. Then the following properties are hold.*

(i) $B \subseteq B^{\overset{\wedge}{(1,2)S_\beta}}$.

(ii) $B^{\overset{\vee}{(1,2)S_\beta}} \subseteq B$.

(iii) *If* $A \subseteq B$*, then* $A^{\overset{\wedge}{(1,2)S_\beta}} \subseteq B^{\overset{\wedge}{(1,2)S_\beta}}$.

(iv) $\left(B^{\overset{\wedge}{(1,2)S_\beta}}\right)^{\overset{\wedge}{(1,2)S_\beta}} = B^{\overset{\wedge}{(1,2)S_\beta}}$.

(v) *If* $A \in$ *(1, 2)*$S_\beta O(X)$*, then* $A = A^{\overset{\wedge}{(1,2)S_\beta}}$.

(vi) $(B^c)^{\overset{\wedge}{(1,2)S_\beta}} = \left(B^{\overset{\vee}{(1,2)S_\beta}}\right)^c$, *(i.e)*$(X\text{–}B)^{\overset{\wedge}{(1,2)S_\beta}} = X\text{–}B^{\overset{\vee}{(1,2)S_\beta}}$

Definition 2.11 ([10]). *In a bitopological space X, a subset B is called*

(i) *generalized* $\underset{(1,2)S_\beta}{\wedge}$ *-set (briefly g.* $\underset{(1,2)S_\beta}{\wedge}$ *-set) of X if* $B^{\overset{\wedge}{(1,2)S_\beta}} \subseteq F$ *whenever* $B \subseteq F$ *and* $F \in (1,2)S_\beta CL(X)$*. The family of all g.* $\underset{(1,2)S_\beta}{\wedge}$ *-sets of X is denoted as* $D^{\overset{\wedge}{(1,2)S_\beta}}$*(X).*

(ii) *generalized* $\underset{(1,2)S_\beta}{\vee}$ *-set (briefly g.* $\underset{(1,2)S_\beta}{\vee}$ *-set) of X if* B^c *is a g.* $\underset{(1,2)S_\beta}{\wedge}$ *-set. The family of all g.* $\underset{(1,2)S_\beta}{\vee}$ *-set of X is denoted as* $D^{\overset{\vee}{(1,2)S_\beta}}$*(X).*

Remark 2.12 ([10]). *In a bitopological space X, every* $\underset{(1,2)S_\beta}{\wedge}$ *-set is g.* $\underset{(1,2)S_\beta}{\wedge}$ *-set and every* $\underset{(1,2)S_\beta}{\vee}$ *- set is g.* $\underset{(1,2)S_\beta}{\vee}$ *-set.*

Proposition 2.13 ([10]). *Let (X,* τ_1*,* τ_2*) be a bitopological space. Then the following properties hold:*

(i) *If* $B_i \in D^{\overset{\wedge}{(1,2)S_\beta}}$ *for all* $i \in I$*, then* $\bigcup_{i\in I} B_i \in D^{\overset{\wedge}{(1,2)S_\beta}}$.

(ii) *If* $B_i \in D^{\overset{\vee}{(1,2)S_\beta}}$ *for all* $i \in I$*, then* $\bigcap_{i\in I} B_i \in D^{\overset{\vee}{(1,2)S_\beta}}$.

Proposition 2.14 ([10]). *Let X be a bitopological space .Then*

(i) *for each* $x \in X$*, either* $\{x\}$ *is a (1, 2)*S_β*-open or* $\{x\}^c$ *is g.* $\underset{(1,2)S_\beta}{\wedge}$ *-set.*

(ii) *for each* $x \in X$*, either* $\{x\}$ *is a (1, 2)*S_β*-open or* $\{x\}$ *is g.* $\underset{(1,2)S_\beta}{\vee}$ *-set.*

3. A New Closure Operator $C^{\overset{\wedge}{(1,2)S_\beta}}$

Definition 3.1. *For any subset B of a bitopological space X, we define* $C^{\overset{\wedge}{(1,2)S_\beta}}(B) = \cap\{G : B \subseteq G$ *and* $G \in D^{\overset{\wedge}{(1,2)S_\beta}}\}$ *and* $Int^{\overset{\vee}{(1,2)S_\beta}}(B) = \cup\{F : F \subseteq B$ *and* $F \in D^{\overset{\vee}{(1,2)S_\beta}}$.

Example 3.2. *Let X = {a, b,c,d} with two topologies* τ_1= {ϕ, X, {a}, {b}, {a, b} }, τ_2 = {ϕ, X, {b,c,d} }. *Then (1, 2)* $S_\beta O(X)$ = {ϕ, X, {a}, {b,c,d} } *and (1, 2)* $S_\beta CL(X)$ = {ϕ, X, {b,c,d}, {a} }. *Now,* $\underset{(1,2)S_\beta}{\wedge} O(X)$ = {ϕ, X, {a}} *and* $D^{\overset{\wedge}{(1,2)S_\beta}}$ = {ϕ, X, {a}, {a,b}, {a,c}, {a,d}, {a, b, c}, {a, b, d}, {a, c, d }}, $D^{\overset{\vee}{(1,2)S_\beta}}$ = {ϕ, X, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d }}. *Then* $C^{\overset{\wedge}{(1,2)S_\beta}}(X)$= {$\phi$, X, {a}, {b}, {a,b}, {a,c}, {a,d}, {a, b, c}, {a, b, d}, {a, c, d }} *and* $Int^{\overset{\wedge}{(1,2)S_\beta}}(X)$ = {ϕ, X, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d }}.

Proposition 3.3. *Let A and B be subsets of a bitopological space X. Then,*

(i) $C^{\overset{\wedge}{(1,2)S_\beta}}(B^c) = (Int^{\overset{\vee}{(1,2)S_\beta}}(B))^c$

(ii) if $A \subseteq B$, *then* $C^{\overset{\wedge}{(1,2)S_\beta}}(A) \subseteq C^{\overset{\wedge}{(1,2)S_\beta}}(B)$.

(iii) if B is a g. $\underset{(1,2)S_\beta}{\wedge}$*-set, then* $C^{\overset{\wedge}{(1,2)S_\beta}}(B) = B$.

(iv) if B is a g. $\underset{(1,2)S_\beta}{\vee}$*-set, then* $Int^{\overset{\vee}{(1,2)S_\beta}}(B) = B$.

Proof. From Definitions 1.11 and 2.1. □

Theorem 3.4. $C^{\overset{\wedge}{(1,2)S_\beta}}$ *is a Kuratowski's closure operator.*

Proof. (i) $C^{\overset{\wedge}{(1,2)S_\beta}}(\phi) = \phi$ is obvious.

(ii) $A \subset C^{\overset{\wedge}{(1,2)S_\beta}}$ (A) is true from the definition 2.1.

(iii) Now to prove $C^{\overset{\wedge}{(1,2)S_\beta}}(A \cup B) = C^{\overset{\wedge}{(1,2)S_\beta}}(A) \cup C^{\overset{\wedge}{(1,2)S_\beta}}(B)$.

Suppose, there exists a point $x \in X$ such that $x \notin C^{\overset{\wedge}{(1,2)S_\beta}}(A \cup B)$. Then there exists a subset $G \in D^{\overset{\wedge}{(1,2)S_\beta}}$ such that $A \cup B \subseteq G$ and $x \notin G$. Then $A \subseteq G$, $B \subseteq G$ and $x \notin G$, which implies $x \notin C^{\overset{\wedge}{(1,2)S_\beta}}(A)$ and $x \notin C^{\overset{\wedge}{(1,2)S_\beta}}(B)$. So, $C^{\overset{\wedge}{(1,2)S_\beta}}(A) \cup C^{\overset{\wedge}{(1,2)S_\beta}}(B) \subseteq C^{\overset{\wedge}{(1,2)S_\beta}}(A \cup B)$.

Conversely, suppose that there exists a point $x \in X$ such that the closure operator $x \notin (C^{\overset{\wedge}{(1,2)S_\beta}}(A) \cup C^{\overset{\wedge}{(1,2)S_\beta}}(B))$. Then there exists two sets G_1 and G_2 in $D^{\overset{\wedge}{(1,2)S_\beta}}$ such that $A \subseteq G_1$ and $B \subseteq G_2$ but $x \notin G_1$ and $x \notin G_2$. Now, let $G = G_1 \cup G_2$. Then by proposition 1.14, $G \in D^{\overset{\wedge}{(1,2)S_\beta}}$. Then $A \cup B \subseteq G$ and $x \notin G$ and so $x \notin C^{\overset{\wedge}{(1,2)S_\beta}}(A \cup B)$. Then $C^{\overset{\wedge}{(1,2)S_\beta}}(A \cup B) \subseteq C^{\overset{\wedge}{(1,2)S_\beta}}(A) \cup C^{\overset{\wedge}{(1,2)S_\beta}}(B)$.

(iv) To prove $C^{\overset{\wedge}{(1,2)S_\beta}}(C^{\overset{\wedge}{(1,2)S_\beta}}(B)) = C^{\overset{\wedge}{(1,2)S_\beta}}(B)$.

Suppose, there exists a point $x \in X$ such that $x \notin C^{\overset{\wedge}{(1,2)S_\beta}}(B)$. Then there exists a $U \in D^{\overset{\wedge}{(1,2)S_\beta}}$ such that $x \notin U$ and $B \subseteq U$. By proposition 2.3, $C^{\overset{\wedge}{(1,2)S_\beta}}(B) \subseteq C^{\overset{\wedge}{(1,2)S_\beta}}(U) = U$. Thus, we have $x \notin C^{\overset{\wedge}{(1,2)S_\beta}}(C^{\overset{\wedge}{(1,2)S_\beta}}(B))$. Hence $C^{\overset{\wedge}{(1,2)S_\beta}}(C^{\overset{\wedge}{(1,2)S_\beta}}(B)) \subseteq C^{\overset{\wedge}{(1,2)S_\beta}}(B)$. Also by (ii), $C^{\overset{\wedge}{(1,2)S_\beta}}(B) \subseteq C^{\overset{\wedge}{(1,2)S_\beta}}(C^{\overset{\wedge}{(1,2)S_\beta}}(B))$. Therefore, $C^{\overset{\wedge}{(1,2)S_\beta}}(C^{\overset{\wedge}{(1,2)S_\beta}}(B)) = C^{\overset{\wedge}{(1,2)S_\beta}}(B)$. □

Definition 3.5. *Let* $\tau^{\overset{\wedge}{(1,2)S_\beta}}$ *be a bitopological space generated by* $C^{\overset{\wedge}{(1,2)S_\beta}}$ *in the usual manner. Then,*

(i) $\tau^{\overset{\wedge}{(1,2)S_\beta}} = \{B : B \subseteq X, C^{\overset{\wedge}{(1,2)S_\beta}}(B^c) = B^c\}$. *Here we also define another family of subsets,*

(ii) $\rho^{\overset{\wedge}{(1,2)S_\beta}} = \{B : C^{\overset{\wedge}{(1,2)S_\beta}}(B) = (B)\}$ *and we can also define that*

(iii) $\rho^{\overset{\wedge}{(1,2)S_\beta}} = \{B : B^c \in \tau^{\overset{\wedge}{(1,2)S_\beta}}\}$

Theorem 3.6. *For a space X, the following hold:*

(i) $\tau^{\overset{\wedge}{(1,2)S_\beta}} = \{B : B \subseteq X, Int^{\overset{\vee}{(1,2)S_\beta}}(B) = B\}$.

(ii) (1, 2)$S_\beta O(X) \subseteq D^{\overset{\wedge}{(1,2)S_\beta}} \subseteq \rho^{\overset{\wedge}{(1,2)S_\beta}}$.

(iii) (1, 2)$S_\beta CL(X) \subseteq D^{\overset{\vee}{(1,2)S_\beta}} \subseteq \tau^{\overset{\wedge}{(1,2)S_\beta}}$.

Proof. (i) Let $A \subseteq X$. Then $A \in \tau^{\overset{\wedge}{(1,2)S_\beta}}$ if and only if $C^{\overset{\wedge}{(1,2)S_\beta}}(A^c) = A^c\}$. By proposition 2.3, $C^{\overset{\wedge}{(1,2)S_\beta}}(A^c) = [Int^{\overset{\vee}{(1,2)S_\beta}}(A)]^c = A^c$, which implies $Int^{\overset{\wedge}{(1,2)S_\beta}}(A) = A$ and so $A \in \tau^{\overset{\wedge}{(1,2)S_\beta}}$.

(ii) Let $B \in$ (1, 2)S_βO(X). Then B is a $\underset{(1,2)S_\beta}{\wedge}$-set and by remark 1.12, B is a g. $\underset{(1,2)S_\beta}{\wedge}$-set. So $B \in D^{\overset{\wedge}{(1,2)S_\beta}}$. Then $C^{\overset{\wedge}{(1,2)S_\beta}}(B) = B$ which implies $B \in \rho^{\overset{\wedge}{(1,2)S_\beta}}$.
Hence, (1,2)S_βO(X)$\subseteq D^{\overset{\wedge}{(1,2)S_\beta}} \subseteq \rho^{\overset{\wedge}{(1,2)S_\beta}}$.

(iii) Let $B \in$ (1, 2)S_βCL(X). By remark 1.12, B is a g. $\underset{(1,2)S_\beta}{\vee}$-set. So $B \in D^{\overset{\vee}{(1,2)S_\beta}}$ and so $Int^{\overset{\vee}{(1,2)S_\beta}}(B) = B$, which implies $C^{\overset{\wedge}{(1,2)S_\beta}}(B^c) = B^c$. So $B \in \tau^{\overset{\wedge}{(1,2)S_\beta}}$.
Hence (1,2)$S_\beta CL(X) \subseteq D^{\overset{\vee}{(1,2)S_\beta}} \subseteq \tau^{\overset{\wedge}{(1,2)S_\beta}}$. □

Proposition 3.7. *If (1, 2)$S_\beta O(X) = \tau^{\wedge_{(1,2)S_\beta}}$, then every singleton set $\{x\}$ of X is $\tau^{\wedge_{(1,2)S_\beta}}$- open.*

Proof. Suppose, $\{x\}$ is not (1, 2)S_β -open, By proposition 1.13, $\{x\}^c$ is a g. $\wedge_{(1,2)S_\beta}$ -set and so $x \in \tau^{\wedge_{(1,2)S_\beta}}$. If $\{x\}$ is (1, 2)S_β -open, then by assumption, $\{x\} \in \tau^{\wedge_{(1,2)S_\beta}}$. □

Proposition 3.8. *Let X be a bitopological space. Then,*

(i) if (1, 2)$S_\beta CL(X) = \tau^{\wedge_{(1,2)S_\beta}}$, then every g. $\wedge_{(1,2)S_\beta}$ -set of X is (1, 2)S_β-open.

(i) if every g. $\wedge_{(1,2)S_\beta}$ -set of X is (1, 2)S_β-open, then $\tau^{\wedge_{(1,2)S_\beta}} = \{B : B \subseteq X, B = B^{\wedge_{(1,2)S_\beta}}\}$.

Proof. (i) Let B be a g. $\wedge_{(1,2)S_\beta}$ -set of X. Then $B \in D^{\wedge_{(1,2)S_\beta}}$ and by theorem 2.6, $B \in \rho^{\wedge_{(1,2)S_\beta}}$ and so $B^c \in \tau^{\wedge_{(1,2)S_\beta}}$. By assumption, $B^c \in$ (1, 2)$S_\beta CL(X)$. Hence $B \in$ (1, 2)S_βO(X).

(ii) Let $A \subseteq X$ and $A \in \tau^{\wedge_{(1,2)S_\beta}}$. Then $C^{\wedge_{(1,2)S_\beta}}(A^c) = A^c = \cap\{G : A^c \subseteq G \text{ and } G \in D^{\wedge_{(1,2)S_\beta}}\} = \cap\{G : A^c \subseteq G and G \in$ (1, 2)$S_\beta O(X)\}$.(by assumption) = $(A^c)^{\wedge_{(1,2)S_\beta}}$. Then by proposition 1.10, $A^c = (A^c)^{\wedge_{(1,2)S_\beta}} = X – A^{\vee_{(1,2)S_\beta}}$. So we get $A = A^{\vee_{(1,2)S_\beta}}$. That is, $A \in \tau^{\wedge_{(1,2)S_\beta}} = \{B : B \subseteq X$ and $B = B^{\vee_{(1,2)S_\beta}}\}$.

□

Remark 3.9. *From definitions 1.9, 2.1 and theorem 2.6, we have that (1, 2)$S_\beta CL(X) \subseteq \vee_{(1,2)S_\beta} O(X) \subseteq D^{\vee_{(1,2)S_\beta}} \subseteq \tau^{\wedge_{(1,2)S_\beta}}$.*

4. Conclusion

In this work, we have defined a new closure operator of $\wedge_{(1,2)S_\beta}$ -set and generated $\tau^{\wedge_{(1,2)S_\beta}}$, $\rho^{\wedge_{(1,2)S_\beta}}$ using (1,2)S_β- open sets in bitopological spaces and studied some of their properties. Also, these findings will help to carry out more theoretical research for future researchers.

References

[1] Alias B.Khalaf, Nehmat K.Ahmed,: S_β-open sets and S_β-continuity in topological spaces, *Thai Journal of Mathematics,* 11(2)(2013), 319-335.

[2] Ameen,Z.A.: A New Class of Semi-open sets in topological spaces, *M.Sc., Thesis, College of Science, Dohuk Univ*, 2007.

[3] Andrijevic, D.: Semi-pre open sets Mat. *Vesnik*, 38(1)(1986), 24-32.

[4] Fututake, T.: On generalized closed sets in Bitopological Spaces, *Bul.Fakuoka Univ. Edn*. 35, Part III (1985), 19-28.

[5] Kelly, J, C., Bitopological spaces proc. *London Math. Soc.*, 13(3)(1963), 71-89.

[6] Lellis Thivagar, M.: Generalization of (1,2)α-continuous functions, *Pure and Applied Mathematicka Sciences*, 28(1991), 55-63.

[7] Maki, H.: Generalized ∧-sets and the associated closure operator, in the special issue in commemoration of *Prof.kazusada Ikeda's, Retirement,* (1986), 139-146.

[8] Navalagi, G, Lellis Thivagar, M And Raja Rajeswari, R. : On some extension of semi-pre open sets in Bitopological spaces, *Mathematical Forum vol XVII,* (2004-2005), 63-75.

[9] Subprabha, V , Durgadevi, N.: A New Approach to (1, 2)Semipre-open sets in Bitopological Spaces, *SSRG-International Journal of Mathematics Trends and Technology* (SSRG-IJMTT), ISSN : 2231-5373 (58-61).

[10] Subprabha, V , Infant Vijula, P, T, Durgadevi, N.: New forms of open and closed sets using (1,2)S_β-Open sets in bitopological spaces, *Malaya Journal of Matematik,* 9(1)(2021), 1040-1042.

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