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# Fuzzy quotient-3 cordial labeling on some path related graphs - Paper I

P. Sumathi[1] and J. Suresh Kumar[2]*

**Abstract**

*In this paper, we discuss the existence of fuzzy quotient-3 cordial labeling on some path related graphs. Let $G(V,E)$ be a simple, finite and planar graph of order $p$ and size $q$. Let $\sigma : V(G) \to [0,1]$ defined by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$ and for each $uv \in E(G)$, define $\mu$: $E(G) \to [0,1]$ by $\mu(uv) = \frac{1}{10}\lceil\frac{3\sigma(u)}{\sigma(v)}\rceil$ where $\sigma(u) \leq \sigma(v)$. When $v_\sigma(i)$ and $v_\sigma(j)$ differ by atmost 1 and $e_\mu(i)$ and $e_\mu(j)$ differ by atmost 1, the graph $G$ is fuzzy quotient-3 cordial graph. where $v_\sigma(i)$ and $e_\mu(i)$ denotes the number of vertices and edges assigned with $i \in \{\frac{\gamma}{10}, \gamma \in Z_4 - \{0\}\}$.*

**Keywords**

Cycle, Path, Fuzzy quotient-3 cordial graph.

**AMS Subject Classification**

05C38, 05C78.

[1] *Department of Mathematics, C. Kandaswami Naidu College for Men, Chennai, Tamil Nadu, India.*
[2] *Department of Mathematics, St. Thomas College of Arts and Science, Chennai, Tamil Nadu, India.*
***Corresponding author**: [2]jskumar.robo@gmail.com
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## Contents



## 1. Introduction

In the field of discrete mathematics the graph theory has the wonderful and remarkable application not only in theoretical mathematics but also in the real time applications. The applications of this graph theory is very wide in all the field of sciences like Biochemistry, Molecular biology and Immune molecular biology on molecules apart from the basic sciences like Physics and Chemistry. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The labeling of graphs was studied in decades in the field of graph theory which is the subfield of the same as it has wonderful application of real time utility and the cases of sciences apart from that with help of labeling some of the complicated problems were simplified and got derived the simple solution for the further applications. Most graph labelling methods trace their origin to one introduced by Rosa in 1967, or one given by Graham and Sloane in 1980. Excellent survey of graph labeling was given by J.A. Gallian [1]. The idea of cordial labeling was introduced by cahit [2]. A. Sundaram, Ponraj and somasundaram [3] introduced the of product cordial labeling. Sundaram and Somasundaram [4] also have introduced the notion of total product cordial labeling. Ponraj, Sivakumar and Sundaram [6] introduced the notion k-Product cordial labeling of graphs. Jeyanthi.P and Maheshwari A introduced 3- Product Cordial labeling [7]. Ponraj R and Adaickalam M. M. proved Quotient cordial labeling of some star related graphs [10]. Sumathi.P, Mahalakshmi A and Rathi A introduced Quotient -3 cordial labeling. Motivated by these labelings we, Sumathi. P, Suresh Kumar. J introduced fuzzy quotient-3 cordial labeling in[11] and proved fuzzy quotient-3 cordial labelling for some star related graphs. In this paper we proved that some path related graphs are fuzzy quotient -3 Cordial.

## 2. Preliminaries

**Definition 2.1.** *A graph G with order p and size q is fuzzy quotient-3 cordial graph if there exists a function $\sigma : V(G) \to [0,1]$ defined by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$ and for each $uv \in E(G)$, define $\mu$: $E(G) \to [0,1]$ by $\mu(uv) = \frac{1}{10}\lceil\frac{3\sigma(u)}{\sigma(v)}\rceil$ where $\sigma(u) \leq \sigma(v)$ such that $v_\sigma(i)$ and $v_\sigma(j)$ differ by atmost 1 and*

*$e_\mu(i)$ and $e_\mu(j)$ differ by atmost 1 where $v_\sigma(i)$ and and $e_\mu(i)$ represents the number of vertices and edges assigned with i $\in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.*

## 3. Main Results

**Theorem 3.1.** *The graph obtained from a path $P_m$ by attaching its one end vertex to a cycle $C_n$ through an edge is fuzzy quotient-3 cordial.*

*Proof.* Let $G$ be a graph obtained from a path $P_m$ by attaching its one end vertex to a cycle $C_n$ through an edge.
The vertex set $V(G) = \{x_\kappa : 1 \leq \kappa \leq n\} \cup \{y_\kappa : 1 \leq \kappa \leq m\}$ and the edge set $E(G) = \{x_\kappa x_{\kappa+1} : 1 \leq \kappa \leq n-1\} \cup \{x_1 x_n\} \cup \{x_n y_1\} \cup \{y_\kappa y_{\kappa+1} : 1 \leq \kappa \leq m-1\}$.
$|V(G)| = |E(G)| = n+m$.
Define $\sigma : V(G) \to [0,1]$ as $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$
The labeling of $x_\kappa\,'s$ and $y_\kappa\,'s$ are as follows.

*Case 1:* $n \equiv 0(mod6)$

$m \equiv 0,1,2,3,4,5(mod6)$
for $1 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 0,1,4,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 2(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$
$\sigma(y_m) = 0.2$

$m \equiv 3(mod6)$
for $1 \leq \kappa \leq m-3$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$
$\sigma(y_{m-2}) = 0.3$
$\sigma(y_{m-1}) = 0.1$
$\sigma(y_m) = 0.2$

*Case 2:* $n \equiv 1(mod6)$

$m \equiv 0,1,2,3,4(mod6)$
for $1 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 5(mod6)$
$\sigma(x_1) = 0.1$
$\sigma(x_2) = 0.3$
$\sigma(x_3) = 0.1$
for $4 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 0,3,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$

$m \equiv 1,4(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$
$\sigma(y_m) = 0.3$

$m \equiv 2(mod6)$
for $1 \leq \kappa \leq m-2$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$
$\sigma(y_{m-1}) = 0.3$
$\sigma(y_m) = 0.2$

*Case 3:* $n \equiv 2(mod6)$

$m \equiv 0,2,3,5(mod6)$
for $1 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 1(mod6)$
$\sigma(x_1) = 0.3$
$\sigma(x_2) = 0.1$
for $3 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$

$m \equiv 4(mod6)$
$\sigma(x_1) = 0.1$
$\sigma(x_2) = 0.3$
$\sigma(x_3) = 0.1$
for $4 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 0,3(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 0,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 2,3(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 1,4(mod6)$
$\sigma(y_m) = 0.3$

$m \equiv 1(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

$m \equiv 2,4,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 0,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 2,3(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 1,4(mod6)$

*Case 4:* $n \equiv 3(mod6)$

$m \equiv 0,1,2,3,4,5(mod6)$
for $1 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

$m \equiv 0(mod6)$
for $1 \leq \kappa \leq m-3$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$
$\sigma(y_{m-2}) = 0.3$
$\sigma(y_{m-1}) = 0.1$
$\sigma(y_m) = 0.2$

$m \equiv 1,2,3,4(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

$m \equiv 5(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$
$\sigma(y_m) = 0.2$

*Case 5:* $n \equiv 4(mod6)$

$m \equiv 0,1,3,4(mod6)$
for $1 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

$m \equiv 2,5(mod6)$
for $1 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

$m \equiv 0,1,3(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 2,5(mod6)$

$m \equiv 2(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 2,5(mod6)$

$m \equiv 4(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 2,5(mod6)$
$\sigma(y_m) = 0.3$

$m \equiv 5(mod6)$
for $1 \leq \kappa \leq m-3$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 2,5(mod6)$
$\sigma(y_{m-2}) = 0.3$
$\sigma(y_{m-1}) = 0.1$
$\sigma(y_m) = 0.2$

*Case 6:* $n \equiv 5(mod6)$

$m \equiv 0,2,3,5(mod6)$
for $1 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

$m \equiv 1,4(mod6)$
for $1 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

$m \equiv 0,2,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 2,3(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 0,5(mod6)$

$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 1,4(mod6)$

$m \equiv 1(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 0,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 2,3(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 1,4(mod6)$

$m \equiv 3(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 2,3(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 0,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 1,4(mod6)$
$\sigma(y_m) = 0.3$

$m \equiv 4(mod6)$
for $1 \leq \kappa \leq m-3$
$\sigma(y_\kappa) = 0.1 \quad if \kappa \equiv 0,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if \kappa \equiv 2,3(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if \kappa \equiv 1,4(mod6)$
$\sigma(y_{m-2}) = 0.3$
$\sigma(y_{m-1}) = 0.1$
$\sigma(y_m) = 0.2$

The number of vertices labeled with $i$, where $i \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ are tabulated below.

*Table 3.1.1.*

| Nature of n and m | $v_\sigma(0.1)$ | $v_\sigma(0.2)$ | $v_\sigma(0.3)$ |
|---|---|---|---|
| $n \equiv 0(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 0(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 0(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 0(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 0(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 0(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 1(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 1(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ |
| $n \equiv 1(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 1(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |

*Table 3.1.1. continued*

| Nature of n and m | $v_\sigma(0.1)$ | $v_\sigma(0.2)$ | $v_\sigma(0.3)$ |
|---|---|---|---|
| $n \equiv 1(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ |
| $n \equiv 1(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 3(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 3(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 3(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 3(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 3(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 3(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 4(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 4(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 4(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 4(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 4(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ |
| $n \equiv 4(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 5(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 5(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 5(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 5(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ |
| $n \equiv 5(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |

*Table 3.1.1. Continued*

| Nature of n and m | $v_\sigma(0.1)$ | $v_\sigma(0.2)$ | $v_\sigma(0.3)$ |
|---|---|---|---|
| $n \equiv 5(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |

The number of edges labeled with $i$, where $i \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ are tabulated below.

*Table 3.1.2.*

| Nature of n and m | $e_\sigma(0.1)$ | $e_\sigma(0.2)$ | $e_\sigma(0.3)$ |
|---|---|---|---|
| $n \equiv 0(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 0(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 0(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 0(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 0(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 0(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ |
| $n \equiv 1(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}+1$ |
| $n \equiv 1(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ |
| $n \equiv 1(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 1(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}+1$ |
| $n \equiv 1(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ |
| $n \equiv 1(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}+1$ |
| $n \equiv 2(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 2(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}+1$ |
| $n \equiv 3(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |

*Table 3.1.2. continued*

| Nature of n and m | $e_\sigma(0.1)$ | $e_\sigma(0.2)$ | $e_\sigma(0.3)$ |
|---|---|---|---|
| $n \equiv 3(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 3(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 3(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 3(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ |
| $n \equiv 3(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 4(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ |
| $n \equiv 4(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ |
| $n \equiv 4(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 4(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}+1$ | $\frac{n+m-1}{3}$ |
| $n \equiv 4(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 4(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 5(mod6)$ $m \equiv 0(mod6)$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ |
| $n \equiv 5(mod6)$ $m \equiv 1(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 5(mod6)$ $m \equiv 2(mod6)$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}+1$ |
| $n \equiv 5(mod6)$ $m \equiv 3(mod6)$ | $\frac{n+m+1}{3}$ | $\frac{n+m+1}{3}-1$ | $\frac{n+m+1}{3}$ |
| $n \equiv 5(mod6)$ $m \equiv 4(mod6)$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ | $\frac{n+m}{3}$ |
| $n \equiv 5(mod6)$ $m \equiv 5(mod6)$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}$ | $\frac{n+m-1}{3}+1$ |

Above tables 3.1.1 and 3.1.2 shows that $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$ for $i \neq j$, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$, It is clear that the graph obtained from a path $P_m$ by attaching its one end vertex to a cycle $C_n$ through an edge satisfies the conditions of fuzzy quotient - 3 cordial labeling. Hence the theorem. □

**Theorem 3.2.** *The graph obtained from two copies of a path $P_m$ by attaching one through an edge to $\frac{n}{2}$ $^{th}$ and other through an edge to $n$ $^{th}$ vertex of an even cycle $C_n$ is fuzzy quotient-3 cordial.*

*Proof.* Let $G$ be a graph obtained from two copies of a path $P_m$ by attaching one through an edge to $\frac{n}{2}$ $^{th}$ and other through an edge to $n$ $^{th}$ vertex of an even cycle $C_n$.
The vertex set $V(G) = \{x_\kappa : 1 \leq \kappa \leq n\} \cup \{y_\kappa : 1 \leq \kappa \leq$

$m\} \cup \{z_\kappa : 1 \le \kappa \le m\}$ and the edge set of
$E(G) = \{x_\kappa x_{\kappa+1} : 1 \le \kappa \le n-1\} \cup \{x_1 x_n\} \cup \{x_n y_1\} \cup \{y_\kappa y_{\kappa+1} : 1 \le \kappa \le m-1\} \cup (x_{\frac{n}{2}} z_1) \cup \{(z_\kappa z_{\kappa+1}) : 1 \le \kappa \le m-1\}$
$|V(G)| = |E(G)| = n + 2m$.
Define $\sigma : V(G) \to [0,1]$ as $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$

The labeling of $x_\kappa\ 's$ are as follows.

*Case 1:* $n \equiv 0(mod12)$

$m \equiv 0,1,3,4(mod6)$
for $1 \le \kappa \le n$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 0,1(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 3,4(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 2,5(mod6)$

$m \equiv 2,5(mod6)$
for $1 \le \kappa \le n$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 0,3(mod6)$

*Case 2:* $n \equiv 2(mod12)$

$m \equiv 0,1,3,4(mod6)$
for $1 \le \kappa \le n$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 0,3(mod6)$

$m \equiv 2(mod6)$
for $1 \le \kappa \le n$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 0,3(mod6)$

$m \equiv 5(mod6)$
for $1 \le \kappa \le n-3$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 0,3(mod6)$
$\sigma(x_{n-2}) = 0.1$
$\sigma(x_{n-1}) = 0.3$
$\sigma(x_n) = 0.1$

*Case 3:* $n \equiv 4(mod12)$

$m \equiv 0,1,2,5(mod6)$
for $1 \le \kappa \le n-1$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 0,3(mod6)$
$\sigma(x_n) = 0.3$

$m \equiv 3(mod6)$
for $1 \le \kappa \le n$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 3,4(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 0,1(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 2,5(mod6)$

$m \equiv 4(mod6)$
for $1 \le \kappa \le n$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 0,3(mod6)$

*Case 4:* $n \equiv 6(mod12)$

$m \equiv 0,1,2,3,4,5(mod6)$
for $1 \le \kappa \le n$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 0,1(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 3,4(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 2,5(mod6)$

*Case 5:* $n \equiv 8(mod12)$

$m \equiv 0,1,3,4(mod6)$
for $1 \le \kappa \le n$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 0,3(mod6)$

$m \equiv 2,5(mod6)$
for $1 \le \kappa \le n-3$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 0,3(mod6)$
$\sigma(x_{n-2}) = 0.1$
$\sigma(x_{n-1}) = 0.3$
$\sigma(x_n) = 0.1$

*Case 6:* $n \equiv 10(mod12)$

$m \equiv 0,2,3,4,5(mod6)$
for $1 \le \kappa \le n$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 3,4(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 0,1(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 2,5(mod6)$

$m \equiv 1(mod6)$
for $2 \le \kappa \le n$
$\sigma(x_\kappa) = 0.1 \quad if\kappa \equiv 3,4(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\kappa \equiv 0,1(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\kappa \equiv 2,5(mod6)$
$\sigma(x_1) = 0.3$

The labeling of $y_\kappa\,'s$ are as follows.

*Case 1:* $n \equiv 0(mod12)$

$m \equiv 1,2,4,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 0(mod6)$
$\sigma(y_1) = 0.1$
$\sigma(y_2) = 0.3$
$\sigma(y_3) = 0.1$
for $4 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 3(mod6)$
for $1 \leq \kappa \leq m-2$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$
$\sigma(y_{m-1}) = 0.3$
$\sigma(y_m) = 0.1$

*Case 2:* $n \equiv 2(mod12)$

$m \equiv 0,1,3,4,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 0,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 2,3(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 1,4(mod6)$

$m \equiv 2(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 2,3(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 0,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 1,4(mod6)$

*Case 3:* $n \equiv 4(mod12)$

$m \equiv 0,1,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 2(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$
$\sigma(y_m) = 0.1$

$m \equiv 3(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 4(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 2,5(mod6)$

*Case 4:* $n \equiv 6(mod12)$

$m \equiv 0(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 1,2,3,4(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 2,5(mod6)$

$m \equiv 5(mod6)$
$\sigma(y_1) = 0.3$
$\sigma(y_2) = 0.1$
for $3 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 2,5(mod6)$

*Case 5:* $n \equiv 8(mod12)$

$m \equiv 0,2,3,4,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 0,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 2,3(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 1,4(mod6)$

$m \equiv 1(mod6)$
$\sigma(y_1) = 0.3$
for $2 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

*Case 6:* $n \equiv 10(mod12)$

$m \equiv 0,3(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 0,5(mod6)$

$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 2,3(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 1,4(mod6)$

$m \equiv 1(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 1,0(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$

$m \equiv 2(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 4,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

The labeling of $z_\kappa\,'s$ are as follows.

*Case 1:* $n \equiv 0(mod12)$

$m \equiv 0(mod6)$
$\sigma(z_1) = 0.1$
$\sigma(z_2) = 0.3$
$\sigma(z_3) = 0.1$
for $4 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 1(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$
$\sigma(z_m) = 0.3$

$m \equiv 2(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 0,5(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 2,3(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 1,4(mod6)$

$m \equiv 3(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 4(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$
$\sigma(z_m) = 0.3$

$m \equiv 5(mod6)$
for $1 \leq \kappa \leq m-2$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 2,3(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 0,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 1,4(mod6)$
$\sigma(z_{m-1}) = 0.2$
$\sigma(z_m) = 0.3$

*Case 2:* $n \equiv 2(mod12)$

$m \equiv 0(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 0,5(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 2,3(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 1,4(mod6)$
$\sigma(z_m) = 0.3$

$m \equiv 1(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 2,5(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$
$\sigma(z_m) = 0.3$

$m \equiv 3(mod6)$
for $1 \leq \kappa \leq m-2$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 2,3(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 0,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 1,4(mod6)$
$\sigma(z_{m-1}) = 0.1$
$\sigma(z_m) = 0.2$

$m \equiv 4(mod6)$
$\sigma(z_1) = 0.1$
$\sigma(z_2) = 0.3$
$\sigma(z_3) = 0.1$
for $4 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

*Case 3:* $n \equiv 4 (mod 12)$

$m \equiv 0,5 (mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 4,5 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 1,2 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3 (mod 6)$
$\sigma(z_m) = 0.2$

$m \equiv 1 (mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 3,4 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 0,1 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 2,5 (mod 6)$
$\sigma(z_m) = 0.2$

$m \equiv 2 (mod 6)$
for $1 \leq \kappa \leq m-2$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 2,3 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 0,5 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 1,4 (mod 6)$
$\sigma(z_{m-1}) = 0.3$
$\sigma(z_m) = 0.2$

$m \equiv 3 (mod 6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 4,5 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 1,2 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3 (mod 6)$

$m \equiv 4 (mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 3,4 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 0,1 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 2,5 (mod 6)$
$\sigma(z_m) = 0.3$

*Case 4:* $n \equiv 6 (mod 12)$

$m \equiv 0 (mod 6)$
for $1 \leq \kappa \leq m-5$
$(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3 (mod 6)$
$\sigma(z_{m-4}) = 0.3$
$\sigma(z_{m-3}) = 0.1$
$\sigma(z_{m-2}) = 0.3$
$\sigma(z_{m-1}) = 0.2$
$\sigma(z_m) = 0.2$

$m \equiv 1 (mod 6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 0,5 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 1,3 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 2,4 (mod 6)$

$m \equiv 2 (mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 4,5 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 1,2 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3 (mod 6)$
$\sigma(z_m) = 0.3$

$m \equiv 3 (mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 3,4 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 0,1 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 2,5 (mod 6)$
$\sigma(z_m) = 0.1$

$m \equiv 4 (mod 6)$
for $1 \leq \kappa \leq m-2$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 2,3 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 0,5 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 1,4 (mod 6)$
$\sigma(z_{m-1}) = 0.2$
$\sigma(z_m) = 0.3$

$m \equiv 5 (mod 6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3 (mod 6)$

*Case 5:* $n \equiv 8 (mod 12)$

$m \equiv 0 (mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 0,5 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 2,3 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 1,4 (mod 6)$
$\sigma(z_m) = 0.3$

$m \equiv 1 (mod 6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 4,5 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 1,2 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3 (mod 6)$

$m \equiv 2 (mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 3,4 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 0,1 (mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 2,5 (mod 6)$
$\sigma(z_m) = 0.3$

$m \equiv 3 (mod 6)$
for $1 \leq \kappa \leq m-2$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 2,3 (mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 0,5 (mod 6)$

$\sigma(z_\kappa) = 0.3 \quad if\,\kappa \equiv 1,4(mod6)$
$\sigma(z_{m-1}) = 0.1$
$\sigma(z_m) = 0.3$

$m \equiv 4(mod6)$
$\sigma(z_1) = 0.1$
$\sigma(z_2) = 0.3$
$\sigma(z_3) = 0.1$
for $4 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 5(mod6)$
for $1 \leq \kappa \leq m-3$
$\sigma(z_\kappa) = 0.1 \quad if\,\kappa \equiv 0,1(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\,\kappa \equiv 3,4(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\,\kappa \equiv 2,5(mod6)$
$\sigma(z_{m-2}) = 0.3$
$\sigma(z_{m-1}) = 0.2$
$\sigma(z_m) = 0.2$

*Case 6:* $n \equiv 10(mod12)$

$m \equiv 0(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 1(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 2(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\,\kappa \equiv 0,5(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\,\kappa \equiv 2,3(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\,\kappa \equiv 1,4(mod6)$

$m \equiv 3(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 4,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\,\kappa \equiv 0,1(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if\,\kappa \equiv 3,4(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if\,\kappa \equiv 2,5(mod6)$

The number of vertices labeled with $i$, where $i \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ are tabulated below.

*Table 3.2.1.*

| Nature of n and m | $v_\sigma(0.1)$ | $v_\sigma(0.2)$ | $v_\sigma(0.3)$ |
|---|---|---|---|
| $n \equiv 0(mod12)$ $m \equiv 0(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 0(mod12)$ $m \equiv 1(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 0(mod12)$ $m \equiv 2(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 0(mod12)$ $m \equiv 3(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 0(mod12)$ $m \equiv 4(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 0(mod12)$ $m \equiv 5(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 2(mod12)$ $m \equiv 0(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 2(mod12)$ $m \equiv 1(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 2(mod12)$ $m \equiv 2(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 2(mod12)$ $m \equiv 3(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n \equiv 2(mod12)$ $m \equiv 4(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 2(mod12)$ $m \equiv 5(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 4(mod12)$ $m \equiv 0(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 4(mod12)$ $m \equiv 1(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 4(mod12)$ $m \equiv 2(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 4(mod12)$ $m \equiv 3(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 4(mod12)$ $m \equiv 4(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 4(mod12)$ $m \equiv 5(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n \equiv 6(mod12)$ $m \equiv 0(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 6(mod12)$ $m \equiv 1(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 6(mod12)$ $m \equiv 2(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n \equiv 6(mod12)$ $m \equiv 3(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |

*Table 3.2.1. continued*

| Nature of n and m | $v_\sigma(0.1)$ | $v_\sigma(0.2)$ | $v_\sigma(0.3)$ |
|---|---|---|---|
| $n\equiv 6(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 6(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 10(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n\equiv 10(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 10(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 10(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n\equiv 10(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 10(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |

The number of edges labeled with $i$, where $i\in\{\frac{r}{10}, r\in Z_4-\{0\}\}$ are tabulated below.

*Table 3.2.2.*

| Nature of n and m | $e_\mu(0.1)$ | $e_\mu(0.2)$ | $e_\mu(0.3)$ |
|---|---|---|---|
| $n\equiv 0(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 0(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 0(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n\equiv 0(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 0(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n\equiv 0(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |

*Table 3.2.2. continued*

| Nature of n and m | $e_\mu(0.1)$ | $e_\mu(0.2)$ | $e_\mu(0.3)$ |
|---|---|---|---|
| $n\equiv 2(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 2(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n\equiv 2(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 2(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 2(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n\equiv 2(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 4(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n\equiv 4(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 4(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 4(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 4(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 4(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 6(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 6(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 6(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n\equiv 6(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 6(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 6(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n\equiv 8(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 8(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |

*Table 3.2.2. continued*

| Nature of n and m | $e_\mu(0.1)$ | $e_\mu(0.2)$ | $e_\mu(0.3)$ |
|---|---|---|---|
| $n \equiv 10(mod12)$ $m \equiv 0(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 10(mod12)$ $m \equiv 1(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 10(mod12)$ $m \equiv 2(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n \equiv 10(mod12)$ $m \equiv 3(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 10(mod12)$ $m \equiv 4(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 10(mod12)$ $m \equiv 5(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |

Above tables 3.2.1 and 3.2.2 shows that $|v_\sigma(i)-v_\sigma(j)| \leq 1$ and $|e_\mu(i)-e_\mu(j)| \leq 1$ for $i \neq j$, where $i,j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$, It is clear that the graph obtained from two copies of a path $P_m$ by attaching one through an edge to $\frac{n}{2}$ $th$ and other through an edge to $n$ $th$ vertex of an even cycle $C_n$ satisfies the conditions of fuzzy quotient - 3 cordial labeling. Hence the theorem. □

**Theorem 3.3.** *The graph obtained from two copies of a path $P_m$ by attaching one through an edge to $\frac{n-1}{2}$ th and other through an edge to n th vertex of an odd cycle $C_n$ is fuzzy quotient-3 cordial.*

*Proof.* Let $G$ be a graph obtained from two copies of a path $P_m$ by attaching one through an edge to $\frac{n-1}{2}$ $th$ and other through an edge to $n$ $th$ vertex of an odd cycle $C_n$

The vertex set $V(G) = \{x_\kappa : 1 \leq \kappa \leq n\} \cup \{y_\kappa : 1 \leq \kappa \leq m\} \cup \{z_\kappa : 1 \leq \kappa \leq m\}$ and the edge set

$E(G) = \{x_\kappa x_{\kappa+1} : 1 \leq \kappa \leq n-1\} \cup x_1x_n \cup x_ny_1 \cup \{y_\kappa y_{\kappa+1} : 1 \leq \kappa \leq m-1\} \cup (x_{\frac{n-1}{2}}z_1) \cup \{(z_\kappa z_{\kappa+1}) : 1 \leq \kappa \leq m-1\}$

$|V(G)| = |E(G)| = n+2m.$

Define $\sigma : V(G) \to [0,1]$as $\sigma(v) = \frac{r}{10}, r?Z_4 - \{0\}$

The labeling of $x_\kappa$ $'s$ are as follows.

*Case 1:* $n \equiv 1,3,7,9(mod12)$

for $1 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

*Case 2:* $n \equiv 5(mod12)$

for $1 \leq \kappa \leq n$
$\sigma(x_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$

*Case 3:* $n \equiv 11(mod12)$

for $1 \leq \kappa \leq n-1$
$\sigma(x_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(x_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(x_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$
$\sigma(x_n) = 0.3$

The labeling of $y_\kappa$ $'s$ are as follows.

*Case 1:* $n \equiv 1(mod12)$

$m \equiv 0(mod6)$
$\sigma(y_1) = 0.3$
for $2 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod6)$

$m \equiv 1(mod6)$
$\sigma(y_1) = 0.3$
for $2 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$

$m \equiv 2,3,4,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$

*Case 2:* $n \equiv 3(mod12)$

$m \equiv 0,1(mod6)$
$\sigma(y_1) = 0.3$
$\sigma(y_2) = 0.1$
for $3 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$

$m \equiv 2(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod6)$
$\sigma(y_m) = 0.2$

$m \equiv 3(mod6)$
for $1 \leq \kappa \leq m-3$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod6)$

$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$
$\sigma(y_{m-2}) = 0.2$
$\sigma(y_{m-1}) = 0.2$
$\sigma(y_m) = 0.3$

$m \equiv 4(mod6)$
for $1 \leq \kappa \leq m-3$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$
$\sigma(y_{m-2}) = 0.3$
$\sigma(y_{m-1}) = 0.2$
$\sigma(y_m) = 0.3$

$m \equiv 5(mod6)$
for $1 \leq \kappa \leq m-4$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$
$\sigma(y_{m-3}) = 0.2$
$\sigma(y_{m-2}) = 0.3$
$\sigma(y_{m-1}) = 0.2$
$\sigma(y_m) = 0.3$

*Case 3:* $n \equiv 5(mod12)$

$m \equiv 0(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 1(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$
$\sigma(y_m) = 0.3$

$m \equiv 2(mod6)$
for $1 \leq \kappa \leq m-2$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$
$\sigma(y_{m-1}) = 0.3$
$\sigma(y_m) = 0.2$

$m \equiv 3(mod6)$
for $1 \leq \kappa \leq m-3$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$
$\sigma(y_{m-2}) = 0.3$
$\sigma(y_{m-1}) = 0.2$
$\sigma(y_m) = 0.1$

$m \equiv 4(mod6)$

for $1 \leq \kappa \leq m-4$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$
$\sigma(y_{m-3}) = 0.3$
$\sigma(y_{m-2}) = 0.2$
$\sigma(y_{m-1}) = 0.1$
$\sigma(y_m) = 0.1$

$m \equiv 5(mod6)$
for $1 \leq \kappa \leq m-5$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$
$\sigma(y_{m-4}) = 0.3$
$\sigma(y_{m-3}) = 0.2$
$\sigma(y_{m-2}) = 0.1$
$\sigma(y_{m-1}) = 0.1$
$\sigma(y_m) = 0.3$

*Case 4:* $n \equiv 7(mod12)$

$m \equiv 0(mod6)$
$\sigma(y_1) = 0.3$
for $2 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 1(mod6)$
$\sigma(y_1) = 0.3$
for $2 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 2,5(mod6)$

$m \equiv 2,3,4,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 0,1(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 3,4(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 2,5(mod6)$

*Case 5:* $n \equiv 9(mod12)$

$m \equiv 0(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$
$\sigma(y_\kappa) = 0.2 \quad if\,\kappa \equiv 4,5(mod6)$
$\sigma(y_\kappa) = 0.3 \quad if\,\kappa \equiv 0,3(mod6)$

$m \equiv 1(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if\,\kappa \equiv 1,2(mod6)$

$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod 6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod 6)$
$\sigma(y_m) = 0.3$

$m \equiv 2(mod 6)$
for $1 \leq \kappa \leq m-2$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod 6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod 6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod 6)$
$\sigma(y_{m-1}) = 0.2$
$\sigma(y_m) = 0.3$

$m \equiv 3(mod 6)$
$\sigma(y_1) = 0.3$
for $2 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod 6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod 6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod 6)$

$m \equiv 4(mod 6)$
$\sigma(y_1) = 0.3$
for $2 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod 6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod 6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod 6)$
$\sigma(y_m) = 0.1$

$m \equiv 5(mod 6)$
$\sigma(y_1) = 0.3$
for $2 \leq \kappa \leq m-2$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod 6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod 6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod 6)$
$\sigma(y_{m-1}) = 0.3$
$\sigma(y_m) = 0.1$

*Case 6:* $n \equiv 11(mod 12)$

$m \equiv 0,3,4(mod 6)$
for $1 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod 6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod 6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod 6)$

$m \equiv 1,2(mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod 6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod 6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod 6)$
$\sigma(y_m) = 0.3$

$m \equiv 5(mod 6)$
$\sigma(y_1) = 0.3$
for $2 \leq \kappa \leq m$
$\sigma(y_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod 6)$
$\sigma(y_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod 6)$
$\sigma(y_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod 6)$

The labeling of $z_\kappa\,'s$ are as follows.

*Case 1:* $n \equiv 1(mod 12)$

$m \equiv 0(mod 6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 1,2(mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 4,5(mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 0,3(mod 6)$

$m \equiv 1(mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod 6)$
$\sigma(z_m) = 0.2$

$m \equiv 2(mod 6)$
$\sigma(z_1) = 0.2$
for $2 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod 6)$

$m \equiv 3,5(mod 6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod 6)$

$m \equiv 4(mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod 6)$
$\sigma(z_m) = 0.3$

*Case 2:* $n \equiv 3(mod 12)$

$m \equiv 0,1(mod 6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod 6)$
$\sigma(z_m) = 0.2$

$m \equiv 2(mod 6)$
$\sigma(z_1) = 0.3$
for $2 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if\, \kappa \equiv 0,1(mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if\, \kappa \equiv 3,4(mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if\, \kappa \equiv 2,5(mod 6)$

$\sigma(z_m) = 0.2$

$m \equiv 3(mod6)$
for $1 \leq \kappa \leq m-2$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$
$\sigma(z_{m-1}) = 0.3$
$\sigma(z_m) = 0.2$

$m \equiv 4(mod6)$
for $1 \leq \kappa \leq m-3$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$
$\sigma(z_{m-2}) = 0.2$
$\sigma(z_{m-1}) = 0.2$
$\sigma(z_m) = 0.2$

$m \equiv 5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

*Case 3:* $n \equiv 5(mod12)$

$m \equiv 0,1,2,3,4,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

*Case 4:* $n \equiv 7(mod12)$

$m \equiv 0(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

$m \equiv 1(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 0,1(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 3,4(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 2,5(mod6)$
$\sigma(z_m) = 0.2$

$m \equiv 2(mod6)$
$\sigma(z_1) = 0.2$
for $2 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 0,1(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 3,4(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 2,5(mod6)$

$m \equiv 3,5(mod6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 0,1(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 3,4(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 2,5(mod6)$

$m \equiv 4(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 0,1(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 3,4(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 2,5(mod6)$
$\sigma(z_m) = 0.3$

*Case 5:* $n \equiv 9(mod12)$

$m \equiv 0,2(mod6)$
$\sigma(z_1) = 0.2$
for $2 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$

$m \equiv 1(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$
$\sigma(z_m) = 0.2$

$m \equiv 3,4,5(mod6)$
$\sigma(z_1) = 0.2$
$\sigma(z_3) = 0.3$
for $3 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 0,1(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 3,4(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 2,5(mod6)$

*Case 6:* $n \equiv 11(mod12)$

$m \equiv 0,2(mod6)$
for $1 \leq \kappa \leq m-1$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$
$\sigma(z_m) = 0.2$

$m \equiv 1(mod6)$
$\sigma(z_1) = 0.2$
for $2 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod6)$
$\sigma(z_m) = 0.2$

$m \equiv 2,3,4(mod 6)$
$\sigma(z_1) = 0.2$
$\sigma(z_2) = 0.2$
for $3 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod 6)$
$\sigma(z_m) = 0.2$

$m \equiv 5(mod 6)$
for $1 \leq \kappa \leq m$
$\sigma(z_\kappa) = 0.1 \quad if \kappa \equiv 1,2(mod 6)$
$\sigma(z_\kappa) = 0.2 \quad if \kappa \equiv 4,5(mod 6)$
$\sigma(z_\kappa) = 0.3 \quad if \kappa \equiv 0,3(mod 6)$

The number of vertices labeled with $i$, where $i \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ are tabulated below.

*Table 3.3.1.*

| Nature of n and m | $v_\sigma(0.1)$ | $v_\sigma(0.2)$ | $v_\sigma(0.3)$ |
|---|---|---|---|
| $n \equiv 1(mod 12)$ $m \equiv 0(mod 6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n \equiv 1(mod 12)$ $m \equiv 1(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 1(mod 12)$ $m \equiv 2(mod 6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 1(mod 12)$ $m \equiv 3(mod 6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 1(mod 12)$ $m \equiv 4(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 1(mod 12)$ $m \equiv 5(mod 6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 3(mod 12)$ $m \equiv 0(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 3(mod 12)$ $m \equiv 1(mod 6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n \equiv 3(mod 12)$ $m \equiv 2(mod 6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 3(mod 12)$ $m \equiv 3(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 3(mod 12)$ $m \equiv 4(mod 6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n \equiv 3(mod 12)$ $m \equiv 5(mod 6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m11}{3}$ |
| $n \equiv 5(mod 12)$ $m \equiv 0(mod 6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 5(mod 12)$ $m \equiv 1(mod 6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n \equiv 5(mod 12)$ $m \equiv 2(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |

*Table 3.3.1. continued*

| Nature of n and m | $v_\sigma(0.1)$ | $v_\sigma(0.2)$ | $v_\sigma(0.3)$ |
|---|---|---|---|
| $n \equiv 5(mod 12)$ $m \equiv 3(mod 6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 5(mod 12)$ $m \equiv 4(mod 6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 5(mod 12)$ $m \equiv 5(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 7(mod 12)$ $m \equiv 0(mod 6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n \equiv 7(mod 12)$ $m \equiv 1(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 7(mod 12)$ $m \equiv 2(mod 6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 7(mod 12)$ $m \equiv 3(mod 6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 7(mod 12)$ $m \equiv 4(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 7(mod 12)$ $m \equiv 5(mod 6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 9(mod 12)$ $m \equiv 0(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 9(mod 12)$ $m \equiv 1(mod 6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 9(mod 12)$ $m \equiv 2(mod 6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 9(mod 12)$ $m \equiv 3(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 9(mod 12)$ $m \equiv 4(mod 6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n \equiv 9(mod 12)$ $m \equiv 5(mod 6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n \equiv 11(mod 12)$ $m \equiv 0(mod 6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n \equiv 11(mod 12)$ $m \equiv 1(mod 6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ |
| $n \equiv 11(mod 12)$ $m \equiv 2(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n \equiv 11(mod 12)$ $m \equiv 3(mod 6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n \equiv 11(mod 12)$ $m \equiv 4(mod 6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n \equiv 11(mod 12)$ $m \equiv 5(mod 6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |

The number of edges labeled with $i$, where $i \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ are tabulated below.

*Table 3.3.2.*

| Nature of n and m | $e_\sigma(0.1)$ | $e_\sigma(0.2)$ | $e_\sigma(0.3)$ |
|---|---|---|---|
| $n\equiv 1(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 1(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 1(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 1(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 1(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 1(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 3(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 3(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n\equiv 3(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 3(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 3(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 3(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 5(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n\equiv 5(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 5(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 5(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ |
| $n\equiv 5(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 5(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 7(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 7(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 7(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 7(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 7(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 7(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |

table 3.3.2 continues

| Nature of n and m | $e_\sigma(0.1)$ | $e_\sigma(0.2)$ | $e_\sigma(0.3)$ |
|---|---|---|---|
| $n\equiv 9(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 9(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 9(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 9(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 9(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 9(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 11(mod12)$ $m\equiv 0(mod6)$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 11(mod12)$ $m\equiv 1(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 11(mod12)$ $m\equiv 2(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |
| $n\equiv 11(mod12)$ $m\equiv 3(mod6)$ | $\frac{n+2m+1}{3}$ | $\frac{n+2m+1}{3}-1$ | $\frac{n+2m+1}{3}$ |
| $n\equiv 11(mod12)$ $m\equiv 4(mod6)$ | $\frac{n+2m-1}{3}$ | $\frac{n+2m-1}{3}+1$ | $\frac{n+2m-1}{3}$ |
| $n\equiv 11(mod12)$ $m\equiv 5(mod6)$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ | $\frac{n+2m}{3}$ |

Above tables 3.3.1 and 3.3.2 shows that $|v_\sigma(i)-v_\sigma(j)|\leq 1$ and $|e_\mu(i)-e_\mu(j)|\leq 1$ for $i\neq j$, where $i,j\in\{\frac{r}{10},r\in Z_4-\{0\}\}$, It is clear that the graph obtained from two copies of a path $P_m$ by attaching one through an edge to $\frac{n-1}{2}$ *th* and other through an edge to $n$ *th* vertex of an odd cycle $C_n$ satisfies the conditions of fuzzy quotient - 3 cordial labeling. Hence the theorem. □

## 4. Conclusion

In this work we have discussed and established the existence of fuzzy qoutient-3 labeling on some path related graph. Investigating this labeling concept on other families of graphs will be our future .

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