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The effect of internal heat generation (absorption) and Prandtl number on MHD mixed convection flow from a vertical flat plate

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Abstract

An investigation is carried out to examine the impacts of internal heat generation (absorption) and Prandtl number on MHD mixed convection boundary layer flow and heat transfer from a vertical flat plate. Proper transformation is used to form a system of coupled non linear partial differential equations for governing both the flow and heat transfer. These conditions have been settled numerically by using an implicit finite difference system called Keller-box method. The present analysis reveals that both velocity and temperature distributions are upsurges as internal heat generation (absorption) parameter increases; henceforth thickness of the thermal and momentum boundary layer increments. The velocity diminishes and temperature increments with the rise in magnetic parameter along the plate surface.

Keywords

Convection, Heat Generation (Absorption), MHD, Prandtl number.

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Contents

1. Introduction

Mixed convection, one of the transport phenomena, is the free and forced convection flow composition. These flow patterns are found at the same time by together an outside forcing mechanism and inside volumetric forces. It is used in numerous geophysical, environmental and energy-related engineering applications including wind-free solar panels, electronic gadgets known as fans nuclear reactors cooled at several stage of crisis and heat exchangers installed in a low velocity environment.

In a mixed convective flow, the impacts of both forced and free convection are found in equivalent order. In several realistic fields, we found significant variations in temperature between the hot body surface and the free stream. These temperature difference cause density gradients in the fluid medium and in presence of gravitational mixed convection influences become significant. The simplest physical model of such flow is the two-dimensional mixed laminar convective flow over a vertical flat plate extensive studies of which had been directed by [1-9]. It has, usually been recognized that($\xi = Gr_x/Re_x^2$, where Gr_x - Grashof number and Re_x - Reynolds number) is the governing parameter for the laminar boundary mixed convective flow, which represents the ratio of buoyancy forces to the inertial forces within the boundary layer. However, forced convection occur when the utmost of ξ goes to zero, which happens at the main edge and the natural convection cutoff, can be come to if ξ turns out to be enormous.

In many technical and physical problems of endothermic or exothermic reaction of fluid, it is exceptionally critical to study the impact of heat generation on these moving fluids, observe Vajravelu[10], Vajravelu and Hadjinicolaou [11]. Westphalia et al., [12] demonstrated that free heat induced convection can be applied to modeling of combustion. Although exact modeling of internal heat generation is very troublesome for most physical circumstances, some simple mathematical models may convey normal behavior. Following it, a great deal of work has been done over the most recent couple of years by many investigators [13-17] on the impact of internal heat generation on free convection flow.

The application of MHD has great impact in many processes

in engineering and industrial. MHD flows and heat transfer were therefore examined in the field of astrophysics, geophysics, electrical transformers, coils for cooling, pumps, meters, bearings and petroleum production. The magnetic field has a propensity to speed up the motion of the fluid, due to the Lorentz force; as a consequence the escalation of the boundary layer is reduced. Several authors [18-24] have investigated the impacts of magnetic fields on boundary layer flow and heat transfer.
On basis of previous observations, we have considered the flow over a vertical flat plate. The main aim of the present investigation is to study the effect of internal heat generation (absorption) and Prandtl number on MHD mixed convection boundary layer flow and heat transfer from a vertical flat plate. Provided that the end product is proportional to the rate of heat transfer, the heat exchange phenomenon that has applicability to real life problems is the most frequent subject of boundary layer flow.

2. Mathematical Formulation

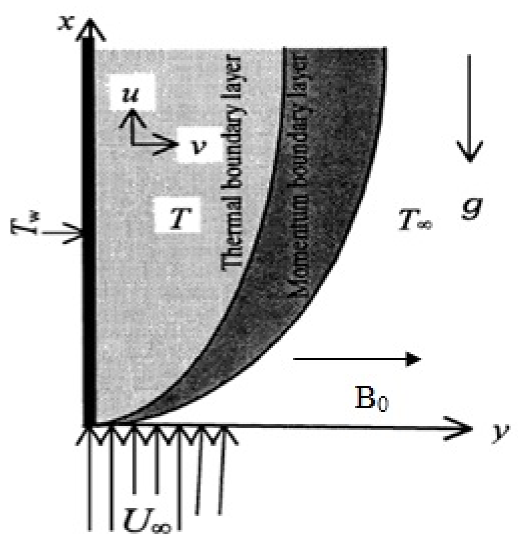


Figure 1 The flow configuration and the coordinate system.

We consider the steady laminar mixed convective flow of a viscous incompressible electrically conducting fluid along a semi-infinite vertical flat plate. The plate is situated at the $x-y$ plane. A uniform magnetic field B_0 is applied normal to the plate in the y-direction and the flow number of magnetic Reynolds is believed to be minute, so that the induced magnetic field can be ignored. The temperature of the plate T_w is uniform and more than the temperature of the free stream T_∞. It is also assume uniform free stream velocity U_∞, parallel to the vertical plate. We further assume that property variation with temperature is limited to density and viscosity with the density considered uniquely to the extent that its effects the buoyancy term in the momentum equation (Boussinesq approximation) only. Figure 1 shows a schematic diagram of the flow domain and the coordinate system.

The boundary layer equations for a fluid's mixed convective flow through the vertical plate are given below, according to the above assumptions.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=g\beta(T-T_\infty)+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho} \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_\infty) \tag{2.3}$$

Here u and v are the components of the fluid along the x and y axes parallel to the plate respectively, g –acceleration due to gravity, ρ -fluid density, C_p specific heat at constant pressure, α - thermal diffusivity, β- thermal expansion coefficient, M- magnetic field parameter, Q - internal heat generation or absorption and T- temperature within the boundary layer.
The boundary conditions to be satisfied by the above equations are

$$\left.\begin{array}{c} u=0, v=0, T=T_w \ at\ y=0 \\ u=0,\ T=T_\infty\ as\ y\to\infty, \end{array}\right\} \tag{2.4}$$

Further,

$$\left.\begin{array}{c} \psi(x,y)=\nu_\infty Re_x^{\frac{1}{2}}(1+\xi)^{\frac{1}{4}}f(\xi,\eta),\ \ \eta=\frac{y}{x}Re_x^{\frac{1}{2}}(1+\xi)^{\frac{1}{4}}, \\ Re_x=\frac{U_\infty x}{\nu_\infty},\ \xi=\frac{Gr_x}{Re_x^{\frac{1}{2}}} \\ Gr_x=\frac{g\beta(T_w-T_\infty)x^3}{\nu_\infty^2},\ \ \theta(\xi,\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \end{array}\right\} \tag{2.5}$$

Where ψ is the stream function that satisfies the continuity equation (2.1) and is defined by,

$$u=\frac{\partial\psi}{\partial y}\quad and\quad v=-\frac{\partial\psi}{\partial x} \tag{2.6}$$

By replacing the above transformations in (2.1) to (2.4), we obtain:

$$f'''+G\left(\frac{\xi}{1+\xi}\right)+f''f\left(\frac{2+3\xi}{4(1+\xi)}\right)-f'^2\left(\frac{\xi}{2(1+\xi)}\right) -Mf'=\xi\left[f'f'_\xi-f''f_\xi\right] \tag{2.7}$$

$$G''+PrQG+\left(\frac{2+3\xi}{4(1+\xi)}\right)G'fPr=Pr\xi\left[f'G_\xi-G'f_\xi\right] \tag{2.8}$$

The heat generation or absorption parameter Q showing up in eqn.(2.8) is the non-dimensional parameter dependent on the quantity of heat produced or ingested per unit volume given by $Q_0(T-T_\infty)$ with Q_0 being constant coefficient that can be negative or positive. The source term represents the heat generation that is scattered wherever when $Q>0$, the heat absorption when $Q<0$ and $Q=0$, if there is no heat generation or absorption.
The transformed boundary conditions are:

$$\left.\begin{array}{l} f' = f = 0, \quad G = 1 \quad \text{at} \quad \eta = 0 \\ f' = F = (1+\xi)^{\frac{-1}{2}} \quad G = 0 \quad \text{as} \quad \eta \to \infty \end{array}\right\} \quad (2.9)$$

Respectively, the local shear stress and the local surface heat flux can be communicated, as

$$\tau_x = \xi^{\frac{-1}{2}}(1+\xi)^{\frac{3}{4}} f''(\xi,0) \quad and \quad (2.10)$$

$$q_x = \xi^{\frac{-1}{2}}(1+\xi)^{\frac{1}{4}} G'(\xi,0) \quad (2.11)$$

3. Results and Discussion

The partial differential equations which are coupled nonlinear (2.7) and (2.8) are solved numerically using a very efficient and accurate implicit finite difference scheme called Keller-box method [25, 26]. In order to measure the reliability, the current research outcomes were equated with the previous study of Raju et.al [6] when Q= 0 and M=0 (i.e., in the absence of the internal heat generation (absorption) and magnetic field) for skin friction (τ_x) and heat transfer (q_x) for a variety of values of ξ , as shown in table 1, for Pr = 0.7. Our results are found to be in excellent conformity for correct four decimal places of accuracy with those of Raju et.al [6].

Table 1. Numerical values of $f''(\xi,0)and - G'(\xi,0)$ compared with [6] for Pr = 0.7 when Q = 0.0 and M = 0.0

ξ	$f''(\xi,0)$		$-G'(\xi,0)$	
	Present	[6]	Present	[6]
0.00000	0.3323	0.3321	0.2931	0.2928
0.29011	0.5742	0.5919	0.3333	0.3373
0.42115	0.6709	0.6889	0.3462	0.3505
0.52992	0.7348	0.7536	0.3547	0.3584
0.62920	0.7897	0.8020	0.3605	0.3639
0.72399	0.8230	0.8404	0.3651	0.3680
1.20059	0.9357	0.9743	0.3765	0.3823
1.77903	0.9888	1.0171	0.3802	0.3828
2.59153	1.0099	1.0282	0.3976	0.3794
3.87298	1.0140	1.0251	0.3770	0.3751
6.16948	1.0079	1.0145	0.3728	0.3706
11.06602	0.9959	1.0002	0.3685	0.3662
24.97999	0.9826	0.9848	0.3635	0.3619
99.99500	0.9693	0.9699	0.3585	0.3576
∞	0.9567	0.9570	0.3536	0.3531

The variance of local shearing stress (τ_x) and local heat flux (q_x) for different values of Q = (-0.5, -0.25, 0.0, 0.25, 0.5) along with $\xi = 1.0$ with Prandtl number = 0.72 is shown in figure 2. With an increase of Q, it shows that q_x increases and τ_x decreases. The rate of growth in τ_x is 35.23 % and 31.6 % in q_x at $\xi = 0.5$.

Figure 3 depicts the effects of internal heat generation (absorption) (Q) parameter on velocity [F] and temperature [G] distributions for fixed Prandtl number (Pr = 0.7). This can be seen that both the temperature and velocity profile upsurges with an enhance of Q (i.e., -0.5, -0.25, 0.0. 0.25, 0.5). It is expected since Q includes extra heat close to the plate region which enables the fluid to travel quicker and subsequently, fluid's velocity and temperature inside the boundary layer region extends. Further, it's also inspected that the thickness of the thermal and momentum boundary layers also increases slightly due to the rise in internal heat generation (absorption) (Q) parameter.

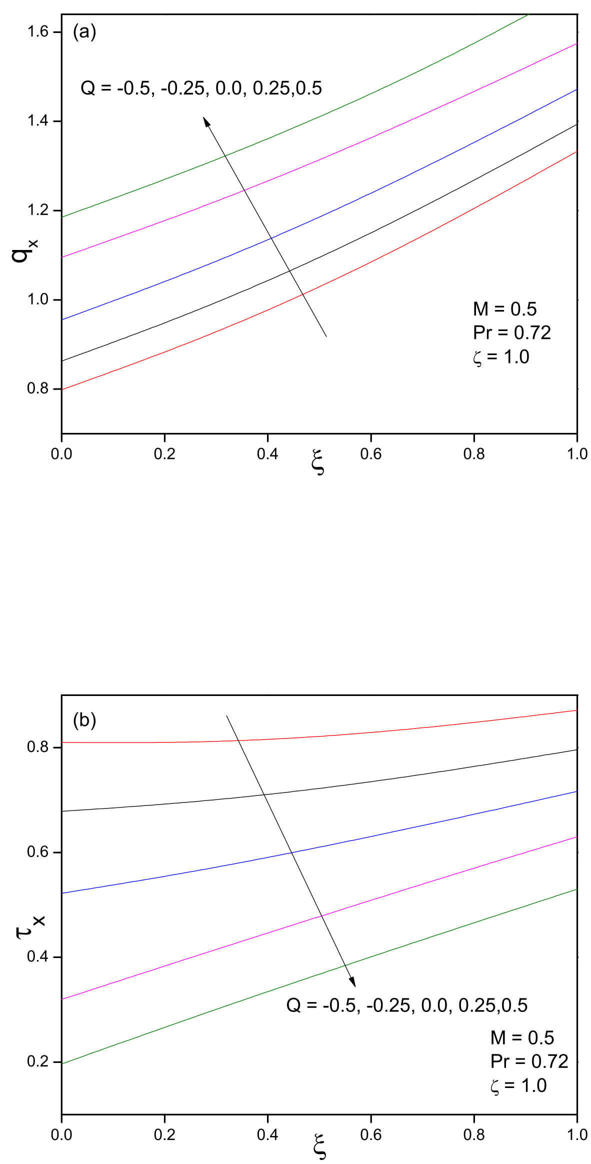


Figure 2. a) The rate of local heat flux and b) the local shearing stress for Q = -0.5,-0.25, 0.0, 0.25, 0.5 at Pr = 0.7 and M = 0.5

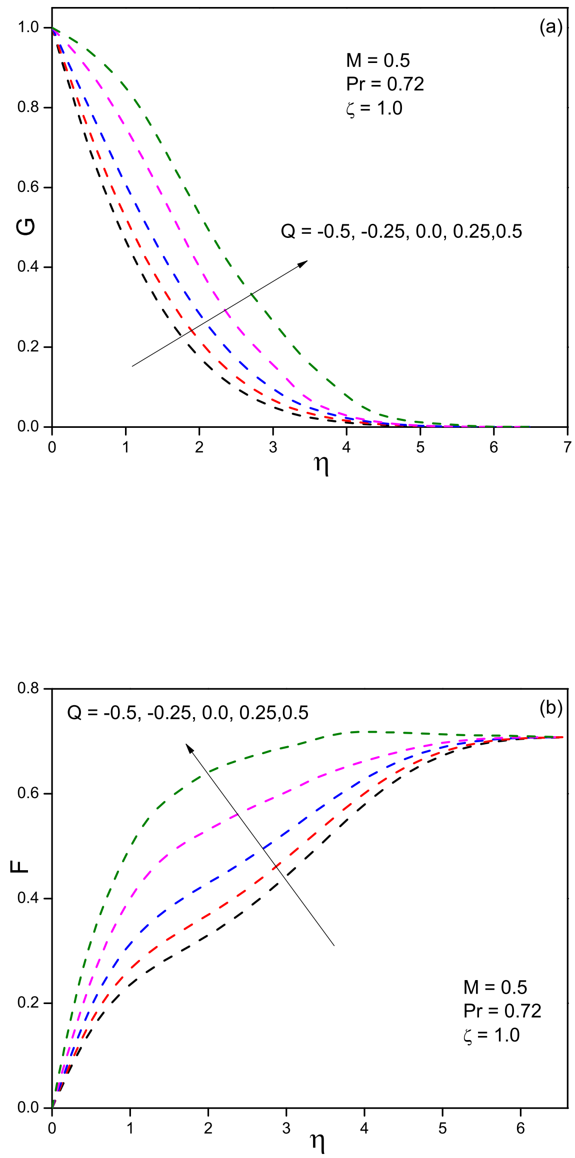


Figure 3 a) The temperature and b) the velocity profiles for Q = -0.5,-0.25, 0.0, 0.25, 0.5 at Pr = 0.72 and M = 0.5

Figure 4, shows how the flow reacts to a magnetic field transition. When the other parameters are kept stable, these figures display the differences of velocity and temperature profiles for expanding values of magnetic field parameter (M). In the figure 4(a) the rise in the parameter M of the magnetic field tends to minimize the velocity profile. The magnetic field related Lorentz force makes the boundary layer thinner by increasing the frequency of magnetic parameters. At free stream velocity, magnetic lines of force pass past the plate. The fluid decelerated by the viscous motion experiences a boost that counteracts the viscous effects from the magnetic field. Furthermore the change in temperature profile (G), in the η-direction also indicates the standard mixed convection boundary wall temperature profile where the thermal boundary layer thickness gradually decreases along the η-direction for M values from 0.0 to 0.5. As a result of excess heating, the magnetic field raises the temperature of the fluid inside the boundary layer and thus decreases the heat flux, as shown in figure 4(b).

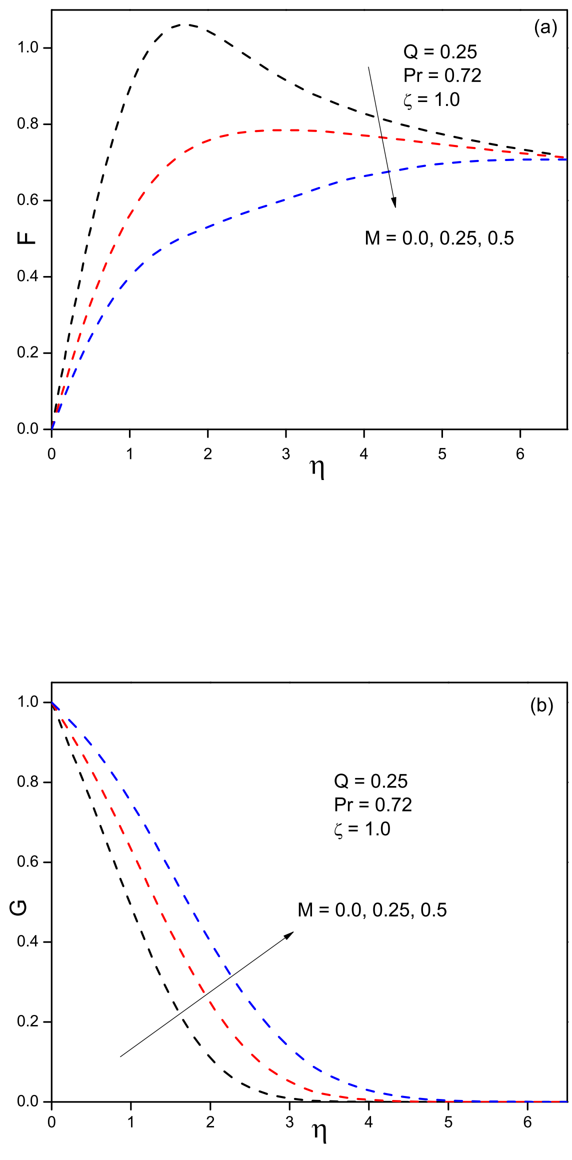
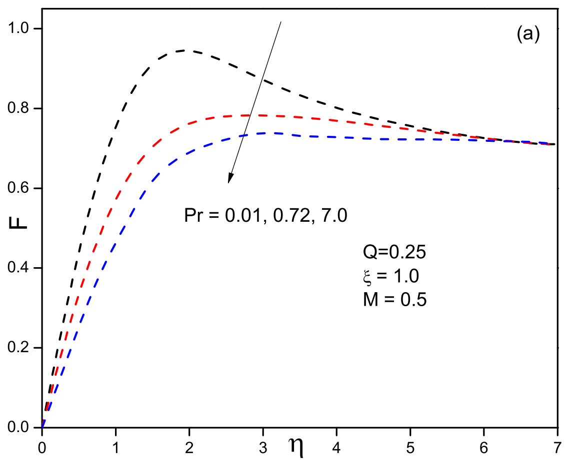


Figure 4. a) The velocity and b) the temperature profiles for M = 0.0, 0.25, 0.5 at Pr = 0.72 and Q = 0.25

The effects of Prandtl number (Pr = 0.01, 0.72, 7.0) for fixed Q = 0.25 and M = 0.5 on velocity profiles are displayed in figure. 5(a) and show the velocity distributions near the wall are significantly lower than the higher Prandtl number. The excess shooting magnitude decreases as the amount of Prandtl number increases. The impact is strong in low Prandtl number (Pr = 0.01) because of the low fluid viscosity, which increases the velocity inside the boundary layer. Whereas the impact of Prandtl number on the temperature profile is portrayed in figure. 5(b). As Prandtl number expands, the mixed convection process become stronger and can carry more heat generated internally, leaving less energy for the back heat flow. This clarifies why the plate surface temperatures in figure. 5(b) are lesser at higher numbers of the Prandtl.



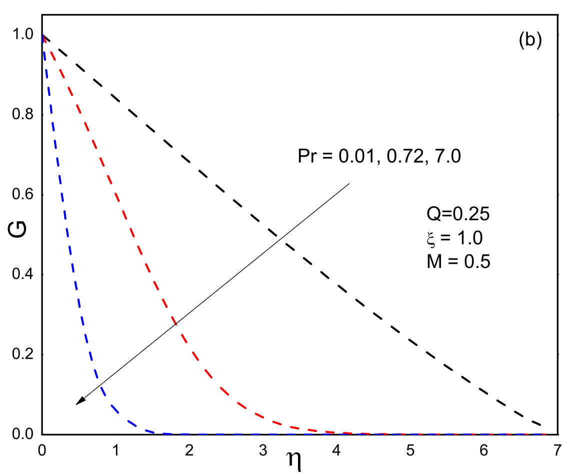


Figure 5. a) The velocity and b) the temperature profiles for Pr = 0.01, 0.72, 7.0 at M = 0.5 and Q = 0.25

Additionally note that, With the expansion of Prandtl number, diffusivity of momentum increments. As Pr expands, thermal diffusivity diminishes while fluid velocity increments, bringing about diminished temperature and velocity profiles. The relative thickness of the boundary layers thermal and momentum is regulated by Prandtl number. Heat diffuses steadily, and thermal boundary layer is lower for greater Prandtl number values, whereas the opposite behavior is observed at lower Pr values.

Conclusion

The effect of an internal heat generation (absorption) on MHD mixed convection flow from a vertical flat plate is considered here. From the current investigation the result can be summarized as follows.

- The local shear stress diminishes and the rate of local heat flux escalates with a rise in values of an internal heat generation (absorption) parameter for the fixed Prandtl number.
- Both velocity and temperature profiles are upsurges as internal heat generation parameter increases, hence thickness of the thermal and momentum boundary layer increases.
- The velocity diminishes and temperature increments with the rise in magnetic parameter along the plate surface.
- Increasing the Prandtl number values for the fixed internal heat generation parameter leads to a reduction in the velocity and temperature profile.

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