Malaya Journal of Matematik, Vol. 7, No. 4, 716-719, 2019

https://doi.org/10.26637/MJM0704/0015

C_m-E- super magic graceful labeling of graphs

Sindhu Murugan[1*] and S. Chandra Kumar[2]

Abstract

A simple graph G admits an H-covering if every edge in $E(G)$ belongs to a subgraph of G isomorphic to H. The graph G is said to be H-magic if there exists a total labeling $f : V(G) \cup E(G) \to \{1,2,\ldots,p+q\}$ such that for every subgraph $H^{'}$ of G isomorphic to H, $\sum_{v \in V(H^{'})} f(v) + \sum_{e \in E(H^{'})} f(e) = M$ for some positive integer M. An H-E-super magic graceful labeling (H-E-SMGL) is a bijection $f : V(G) \cup E(G) \to \{1,2,\ldots,p+q\}$ with the property $f(E(G)) = \{1,2,\ldots,q\}$ such that $\sum_{v \in V(H^{'})} f(v) - \sum_{e \in E(H^{'})} f(e) = M$ for some positive integer M. In this paper, we introduce H-E-SMGL and study C_m-E-super magic graceful labeling of generalized book graph.

Keywords

H-covering, H-magic labeling, H-E-super magic labeling, H-E- super magic graceful labeling.

AMS Subject Classification

05C78.

[1] *Research Scholar, Reg No-18213162092011, Scott Christian College(Autonomous), Nagercoil-629003. Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India.*

[2] *Department of Mathematics, Scott Christian College(Autonomous), Nagercoil-629003, Tamil Nadu, India.*

**Corresponding author*: [1] msindhu_87@yahoo.co.in; [2]kumar.chandra82@yahoo.in

Article History: Received **06** July **2019**; Accepted **15** October **2019**

Contents

1. Introduction

Throughout this paper, we consider only finite, simple and undirected graphs. The set of vertices and edges of a graph $G(p,q)$ will be denoted by $V(G)$ and $E(G)$ respectively, $p = |V(G)|$ and $q = |E(G)|$.

A labeling of a graph G is a mapping that carries a set of graph elements, usually vertices and or edges into a set of numbers, usually integers. Many kinds of labelings have been defined and studied by many authors and an excellent survey of graph labelings can be found in [1].

In 1963, Sedlàček [7] introduced the concept of magic labeling in graphs. A graph G is magic if the edges of G can be labeled by the set of numbers $\{1,2,\ldots,q\}$ so that the sum of labels of all the edges incident with any vertex is the same [5].

A covering of G is a family of subgraphs $H_1, H_2, \ldots, H_h$ such that each edge of $E(G)$ belongs to at least one of the subgraphs $H_i, 1 \leq i \leq h$. Then, it is said that G admits an $(H_1, H_2, \ldots, H_h)$ covering. If every H_i is isomorphic to a given graph H, then G admits an H-covering. Suppose G admits an H-covering. A total labeling $f : V(G) \cup E(G) \to \{1,2,3,\ldots,|V(G)|+|E(G)|\}$ is called an H-magic labeling of G if there exists a positive integer M (called the magic constant) such that for every subgraph $H^{'}$ of G isomorphic to H, $\sum_{v \in V(H^{'})} f(v) + \sum_{e \in E(H^{'})} f(e) = M$. A graph that admits such a labeling is called H-magic. The function f is said to be H-E-super magic labeling if $f(E(G)) = \{1,2,\ldots,q\}$.

The notion of H-magic labeling was introduced by Gutierrez and Llad [2] in 2005.

In 2007, Llado and Moragas [4] studied some C_n-supermagic graphs.

In 1967, Rosa [6] introduced a labeling called β-valuation. Golomb [3] called such labeling as graceful. An injection f from the vertices of G to { 0, 1, 2, ..., q } is called a graceful labeling of G if when we assign each edge uv the label $|f(u) - f(v)|$, the resulting edge labels are distinct.

For further information about H-E-super magic graphs, see [8].

By using definitions of H-E-super magic labeling and graceful labeling, we define a new labeling called H-E-super magic graceful. An H-E-super magic graceful labeling (H-

E-SMGL) is a bijection $f : V(G)\cup E(G) \to \{1,2,\ldots,p+q\}$ with the property $f(E(G)) = \{1,2,\ldots,q\}$ and $\sum_{v\in V(H')} f(v) - \sum_{e\in E(H')} f(e) = M$ for some positive integer M. In this paper, we introduce H-E-SMGL and study C_m-E-super magic graceful labeling of generalized book graph.

2. C_m-E-Super magic graceful graphs

A book graph $B_n = K_{1,n} \times K_2$, is defined as follows:
The vertex set $V(B_n) = \{u_1,u_2\}\cup\{v_i,w_i : 1\le i\le n\}$ and the edge set
$E(B_n) = \{u_1,u_2\}\cup\{u_1w_i, u_2v_i, v_iw_i : 1\le i\le n\}$.
The generalized book graph $B_{n,m}$ with vertex and edge sets by
$V(B_{n,m}) = \{u_i : 1\le i\le m-2\}\cup\{v_i,w_i : 1\le i\le n\}$ and
$E(B_{n,m}) = \{u_iu_{i+1} : \text{for} 1\le i\le m-3\}\cup\{u_iv_j : 1\le j\le n, i = m-2\}$
$\cup\{u_1w_i : 1\le i\le n\}\cup\{v_iw_i : 1\le i\le n\}$.
Number of vertices and edges of $B_{n,m}$ is $|V(B_{n,m})| = 2n+m-2$ and $|E(B_{n,m})| = 3n+m-3$. The following theorems shows that the generalized book graph $B_{n,m}$ is $C_m - E$-super magic graceful (SMG).

Theorem 2.1. *For an odd integer $n\ge 3$ and $m\ge 4$ the generalized book graph $B_{n,m}$ is $C_m - E$-SMG.*

Proof. We define a total labeling $f : V(B_{n,m})\cup E(B_{n,m}) \to \{1,2,3,\ldots,2m+5(n-1)\}$ as follows:

$$f(u) = \begin{cases} 3n+m+i-3 & if\ u = u_i\ for\ 1\le i\le m-2; \\ & \{3n+m-2, 3n+m-1,\ldots,3n+2m-5\} \\ 3n+2m-5+i & if\ u = v_i\ for\ 1\le i\le n \\ & \{3n+2m-4, 3n+2m-3,\ldots,4n+2m-5\} \\ 5n+2m-4-i & if\ u = w_i\ for\ 1\le i\le n \\ & \{5n+2m-5, 5n+2m-6,\ldots,4n+2m-4\}, \end{cases}$$

$$f(e) = \begin{cases} i & if\ e = u_iu_{i+1}\ for\ 1\le i\le m-3; \{1,2,\ldots,m-3\} \\ m-3+i & if\ e = u_{m-2}v_i\ for\ 1\le i\le n; \{m-2, m-1,\ldots,m+n-3\} \\ \frac{1}{2}(n+2m-7+2i) & if\ e = v_iw_i\ for\ \frac{n+3}{2}\le i\le n; \{m+n-2,\ldots,\frac{1}{2}(3n+2m-7)\} \\ \frac{1}{2}(3n+2m-7+2i) & if\ e = v_iw_i\ for\ 1\le i\le \frac{n+1}{2}; \\ & \{\frac{1}{2}(3n+2m-5),\ldots,(2n+m-3)\} \\ 3n+m-1-2i & if\ e = u_1w_i\ for\ 1\le i\le \frac{n+1}{2}; \\ & \{2n+m-2, 2n+m,\ldots,3n+m-3\} \\ 4n+m-1-2i & if\ e = u_1w_i\ for\ \frac{n+3}{2}\le i\le n; \\ & \{2n+m-1, 2n+m+1,\ldots,3n+m-4\} \end{cases}$$

Now we prove f is a $C_m - E$-super magic graceful labeling. For $1\le i\le n$, let $C_m^{(i)}$ be the sub cycle of the graph $B_{n,m}$ with $V(C_m^{(i)}) = \{u_j : 1\le j\le m-2\}\cup\{v_i,w_i\}$ and
$E(C_m^{(i)}) = \{u_ju_{j+1} : 1\le j\le m-3\}\cup\{u_1w_i, v_iw_i\}\cup\{u_{m-2}v_i\}$.
Case 1: Suppose $1\le i\le \frac{n+1}{2}$. Then $\sum_{v\in V(C_m)} f(v) - \sum_{e\in E(C_m)} f(e)$
$= \left[\sum_{j=1}^{m-2} f(u_j) + f(v_i) + f(w_i)\right] - \left[\sum_{j=1}^{m-3} f(u_ju_{j+1}) + f(u_{m-2}v_i) + f(u_1w_i) + f(v_iw_i)\right]$
$= [f(u_1)+f(u_2)+\cdots+f(u_{m-2})+f(v_i)+f(w_i)] - [f(u_1u_2) + f(u_2u_3)+\cdots+f(u_{m-3}u_{m-2})+f(u_{m-2}v_i)+f(v_iw_i)+f(u_1w_i)]$
$= [(3n+m-3+1)+(3n+m-3+2)+\cdots+(3n+m-3+m-2)+(3n+2m-5+i)+(5n+2m-4-i)] - [(1+2+\cdots+m-3)+(m+i-3)+\frac{1}{2}(3n+2m-7+2i)+(3n+m-1-2i)]$
$= \frac{1}{2}[6mn+2m^2-6m-12n-4m+12+m^2-3m+2+16n+8m-18] - \frac{1}{2}[m^2-5m+6+9n+6m-15]$
$= \frac{1}{2}[2m^2+6mn-6m-5n+5]$, which is an integer since n is odd.
Case 2: Suppose $\frac{n+3}{2}\le i\le n$. Then $\sum_{v\in V(C_m)} f(v) - \sum_{e\in E(C_m)} f(e)$
$= \left[\sum_{j=1}^{m-2} f(u_j) + f(v_i) + f(w_i)\right] - \left[\sum_{j=1}^{m-3} f(u_ju_{j+1}) + f(u_{m-2}v_i) + f(u_1w_i) + f(v_iw_i)\right]$
$= [f(u_1)+f(u_2)+\cdots+f(u_{m-2})+f(v_i)+f(w_i)] - [f(u_1u_2) + f(u_2u_3)+\cdots+f(u_{m-3}u_{m-2})+f(u_{m-2}v_i)+f(v_iw_i)+f(u_1w_i)]$
$= [(3n+m-3+1)+(3n+m-3+2)+\cdots+(3n+m-3+m-2)+(3n+2m-5+i)+(5n+2m-4-i)] - [(1+2+\cdots+m-3)+(m-3+i)+\frac{1}{2}(n+2m-7+2i)+(4n+m-1-2i)]$
$= \frac{1}{2}[6mn+2m^2-6m-12n-4m+12+m^2-3m+2+16n+8m-18] - \frac{1}{2}[m^2+m+9n-9]$
$= \frac{1}{2}[2m^2+6mn-6m-5n+5]$.
Thus the generalized book graph $B_{n,m}$ is $C_m - E$-SMG with magic constant $\frac{1}{2}[2m^2+6mn-6m-5n+5]$ when n is odd. □

Example 2.2. Consider the following graph $G = B_{7,5}$.
Here, $V(G) = \{u_i : 1\le i\le 3\}\cup\{v_i : 1\le i\le 7\}\cup\{w_i : 1\le i\le 7\}$ and
$E(G) = \{u_iu_{i+1} : \text{for} 1\le i\le 2\}\cup\{u_3v_i : 1\le i\le 7\}\cup\{u_1w_i : 1\le i\le 7\}\cup\{v_iw_i : 1\le i\le 7\}$.
As discussed in Theorem 2.1, we define a total labeling $f : V(G)\cup E(G) \to \{1,2,3,\ldots,40\}$ as follows:

$$f(u) = \begin{cases} 23+i & if\ u = u_i\ for\ 1\le i\le 3; \{24,25,26\} \\ 26+i & if\ u = v_i\ for\ 1\le i\le 7; \{27,28,\ldots,33\} \\ 41-i & if\ u = w_i\ for\ 1\le i\le 7; \{40,39,\ldots,34\}, \end{cases}$$

$$f(e) = \begin{cases} i & if\ e = u_iu_{i+1}\ for\ 1\le i\le 2; \{1,2\} \\ 2+i & if\ e = u_3v_i\ for\ 1\le i\le 7; \{3,4,\ldots,9\} \\ 5+i & if\ e = v_iw_i\ for\ 5\le i\le 7; \{10,11,12\} \\ 12+i & if\ e = v_iw_i\ for\ 1\le i\le 4; \{13,14,\ldots,16\} \\ 25-2i & if\ e = u_1w_i\ for\ 1\le i\le 4; \{23,21,19,17\} \\ 32-2i & if\ e = u_1w_i\ for\ 5\le i\le 7; \{22,20,18\} \end{cases}$$

Now we prove f is a C_m-E-super magic graceful labeling. For $1\le i\le 7$ be the sub cycle of the graph $B_{7,5}$ with $V(C_m^{(i)}) = \{u_j : 1\le j\le 3\}\cup\{v_i,w_i\}$ and
$E(C_m^{(i)}) = \{u_ju_{j+1} : 1\le j\le 2\}\cup\{u_1w_i, v_iw_i\}\cup\{u_3v_i\}$.
Case 1: Suppose $1\le i\le 4$. Then $\sum_{v\in V(C_m^{(i)})} f(v) - \sum_{e\in E(C_m^{(i)})} f(e)$
$= \left[\sum_{j=1}^{3} f(u_j) + f(v_i) + f(w_i)\right] - \left[\sum_{j=1}^{2} f(u_ju_{j+1}) + f(u_3v_i) + f(u_1w_i) + f(v_iw_i)\right]$
$= [f(u_1)+f(u_2)+f(u_3)+f(v_i)+f(w_i)] - [f(u_1u_2)+f(u_2u_3)+f(u_3v_i)+f(v_iw_i)+f(u_1w_i)] = [(3(7)+5-3+1)+(3(7)+5-3+2)+(3(7)+5-3+3)+(3(7)+2(5)-5+i)+(5(7)+2(5)-4-i)] - [(1+2)+(5+i-3)+(3(7)+5-1-2i)+\frac{1}{2}(3(7)+2(5)-7+2i)]$
$= 142-42 = 100.$

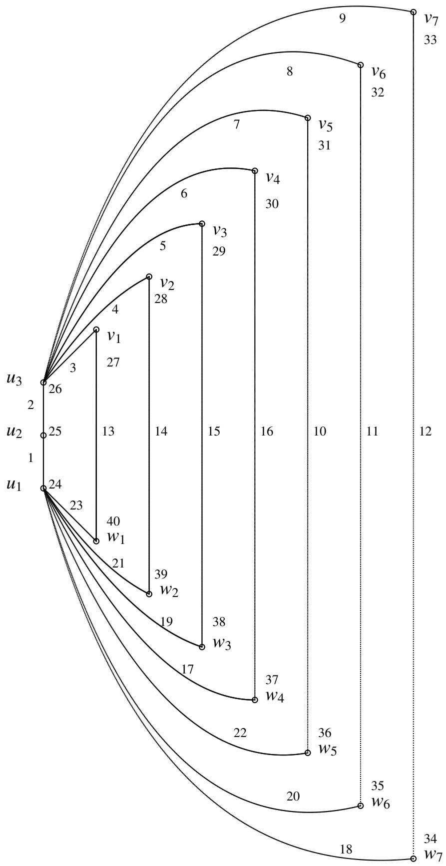

Figure 1: G

Case 2: Suppose $5 \leq i \leq 7$. Then $\sum_{v \in V(C_m^{(i)})} f(v) - \sum_{e \in E(C_m^{(i)})} f(e)$
$= \left[\sum_{j=1}^{3} f(u_j) + f(v_i) + f(w_i) \right] - \left[\sum_{j=1}^{2} f(u_j u_{j+1}) + f(u_3 v_i) + f(u_1 w_i) + f(v_i w_i) \right]$
$= [f(u_1) + f(u_2) + f(u_3) + f(v_i) + f(w_i)] - [f(u_1u_2) + f(u_2u_3) + f(u_3v_i) + f(v_iw_i) + f(u_1w_i)]$
$= [(3(7)+5-3+1) + (3(7)+5-3+2) + (3(7)+5-3+3) + (3(7)+2(5)-5+i) + (5(7)+2(5)-4-i)] - [(1+2) + (2+i) + \frac{1}{2}(7+2(5)-7+2i) + (4(7)+5-1-2i)]$
$= 142 - 42 = 100.$
The graph $B_{7,5}$ is $C_m - E$-super magic graceful.

Theorem 2.3. *For an even integer $n \geq 3$ and $m \geq 4$ the generalized book graph $B_{n,m}$ is C_m-E-SMG.*

Proof. We define a total labeling $f : V(B_{n,m}) \cup E(B_{n,m}) \to \{1, 2, 3, \dots, 2m + 5(n-1)\}$ as follows:

$$f(u) = \begin{cases} 3n+m+i-3 & if\ u = u_i\ for\ 1 \leq i \leq m-2; \\ & \{3n+m-2, 3n+m-1, \dots, 3n+2m-5\} \\ 3n+2m-5+i & if\ u = v_i\ for\ 1 \leq i \leq n \\ & \{3n+2m-4, 3n+2m-3, \dots, 4n+2m-5\} \\ 5n+2m-4-i & if\ u = w_i\ for\ 1 \leq i \leq n \\ & \{5n+2m-5, 5n+2m-6, \dots, 4n+2m-4\}, \end{cases}$$

$$f(e) = \begin{cases} i & if\ e = u_{m-2}v_i\ for\ 1 \leq i \leq \frac{n}{2}; \{1, 2, \dots, \frac{n}{2}\} \\ \frac{n+2}{2} & if\ e = u_1u_2; \{\frac{n+2}{2}\} \\ i+1 & if\ e = u_{m-2}v_i\ for\ \frac{n+2}{2} \leq i \leq n; \{\frac{n+4}{2}, \dots, n+1\} \\ n+i & if\ e = u_iu_{i+1}\ for\ 2 \leq i \leq m-3; \{n+2, \dots, n+m-3\} \\ \frac{1}{2}(n+2m-6+2i) & if\ e = v_iw_i\ for\ \frac{n+2}{2} \leq i \leq n; \{n+m-2, \dots, \frac{1}{2}(3n+2m-6)\} \\ \frac{1}{2}(3n+2m-6+2i) & if\ e = v_iw_i\ for\ 1 \leq i \leq \frac{n}{2}; \{\frac{1}{2}(3n+2m-4), \dots, 2n+m-3\} \\ 4n+m-2-2i & if\ e = u_1w_i\ for\ \frac{n+2}{2} \leq i \leq n; \{3n+m-4, \dots, 2n+m-2\} \\ 3n+m-1-2i & if\ e = u_1w_i\ for\ 1 \leq i \leq \frac{n}{2}; \{3n+m-3, \dots, 2n+m-1\}. \end{cases}$$

Now we prove f is a C_m-E-super magic graceful labeling.
Case 1: Suppose $1 \leq i \leq \frac{n}{2}$. Then $\sum_{v \in V(C_m)} f(v) - \sum_{e \in E(C_m)} f(e)$
$= \left[\sum_{j=1}^{m-2} f(u_j) + f(v_i) + f(w_i) \right] - \left[f(u_1u_2) + \sum_{j=2}^{m-3} f(u_ju_{j+1}) + f(u_{m-2}v_i) + f(u_1w_i) + f(v_iw_i) \right]$
$= [f(u_1) + f(u_2) + \dots + f(u_{m-2}) + f(v_i) + f(w_i)] - [f(u_1u_2) + f(u_2u_3) + \dots + f(u_{m-3}u_{m-2}) + f(u_{m-2}v_i) + f(v_iw_i) + f(u_1w_i)]$
$= [(3n+m-3+1) + (3n+m-3+2) + \dots + (3n+m-3+m-2) + (3n+2m-5+i) + (5n+2m-4-i)] - [\frac{n+2}{2} + (n+1) + (n+2) + \dots + (n+m-3) + (-n-1) + (i) + \frac{1}{2}(3n+2m-6+2i) + (3n+m-1-2i)]$
$= \frac{1}{2}[6mn + 2m^2 - 6m - 12n - 4m + 12 + m^2 - 3m + 2 + 16n + 8m - 18] - \frac{1}{2}[m^2 + 2mn + 2n - m - 2]$
$= m^2 + 2mn - 2m + n - 1.$
Case 2: Suppose $\frac{n+2}{2} \leq i \leq n$. Then $\sum_{v \in V(C_m)} f(v) - \sum_{e \in E(C_m)} f(e)$
$= \left[\sum_{j=1}^{m-2} f(u_j) + f(v_i) + f(w_i) \right] - \left[f(u_1u_2) + \sum_{j=2}^{m-3} f(u_ju_{j+1}) + f(u_{m-2}v_i) + f(u_1w_i) + f(v_iw_i) \right]$
$= [f(u_1) + f(u_2) + \dots + f(u_{m-2}) + f(v_i) + f(w_i)] - [f(u_1u_2) + f(u_2u_3) + \dots + f(u_{m-3}u_{m-2}) + f(u_{m-2}v_i) + f(v_iw_i) + f(u_1w_i)]$
$= [(3n+m-3+1) + (3n+m-3+2) + \dots + (3n+m-3+m-2) + (3n+2m-5+i) + (5n+2m-4-i)] - [\frac{n+2}{2} + (n+1) + (n+2) + \dots + (n+m-3) + (-n-1) + (i+1) + (4n+m-2-2i + \frac{1}{2}(n+2m-6+2i))]$
$= \frac{1}{2}[6mn + 2m^2 - 6m - 12n - 4m + 12 + m^2 - 3m + 2 + 16n + 8m - 18] - \frac{1}{2}[m^2 + 2mn + 2n - m - 2]$
$= m^2 + 2mn - 2m + n - 1.$
The generalized book graph $B_{n,m}$ is C_m-E-SMG when n is even. □

Example 2.4. Consider the following graph $G = B_{8,5}$.
Here, $V(G) = \{u_i : 1 \leq i \leq 3\} \cup \{v_i : 1 \leq i \leq 8\} \cup \{w_i : 1 \leq i \leq 8\}$ and
$E(G) = \{u_iu_{i+1} : \text{for} 1 \leq i \leq 2\} \cup \{u_3v_i : 1 \leq i \leq 8\} \cup \{u_1w_i : 1 \leq i \leq 8\} \cup \{v_iw_i : 1 \leq i \leq 8\}$.
As discussed in Theorem 2.3, we define a total labeling $f : V(G) \cup E(G) \to \{1, 2, 3, \dots, 45\}$ as follows:

$$f(u) = \begin{cases} 26+i & if\ u = u_i\ for\ 1 \leq i \leq 3; \{27, 28, 29\} \\ 29+i & if\ u = v_i\ for\ 1 \leq i \leq 8; \{30, 31, \dots, 37\} \\ 46-i & if\ u = w_i\ for\ 1 \leq i \leq 8; \{45, 44, \dots, 38\}, \end{cases}$$

$$f(e) = \begin{cases} i & if\ e = u_3v_i\ for\ 1 \leq i \leq 4; \{1, 2, 3, 4\} \\ 5 & if\ e = u_1u_2; \{5\} \\ i+1 & if\ e = u_3v_i\ for\ 5 \leq i \leq 8; \{6, 7, 8, 9\} \\ 8+i & if\ e = u_iu_{i+1}\ for\ 2 \leq i \leq 2; \{10\} \\ 6+i & if\ e = v_iw_i\ for\ 5 \leq i \leq 8; \{11, 12, 13, 14\} \\ 14+i & if\ e = v_iw_i\ for\ 1 \leq i \leq 4; \{15, 16, 17, 18\} \\ 35-2i & if\ e = u_1w_i\ for\ 5 \leq i \leq 8; \{25, 23, 21, 19\} \\ 28-2i & if\ e = u_1w_i\ for\ 1 \leq i \leq 4; \{26, 24, 22, 20\} \end{cases}$$

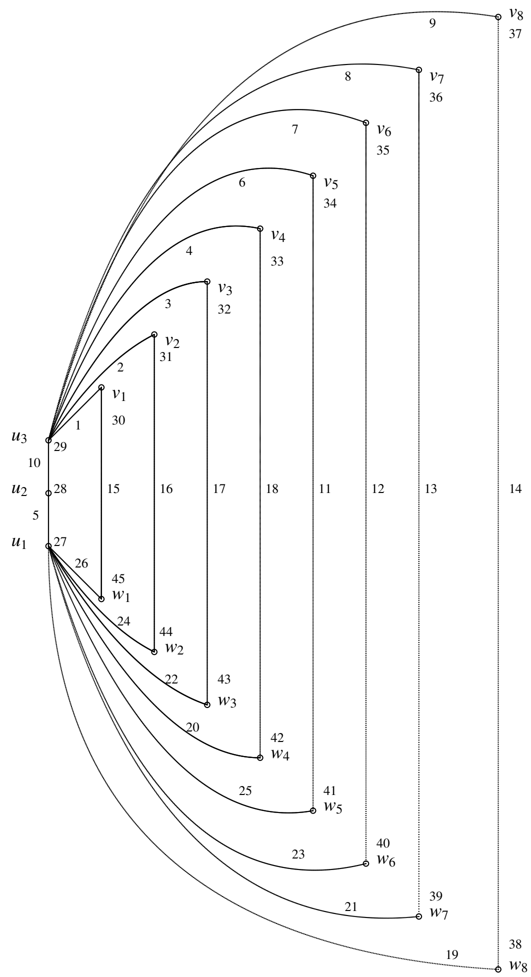

Figure 2: G

Now we prove f is a C_m-E-super magic graceful labeling. For $1 \le i \le 8$ be the sub cycle of the graph $B_{8,5}$ with $V(C_m^{(i)}) = \{u_j : 1 \le j \le 3\} \cup \{v_i, w_i\}$ and
$E(C_m^{(i)}) = \{u_ju_{j+1} : 1 \le j \le 2\} \cup \{u_1w_i, v_iw_i\} \cup \{u_3v_i\}$.

Case 1: Suppose $1 \le i \le 4$. Then $\sum\limits_{v \in V(C_m^{(i)})} f(v) - \sum\limits_{e \in E(C_m^{(i)})} f(e)$ =

$$= \left[\sum_{j=1}^{3} f(u_j) + f(v_i) + f(w_i)\right] - \left[f(u_1u_2 + \sum_{j=2}^{2} f(u_ju_{j+1}) + f(u_3v_i) + f(u_1w_i) + f(v_iw_i)\right]$$

$= [f(u_1) + f(u_2) + f(u_3) + f(v_i) + f(w_i)] - [5 + f(u_2u_3) + f(u_3v_i) + f(u_1w_i) + f(v_iw_i)]$
$= [(3(8)+5+1-3) + (3(8)+5+2-3) + (3(8)+5+3-3) + (3(8)+2(5)-5+i) + (5(8)+2(5)-4-i)] - [(5) + (n+2) + i + (3(8)+5-1-2i) + \frac{1}{2}(3(8)+2(5)-6+2i)]$
$= 159 - 57 = 102.$

Case 2: Suppose $5 \le i \le 8$. Then $\sum\limits_{v \in V(C_m^{(i)})} f(v) - \sum\limits_{e \in E(C_m^{(i)})} f(e)$

$$= \left[\sum_{j=1}^{3} f(u_j) + f(v_i) + f(w_i)\right] - \left[f(u_1u_2) + \sum_{j=2}^{2} f(u_ju_{j+1}) + f(u_{m-2}v_i) + f(u_1w_i) + f(v_iw_i)\right]$$

$= [f(u_1) + f(u_2) + f(u_3) + f(v_i) + f(w_i)] - [f(u_1u_2) + f(u_2u_3) + f(u_{m-2}v_i) + f(u_1w_i) + f(v_iw_i)]$
$= [(3(8)+5+1-3) + (3(8)+5+2-3) + (3(8)+5+3-3) + (3(8)+2(5)-5+i) + (5(8)+2(5)-4-i)] - [\frac{8+2}{2} + (n+2) + (i+1) + (4(8)+5-2-2i) + \frac{1}{2}(8+2(5)-6+2i)]$
$= 159 - 57 = 102.$

The graph $B_{8,5}$ is $C_m - E$-super magic graceful.

Corollary 2.5. *For any integer $n \ge 2$, the book graph B_n is C_4-supermagic.*

References

[1] J.A. Gallian, *A Dynamic Survey of Graph Labeling*, Electron. J. Combin., (2017), #DS6.

[2] A. Gutierrez, A. Llad,*Magic coverings*, J. Combin. Math. Combin. Comput. 55 (2005) 43-56.

[3] S. W. Golomb,*How to number a graph, in Graph Theory and Computing*, R. C. Road, ed., Academic Press, New York (1972), 23-37.

[4] A. Llad, J. Moragas,*Cycle- magic graphs*, Discrete Math. 307 (23) (2007) 2925-2933.

[5] G. Marimuthu and M.Balakrishnan, *E-super vertex magic labeling of graphs*, Discrete Appl. Math. **160** (2012) 1766-1774.

[6] A. Rosa,*On certain valuations of the vertices of a graph*, Theory of Graphs (Inter. Nat. Sym. Rome, July 1966), Gordon and Breach, N.Y. and Dunod Paris, (1967), 349-355.

[7] Sedlàček, *Problem 27, in Theory of Graphs and its Applications*, Proc. Symposium Smolenice, June (1963) 163-167.

[8] S. StalinKumar, G. Marimuthu, *H-E-super Magic decomposition of complete bipartite graphs,* Electron. Notes Discrete Math. 48(2015), 297-300.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆