Malaya Journal of Matematik, Vol. 9, No. 1, 1204-1206, 2021

https://doi.org/10.26637/MJM0901/0207

Optimization using Hamiltonian cycle

K. Sushma [1] and Uma Dixit [2*]

Abstract

Several researchers have worked on problem of optimization. There are fuzzy time minimization Assignment problem, which are form of transportation model. In this paper, the fuzzy time minimization assignment problem is formulated as weighted graph and optimization is achieved by reducing it to a Hamiltonian cycle.

Keywords

Optimization, assignment problem, fuzzy number, weighted graph, Hamiltonian.

AMS Subject Classification

05C45.

[1] *Department of Mathematics, University College of Science, Osmania University, Hyderabad-500007, Telangana, India.*

[2] *Department of Mathematics, University Post Graduate College, Secunderabad, Osmania University, Hyderabad-500003, Telangana, India.*

***Corresponding author**: [1] sushmarasala@gmail.com; [2] umadixit@gmail.com

Article History: Received **12** October **2020**; Accepted **06** March **2021**

Contents

1. Introduction

The assignment problem is one of the earliest methods of linear programming problem. An assignment problem deals with situation about what happens to the effectiveness function when each of number of origins are associated with each of same number of destinations. These are special case of transportation model. The transportation model which deals with transportation costs, supply and demand was originally developed by Hitchcock (1941).

An assignment plan is optimal if it optimizes the total cost or effectiveness of doing all the jobs. Different methods have been presented for assignment problem and various articles have been published on the subject. To deal with imprecise information in making decisions, Zadeh [1] introduced the notion of fuzziness. Further studies about decision making in fuzzy environment was done by Bellman and Zadeh [7]. Later, Zimmermann [3] showed that solutions obtained by fuzzy linear programming were effectively accurate. R. Nagarajan and A. Solairaju [4] had given an algorithm to solve fuzzy assignment problem using Robust ranking technique.

This paper attempts to adopt a method to minimize fuzzy cost by using weighted graph and reducing it to Hamiltonian cycle using Hungarian method.

2. Preliminaries

We require few definitions in graphical terminology:

2.1 Graph : A graph is an ordered pair $G=(V,E)$ consisting set V of vertices or nodes together with a set E of edges or links, which are $2-$elements subset of V (that is an edge is related with two vertices, and the relation is represented as an unordered pair of the vertices with respect to the particular edge).

2.2 Order and Size of graph: The number of vertices in graph is called order and number of edges is called size of graph.

2.3 Cycle graph : A cycle graph of order n is a connected graph whose edges form a cycle of length n.

2.4 Weighted graph: Weighted graph is a graph in which each branch is given a numerical weight. A weighted graph is therefore a special type of labeled graph in which the labels are numbers.

2.5 Complete graph: A simple nondirected graph with n mutually adjacent vertices is called a complete graph on n vertices.

2.6 Minimal spanning tree: A spanning tree T of a graph G is a sub graph containing all the vertices of G. It is a minimal set of edges that connects all the vertices of G without creating any cycles or loops. Out of all the spanning trees of G, the minimum spanning tree is one with least weight.

2.7 Hamiltonian path: A Hamiltonian path is a path in an undirected or directed graph that touches each vertex exactly once.
2.8 Hamiltonian cycle: A Hamiltonian cycle is a Hamiltonian path that is a cycle.

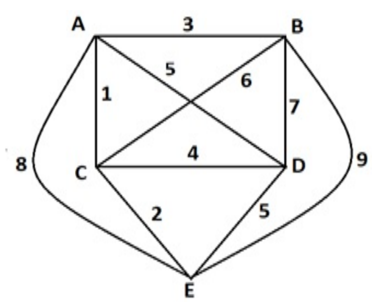
3. Hungarian Method - Algorithm

Step 1. If the matrix is $n \times n$, proceed to next step. For $m \times n$, a dummy row or column is added to make the matrix square.
Step 2. Subtract the entries of each row by the row minimum. These operations create at least one zero in each rows.
Step 3. Subtract the entries of each column by the column minimum. These operations create at least one zero in each rows and each column.
Step 4. Draw the minimum number of lines to cover all the zero's of the matrix. If the number of drawn lines less than n, then the complete assignment is not possible, while if the number of lines is exactly equal to n, then the complete assignment is obtained.
Step 5. If a complete assignment is not possible in Step 4, then select the smallest element (say k) which is not covered by the line in Step 4. Now subtract k from all uncovered elements and add k to all elements which are covered by both horizontal and vertical line. If still a complete optimal assignment is not achieved, then use Steps 4 and 5 iteratively until the number of lines crossing all zero's becomes equal to the order of matrix, which indicates an optimal assignment is obtained.

4. Main Results

We illustrate the proposed method by solving a fuzzy assignment problem, using weighted graph, the above mentioned algorithm and reducing to Hamiltonian cycle.

Problem: A school bus driver has to pickup students from different locations in city. The stops and the distance travelled in between is given in kilometers and is depicted using a weighted graph. Optimize the problem so that the transportation cost to the management of school is less and the bus driver takes minimum time to reach his destination.



Proof. In conformation to model, the fuzzy assignment problem can be formulated in the following way. The vertices are representing the bus stops, the edges of graph are the routes and the weights on edges are showing distance in kilometers. We shall now represent it in matrix form as assignment problem.

We shall use the symbol $**$ when there is no route between stops. We shall now first use steps in Hungarian algorithm

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A\\B\\C\\D\\E \end{array} & \begin{bmatrix} ** & 3 & 1 & 5 & 8 \\ 3 & ** & 6 & 7 & 9 \\ 1 & 6 & ** & 4 & 2 \\ 5 & 7 & 4 & ** & 5 \\ 8 & 9 & 2 & 5 & ** \end{bmatrix} \end{array}$$

and optimize the problem.
Step 1: The matrix is square matrix, so we proceed to next step.
Step 2: We find the row minimum element and then subtract each element of the row with corresponding row minimum element. We then get

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A\\B\\C\\D\\E \end{array} & \begin{bmatrix} ** & 2 & 0 & 4 & 7 \\ 0 & ** & 3 & 4 & 6 \\ 0 & 5 & ** & 3 & 1 \\ 1 & 3 & 0 & ** & 1 \\ 6 & 7 & 0 & 3 & ** \end{bmatrix} \end{array}$$

Step 3: Similarly we find the column minimum element and then subtract each element of the column with corresponding column minimum element in columns in which zero is not present and obtain

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A\\B\\C\\D\\E \end{array} & \begin{bmatrix} ** & 0 & 0 & 1 & 6 \\ 0 & ** & 3 & 1 & 5 \\ 0 & 3 & ** & 0 & 0 \\ 1 & 1 & 0 & ** & 0 \\ 6 & 5 & 0 & 0 & ** \end{bmatrix} \end{array}$$

Step 4: Draw the minimum number of lines to cover all the zero's of the matrix.

Substep- Examine rows successively until a row with exactly one unmarked zero is found. Mark 0 indicating that an assignment will be made there. Mark X all other zeros in the same column showing that they cannot be used for making other assignments. Similarly examine columns for single unmarked zeros making them 0 and also marking any other zeros in their rows.

$$
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix}
** & \mathbf{0} & X & 1 & 6 \\
X & ** & 3 & 1 & 5 \\
\mathbf{0} & 3 & ** & X & X \\
1 & 1 & \mathbf{0} & ** & X \\
6 & 5 & X & \mathbf{0} & **
\end{bmatrix}
\end{array}
$$

Substep- Mark the rows for which assignment has not been made. It is row 2. And mark columns which have marked zero corresponding to the ticked row. It is column 1. Corresponding to the ticked column, tick the row which has marked zero. It is row 3. Then draw the lines on unticked row and columns.

$$
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix}
** & \mathbf{0} & X & 1 & 6 \\
X & ** & 3 & 1 & 5 \\
\mathbf{0} & 3 & ** & X & X \\
1 & 1 & \mathbf{0} & ** & X \\
6 & 5 & X & \mathbf{0} & **
\end{bmatrix}
\end{array}
$$

Step 5: Select the smallest element ($k = 1$) which is not covered by the line in Step 4. Now subtract $k = 1$ from all uncovered elements and add $k = 1$ to all elements which are covered by both horizontal and vertical line.

Observe for a zero in each row or column and modify the matrix using above rules.

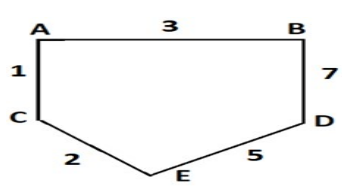
$$
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix}
** & \mathbf{0} & X & 1 & 6 \\
X & ** & 2 & \mathbf{0} & 4 \\
\mathbf{0} & 2 & ** & X & X \\
2 & 1 & \mathbf{X} & ** & \mathbf{0} \\
7 & 5 & \mathbf{0} & \mathbf{X} & **
\end{bmatrix}
\end{array}
$$

From the above matrix we conclude that $A \to B \to D \to E \to C \to A$ is the required sequence path of the bus driver and traveled the stops exactly once and reached to destination with shortest distance.

The shortest distance is 3+7+5+2+1=18 Km.

This is the required shortest distance for the optimization problem.

We represent the shortest path as Hamiltonian cycle.



□

References

[1] L. A. Zadeh, Fuzzy sets, *Information and Control,* 8(1965), 338–353.

[2] Hamdy A. Taha, *Operations Research: An Introduction*, 8th Ed., 2007.

[3] H.J. Zimmerman, Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems*, (1978), 45–55.

[4] R. Nagarajan and A. Solairaju, A computing improved fuzzy optimal Hungarian assignment problem with fuzzy cost under Robust ranking technique, *Int. Journal of Computer Application*, 6(4)(2010), 6–13.

[5] H. J. Zimmermann, *Fuzzy Set Theory and its Applications*, Third Ed., kluwer Academic, Boston, 1996.

[6] Hadi Basirzadeh, Ones assignment method for solving assignment problems, *Applied Mathematical Sciences*, 6(47)(2012), 2345–2355.

[7] R.E. Bellman and L.A. Zadeh, Decision making in a fuzzy environment, *Management Sci.,* 17(1970), 141–164.

[8] D.S. Dinagar and K. Palanivel, The transportation problem in fuzzy environment, *International Journal of Algorithms, Computing and Mathematics*, (2009), 65–71.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆