



# Relatively prime restrained geodetic number of graphs

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## Abstract

In this paper we introduce relatively prime restrained geodetic set of graphs  $G$ . A set  $S \subseteq V(G)$  is said to be relatively prime restrained geodetic set in  $G$  if  $S$  is a relatively prime geodetic set and  $\langle V(G) - S \rangle$  has no isolated vertices. The relatively prime restrained geodetic set is denoted by  $g_{rpr}(G)$ - set. The minimum cardinality of relatively prime restrained geodetic set is the relatively prime restrained geodetic number and it is denoted by  $g_{rpr}(G)$ .

## Keywords

Geodetic set, Geodetic Number, Restrained, Relatively prime, Line graph.

## AMS Subject Classification

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## 1. Introduction

By a graph  $G = (V, E)$  we mean a finite, connected, undirected graph with neither loops nor multiple edges. The order  $|V|$  and size  $|E|$  of  $G$  are denoted by  $p$  and  $q$  respectively. For graph theoretic terminology we refer to West [7]. In a connected graph  $G$ , the distance between two vertices  $x$  and  $y$  is denoted by  $d(x, y)$  and is defined as the length of a shortest  $x - y$  path in  $G$ . If  $e = \{u, v\}$  is an edge of a graph  $G$  with  $\deg(u) = 1$  and  $\deg(v) > 1$ , then we call  $e$  a pendant edge,  $u$  a pendent vertex and  $v$  a support vertex. A set of vertices is said to be independent if no two vertices in it are adjacent. A vertex  $v$  of  $G$  is said to be an extreme vertex if the subgraph induced by its neighborhood is complete. For any set  $S$  of points of  $G$ , the induced subgraph  $\langle S \rangle$  is the maximal subgraph of  $G$  with point set  $S$ . Thus two points of  $S$  are adjacent in  $\langle S \rangle$  if and only if they are adjacent in  $G$ . An acyclic connected

graph is called a tree. An  $x - y$  path of length  $d(x, y)$  is called geodesic. A vertex  $v$  is said to lie on a geodesic  $P$  if  $v$  is an internal vertex of  $P$ . The closed interval  $I[x, y]$ , consists of  $x, y$  and all vertices lying on some  $x - y$  geodesic of  $G$  and for a non empty set  $S \subseteq V(G)$ ,  $I[S] = \bigcup_{x, y \in S} I[x, y]$ .

A set  $S \subseteq V(G)$  in a connected graph is a geodetic set of  $G$  if  $I[S] = V(G)$ . The geodetic number of  $G$  denoted by  $g(G)$ , is the minimum cardinality of a geodetic set of  $G$ . The geodetic number of a disconnected graph is the sum of the geodetic number of its components. A geodetic set of cardinality  $g(G)$  is called  $g(G) -$  set. Various concepts inspired by geodetic set are introduced in [2, 4].

## 2. Definitions and Known results

**Definition 2.1.** [5] The line graph  $L(G)$  of a graph  $G$  is the graph whose vertices are the edges of  $G$  and two vertices in  $L(G)$  are adjacent if the corresponding edges of  $G$  are adjacent.

**Definition 2.2.** [1] A geodetic set  $S \subseteq V(G)$  of a graph  $G = (V, E)$  is a restrained geodetic set if the subgraph  $G[V - S]$  has no isolated vertex. The minimum cardinality of a restrained geodetic set is the restrained geodetic number.

**Definition 2.3.** [3] The total graph  $T(G)$  of a graph  $G$  is a graph such that the vertex set of  $T$  corresponds to the vertices

and edges of  $G$  and two vertices adjacent in  $T$  iff if their corresponding elements are either adjacent or incident in  $G$ .

**Theorem 2.4.** [6] Every end vertices of a graph  $G$  belongs to relatively prime geodetic set of  $G$ .

**Theorem 2.5.** [6] Every relatively prime geodetic set of a graph contains its extreme vertices.

**Theorem 2.6.** [6] For a star graph  $K_{1,n}$ ,  $g_{rp}(K_{1,n})$

$$= \begin{cases} 3 & \text{for } n = 2 \\ 0 & \text{for } n \geq 3 \end{cases} .$$

**Theorem 2.7.** [6] For a connected graph  $G$  of order  $n$  if  $g_{rp}(G)$  exists, then  $g(G) \leq g_{rp}(G) \leq n$ .

**Theorem 2.8.** [6] For a wheel graph  $W_{1,n}(n \geq 3)$ ,

$$g_{rp}(W_{1,n}) = \begin{cases} 4 & \text{if } n = 3 \\ 3 & \text{if } n = 4 \\ 0 & \text{otherwise} \end{cases} .$$

**Definition 2.9.** [6] Let  $G$  be a connected graph. A set  $S \subseteq V$  is said to be a relatively prime geodetic set if it is a geodetic set with at least three elements and the shortest distance between any two pairs of vertices in  $S$  is relatively prime. The relatively prime geodetic set of  $G$  is denoted by  $g_{rp}(G)$ -set. The cardinality of a minimum relatively prime geodetic set is the relatively prime geodetic number and it is denoted by  $g_{rp}(G)$ .

### 3. Relatively Prime Restrained Geodetic Number of Graphs

**Definition 3.1.** A set  $S \subseteq V(G)$  is said to be relatively prime restrained geodetic set in  $G$  if  $S$  is a relatively prime geodetic set and  $\langle V(G) - S \rangle$  has no isolated vertices. The relatively prime restrained geodetic set is denoted by  $g_{rpr}(G)$ -set. The minimum cardinality of relatively prime restrained geodetic set is the relatively prime restrained geodetic number and it is denoted by  $g_{rpr}(G)$ .

**Example 3.2.** Consider the graph in Figure 1. The set  $S = \{v_4, v_5\}$  is a minimum geodetic set and  $S' = \{v_4, v_5, v_6\}$  is a minimum relatively prime geodetic set. But  $\langle V(G) - S' \rangle = K_3 \cup K_1$  has isolated vertex of  $v_7$  and hence  $S'$  cannot be a relatively prime restrained geodetic number. Now consider  $S'' = \{v_4, v_5, v_6, v_7\}$ . Then  $S''$  is a relatively prime geodetic set and  $\langle V(G) - S'' \rangle = K_3$  has no isolated vertices. Hence  $S''$  is a relatively prime restrained geodetic set of  $G$ . Moreover it has the minimum cardinality with this property and hence  $g_{rpr}(G) = 4$ .

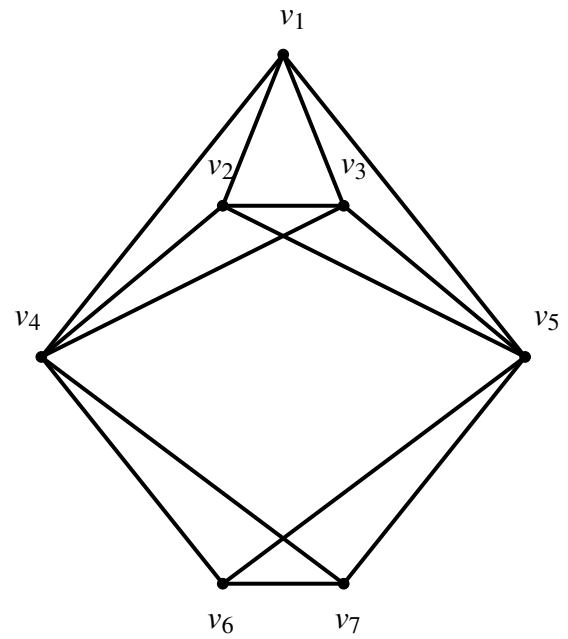


Figure 1

**Theorem 3.3.** Let  $G$  be a connected graph of order  $n$ . Then

- Each relatively prime restrained geodetic set of  $G$  contains its extreme vertices.
- Each end vertex of  $G$  belongs to relatively prime restrained geodetic set of  $G$ .

*Proof.* Let  $G$  be a connected graph of order  $n$ . By definition, each relatively prime restrained geodetic set is a relatively prime geodetic set.

(i) Hence by Theorem 2.4, each relatively prime restrained geodetic set of  $G$  contains its extreme vertices.

(ii) Further by Theorem 2.5, each end vertex of  $G$  belongs to relatively prime restrained geodetic set of  $G$ .  $\square$

**Theorem 3.4.** For the complete graph  $K_n (n \geq 3)$ ,  $g_{rpr}(K_n) = n$ .

*Proof.* In a complete graph  $K_n$ , every vertex is an extreme vertex and by Theorem 3.3, the vertex set  $V(K_n)$  is the unique relatively prime restrained geodetic set of  $K_n$ . Hence  $g_{rpr}(K_n) = n$ .  $\square$

**Theorem 3.5.** Let  $G$  be connected graph of order  $n$ . If  $g_{rpr}(G)$  exists, then  $g(G) \leq g_{rp}(G) \leq g_{rpr}(G) \leq n$ .

*Proof.* Let  $G$  be a connected graph of order  $n$ , such that  $g_{rpr}(G)$  exists. Since every relatively prime restrained geodetic set is a relatively prime geodetic set, it follows that  $g_{rp}(G) \leq g_{rpr}(G)$ . Also any relatively prime restrained geodetic set can have atmost  $n$  vertices and hence  $g_{rpr}(G) \leq n$ . By Theorem 2.7,  $g(G) \leq g_{rp}(G)$ . Hence  $g(G) \leq g_{rp}(G) \leq g_{rpr}(G) \leq n$ .  $\square$



**Remark 3.6.** For the complete graph  $K_n$  ( $n \geq 3$ ),  $g(K_n) = g_{rp}(K_n) = g_{rpr}(K_n) = n$ . Hence all the inequalities in Theorem 3.5 become sharp. Now consider the graph  $G$  given in Figure 1. The set  $S = \{v_4, v_5\}$  is a geodetic set of  $G$  and so  $g(G) = 2$ . The set  $S' = \{v_4, v_5, v_6\}$  is a minimum relatively prime geodetic set of  $G$  and so  $g_{rp}(G) = 3$ . The set  $S'' = \{v_4, v_5, v_6, v_7\}$  is a minimum relatively prime nonsplit geodetic set of  $G$  and so  $g_{rpr}(G) = 4$ . Thus  $g(G) < g_{rp}(G) < g_{rpr}(G) < n$ , and hence all the inequalities in Theorem 3.5 become strict.

**Theorem 3.7.** For cycle  $C_n$  of even order  $n$ ,  $g_{rpr}(C_n) =$

$$\begin{cases} 3 & \text{if } n \geq 8 \\ 0 & \text{if } n = 4, 6 \end{cases}$$

*Proof.* Let  $v_1 v_2 \dots v_n v_1$  be the cycle  $C_n$  of order  $n$ . Clearly  $S = \{v_i, v_{i+\frac{n}{2}}\}$  where the suffices modulo  $n$  is a minimum geodetic set of  $C_n$  and so  $g(C_n) = 2$ .

Case 1.  $n = 4$

Any minimum geodetic set of  $C_4$  is  $\{v_i, v_{i+2}\}$  where the suffices modulo 4. To get a relatively prime geodetic set, we add one more vertex with  $\{v_i, v_{i+2}\}$ . The possible relatively prime restrained geodetic set are  $\{v_i, v_{i+1}, v_{i+2}\}$  and  $\{v_i, v_{i+2}, v_{i+3}\}$  where the suffices modulo 4. In each case  $\langle V(C_n) - S \rangle = K_1$  has an isolated vertex and so  $S$  is not a restrained geodetic set. The only possible restrained geodetic sets with at least four vertices are either  $\{v_i, v_{i+1}, v_{i+2}, v_{i+3}\}$  or  $\{v_1, v_2, v_3, v_4\}$  where the suffices modulo 4, which are not relatively prime and hence  $g_{rpr}(C_4) = 0$ .

Case 2.  $n = 6$

Any minimum geodetic set of  $C_6$  is  $\{v_i, v_{i+3}\}$  where the suffices modulo 6. To get a relatively prime geodetic set, we add one more vertex with  $\{v_i, v_{i+3}\}$ . The possible relatively prime restrained geodetic set are  $\{v_i, v_{i+1}, v_{i+3}\}$ ,  $\{v_i, v_{i+2}, v_{i+4}\}$ ,  $\{v_i, v_{i+3}, v_{i+4}\}$  and  $\{v_i, v_{i+3}, v_{i+5}\}$  where the suffices modulo 6. In each case  $\langle V(C_n) - S \rangle = K_2 \cup K_1$  has an isolated vertex and so  $S$  is not a restrained geodetic set. The only possible restrained geodetic sets with at least four vertices are either  $\{v_i, v_{i+1}, v_{i+2}, v_{i+3}\}$  or  $\{v_1, v_2, v_3, v_4, v_5, v_6\}$  where the suffices modulo 6, which are not relatively prime and hence  $g_{rpr}(C_6) = 0$ .

Case 3.  $n \geq 8$

Clearly  $S = \{v_i, v_{i+\frac{n}{2}}\}$  where the suffices modulo  $n$  is a minimum geodetic set of  $C_n$  and so  $g(C_n) = 2$ . To get relatively prime geodetic we must add one more vertex to  $S$ . Let  $S' = \{v_i, v_{i+1}, v_{i+\frac{n}{2}}\}$  where the suffices modulo  $n$ . Then  $S'$  is a geodetic set and  $\langle V(C_n) - S' \rangle = K_{\lceil \frac{n}{3} \rceil} \cup K_{\lceil \frac{n}{3} + 1 \rceil}$  which has no isolated vertex implies that  $S'$  is a restrained geodetic set. Now  $d(v_i, v_{i+1}) = 1, d(v_i, v_{i+\frac{n}{2}}) = \frac{n}{2}, d(v_{i+1}, v_{i+\frac{n}{2}}) = \frac{n}{2} - 1$  and  $(1, \frac{n}{2}) = (\frac{n}{2}, \frac{n}{2} - 1) = 1$ . Therefore,  $S'$  is a minimum relatively prime restrained geodetic set of  $C_n$  and hence  $g_{rpr}(C_n) = 3$ .

The result follows from cases 1, 2 and 3. □

**Theorem 3.8.** For the path  $P_n$  of order ( $n \geq 3$ ),  $g_{rpr}(P_n) =$

$$\begin{cases} 0 & \text{if } n = 4 \\ 3 & \text{otherwise} \end{cases}$$

*Proof.* Let  $v_1 v_2 \dots v_n$  be the path  $P_n$ . By Theorem 2.4, the end vertices  $v_1$  and  $v_n$  must in any relatively prime geodetic set.

Case 1.  $n = 3$

Clearly  $S = \{v_1, v_2, v_3\}$  is the only relatively prime restrained geodetic set of  $P_3$  and hence  $g_{rpr}(P_3) = 3$ .

Case 2.  $n = 4$

Any relatively prime geodetic set  $S$  with three vertices is either  $\{v_1, v_2, v_4\}$  or  $\{v_1, v_3, v_4\}$ . Since  $\langle V(P_4) - S \rangle = K_1$ ,  $S$  cannot be restrained geodetic set of  $P_4$ . The only restrained geodetic set  $S$  of  $P_4$  is  $V(P_4)$  which is not relatively prime since  $(d(v_1, v_3), d(v_2, v_4)) = (2, 2) = 2 \neq 1$ . This gives  $g_{rpr}(P_4) = 0$ .

Case 3.  $n \geq 5$

The set  $S = \{v_1, v_2, v_n\}$  is a minimum geodetic set with three vertices of  $P_n$ . Now  $d(v_1, v_2) = 1, d(v_2, v_n) = n - 2, d(v_1, v_n) = n - 1$  and  $(1, n - 2) = (n - 1, n - 2) = 1$ . Also  $\langle V(P_n) - S \rangle$  is the path  $v_3, v_4, \dots, v_{n-1}$ , which has no isolated vertex. Hence  $S$  is a minimum relatively prime restrained geodetic set of  $P_n$  and hence  $g_{rpr}(P_n) = 3$ .

The result follows from cases 1, 2 and 3. □

**Theorem 3.9.** For a star graph  $K_{1,n}$ ,  $g_{rpr}(K_{1,n}) =$

$$\begin{cases} 3 & \text{if } n = 2 \\ 0 & \text{if } n \geq 3 \end{cases}$$

*Proof.* Let  $v, v_1, v_2, \dots, v_n$  be the vertices of a star graph  $K_{1,n}$  with central vertex  $v$ . We consider the following two cases depends on  $n$ .

Case 1.  $n = 2$

In this case  $K_{1,2} = P_3$ . By Theorem 3.8,  $g_{rpr}(K_{1,3}) = 3$ .

Case 2.  $n \geq 3$

By Theorem 3.3(ii), the end vertices  $v_1, v_2, \dots, v_n$  must be in any relatively prime restrained geodetic set  $S$ . If  $S = \{v_1, v_2, \dots, v_n\}$ , then  $\langle V(K_{1,n}) - S \rangle = K_1$  and hence  $S$  cannot be a restrained geodetic set. The only possibility for  $S$  is  $V(K_{1,n})$ . In this case,  $(d(v_1, v_2), d(v_1, v_3)) = (2, 2) \neq 1$  and hence  $K_{1,n}$  has no relatively prime restrained geodetic set. Thus  $g_{rpr}(K_{1,n}) = 0$ .

The result follows from cases 1 and 2. □

**Theorem 3.10.** For a jelly fish graph  $J(m, n)$ ,  $g_{rpr}(J(m, n)) =$

$$\begin{cases} 3 & \text{if } m = n = 1 \\ 0 & \text{otherwise} \end{cases}$$

*Proof.* Consider a 4-cycle  $v_1, v_2, v_3, v_4$  and join  $v_1$  and  $v_3$  with an edge. Append  $m$  pendent edges to  $v_2$  and  $n$  pendent edges to  $v_4$ . The resultant graph is  $J(m, n)$  with vertex set  $V(J(m, n)) = \{v_k, v_{2i}, v_{4j} / 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq 4\}$  and edge set  $E(J(m, n)) = \{v_k v_{k+1}, v_1 v_4, v_1 v_3, v_2 v_{2i}, v_4 v_{4j} / 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq 4\}$

Case 1.  $m = n = 1$

Clearly  $S = \{v_{21}, v_2, v_{41}\}$  is a geodetic set. Now  $d(v_{21}, v_2) = 1, d(v_2, v_{41}) = 4, d(v_{21}, v_{41}) = 3$  and  $(1, 4) = (4, 3) = (1, 3) =$



1. Also  $\langle V(J(m,n)) - S \rangle = K_3$  which has no isolated vertices and hence  $S$  is a minimum relatively prime restrained geodetic set. Therefore,  $g_{rpr}(J(m,n)) = 3$ .

Case 2.  $m = 1, n \geq 2$  or  $m \geq 2, n = 1$

By Theorem 3.3(ii),  $v_{21}, v_{41}, \dots, v_{4n}$  must be in relatively prime restrained geodetic set. Now  $(d(v_{21}, v_{41}), d(v_{21}, v_{42})) = (4, 4) = 4 \neq 1$  and so  $J(m,n)$  cannot have a relatively prime restrained geodetic set. Thus  $g_{rpr}(J(m,n)) = 0$

Case 3.  $m, n \geq 2$

By Theorem 3.3(ii),  $v_{21}, v_{22}, \dots, v_{2m}, v_{41}, v_{42}, \dots, v_{4n}$  must be in any relatively prime restrained geodetic set. Since  $d(v_{2i}, v_{4j}) = 4$  for  $1 \leq i \leq m, 1 \leq j \leq n$ , we have  $(d(v_{21}, v_{41}), d(v_{22}, v_{42})) = 4 \neq 1$ , and hence  $J(m,n)$  has no relatively prime restrained geodetic set. Thus  $g_{rpr}(J(m,n)) = 0$ .

The result follows from cases 1, 2 and 3. □

**Theorem 3.11.** For a fan graph  $F_{1,n}, g_{rpr}(F_{1,n}) = \begin{cases} 3 & \text{if } n = 2 \\ 4 & \text{if } n = 3 \\ 0 & \text{otherwise} \end{cases}$

*Proof.* Let  $P_n$  be the path  $v_1v_2\dots v_n$  and  $K_1$  be the vertex  $v$ . By definition,  $F_{1,n} = P_n + K_1$ . Then  $V(F_{1,n}) = \{v, v_i/1 \leq i \leq n\}$  and  $E(F_{1,n}) = \{vv_i, v_iv_{i+1}, vv_n/1 \leq i \leq n-1\}$ . Now we consider the following three cases depends on  $n$ .

Case 1.  $n = 2$

In this case  $F_{1,2} \cong K_3$ . By Theorem 3.5,  $g_{rpr}(F_{1,2}) = 3$ .

Case 2.  $n = 3$

Clearly  $S = \{v_i, v_{i+2}\}$  is a minimum geodetic set of  $F_{1,n}$  and so  $g(F_{1,n}) = 2$ . To get relatively prime geodetic we must add one more vertex to  $S$ . Let  $S' = \{v_i, v_{i+2}, v\}$ . Then  $S'$  is a geodetic set and  $\langle V(F_{1,n}) - S' \rangle = K_1$  which has an isolated vertex and so  $S$  is not a restrained geodetic set. The only possible restrained geodetic sets with at least four vertices are  $\{v_i, v_{i+1}, v_{i+2}, v\}$  is a minimum restrained geodetic set. Now  $d(v_i, v_{i+1}) = 1, d(v_i, v_{i+2}) = 2, d(v_{i+1}, v_{i+2}) = 1, d(v_i, v) = 1, d(v_{i+1}, v) = 1, d(v_{i+2}, v) = 1$  and  $(1, 2) = (1, 1) = 1$ . Also  $\langle V(F_{1,n}) - S \rangle$  which has no isolated vertex. Hence  $S$  is a minimum relatively prime restrained geodetic set of  $F_{1,n}$ . Therefore,  $g_{rpr}(F_{1,n}) = 4$ .

Case 3.  $n \geq 4$

Any minimum geodetic set  $S$  of  $F_{1,n}$  must contain  $\lfloor \frac{n}{2} \rfloor + 1$  vertices of  $\{v_i/1 \leq i \leq n\}$  and for any pair of vertices  $u, v \in S$ , we have  $d(u, w)$  is 1 for only one pair and 2 for the remaining pairs. This implies that  $F_{1,n}$  has no relatively prime restrained geodetic set and so  $g_{rpr}(F_{1,n}) = 0$ .

The result follows from cases 1, 2 and 3. □

**Theorem 3.12.** For the ladder graph  $L_n, g_{rpr}(L_n) = 3$ .

*Proof.* Let  $L_n$  be a ladder graph. Then the vertex  $V(L_n) = \{u_i, v_i/1 \leq i \leq n\}$  and  $E(L_n) = \{v_iv_{i+1}, u_iu_{i+1}, v_iu_i, v_nu_n/1 \leq i \leq n-1\}$ . Clearly  $S = \{v_1, u_n\}$  is a minimum geodetic set. To get a relatively prime restrained geodetic set we must add one more vertex to  $S$ . Now  $S' = \{v_1, u_1, u_n\}$  is a geodetic set and  $\langle V(L_n) - S' \rangle$  has no isolated vertex. Also

$d(v_1, u_1) = 1, d(v_1, u_n) = n, d(v_2, u_n) = n - 1$  and  $(1, n) = (1, n - 1) = (n, n - 1) = 1$ . Hence  $S'$  is a minimum relatively prime restrained geodetic set of  $L_n$  and so  $g_{rpr}(L_n) = 3$ . □

**Theorem 3.13.** For cycle  $C_n$  of odd order  $n, g_{rpr}(T(C_n)) = 3$ .

*Proof.* Let  $v_1v_2\dots v_nv_1$  be a cycle of order  $n$  and  $T(C_n)$  be the total graph of cycle graph. Denote  $e_i$  by  $v_iv_{i+1}, 1 \leq i \leq n$  where the suffices modulo  $n$ . Then  $V(T(C_n)) = \{v_i, e_i/1 \leq i \leq n\}$  and  $E(T(C_n)) = \{v_iv_{i+1}, e_ie_{i+1}, v_ie_i, v_ie_{i+1}/1 \leq i \leq n\}$ . For  $1 \leq i \leq n, S_i = \{v_i, e_{i+\lfloor \frac{n}{2} \rfloor}, e_{i+\lceil \frac{n}{2} \rceil}\}$  is a geodetic set and  $\langle V(T(C_n)) - S_i \rangle$  has no isolated vertex. Also  $d(v_i, e_{i+\lfloor \frac{n}{2} \rfloor}) = \lfloor \frac{n}{2} \rfloor, d(v_i, e_{i+\lceil \frac{n}{2} \rceil}) = \lceil \frac{n}{2} \rceil, d(e_{i+\lfloor \frac{n}{2} \rfloor}, e_{i+\lceil \frac{n}{2} \rceil}) = 1$  and  $(1, \lfloor \frac{n}{2} \rfloor) = (1, \lceil \frac{n}{2} \rceil) = (\lfloor \frac{n}{2} \rfloor, \lceil \frac{n}{2} \rceil) = 1$ . Hence  $S_i$  is a minimum relatively prime restrained geodetic set of  $T(C_n)$  and so  $g_{rpr}(T(C_n)) = 3$ . □

**Theorem 3.14.** Let  $L(C_n)$  be the line graph of  $C_n$  of even order  $n$ . Then  $g_{rpr}(L(C_n)) = 3$  for  $n \geq 8$ .

*Proof.* We have  $L(C_n) = C_n$ . Now the theorem follows from Theorem 3.7. □

**Theorem 3.15.** Let  $L(P_n)$  be the line graph of  $P_n$ . Then  $g_{rpr}(L(P_n)) = 3$ , for  $n \geq 4, n \neq 5$ .

*Proof.* We have  $L(P_n) = P_{n-1}$ . By Theorem 3.8,  $g_{rpr}(L(P_n)) = g_{rpr}(P_{n-1}) = 3$  for  $n - 1 \geq 3$ , and  $n - 1 \neq 4$  and hence  $n \geq 4$ , and  $n \neq 5$ . □

**Theorem 3.16.** Let  $L(K_{1,n})$  be the line graph of  $K_{1,n}$ . Then  $g_{rpr}(L(K_{1,n})) = n$ .

*Proof.* We have  $L(K_{1,n}) = K_n$ . By Theorem 3.4,  $g_{rpr}(L(K_{1,n})) = g_{rpr}(K_n)$ , hence  $L(K_{1,n}) = n$ . □

## 4. Conclusion

In this paper, we have found the relatively prime restrained geodetic number of some standard graphs like cycle graph, path graph, wheel graph, jelly fish graph, star graph complete graph, fan graph and ladder graph.

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