Malaya Journal of Matematik, Vol. 7, No. 4, 735-745, 2019

https://doi.org/10.26637/MJM0704/0018

Generalized interval valued fuzzy ideals of KU-algebra

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Abstract

In this paper, we introduced the concept of "belongs to" relation $(\in_{\hat{a}})$ between interval valued fuzzy point to an interval valued fuzzy set with respect to an interval $\hat{a}$ and "quasi-coincident with " relation $(q_{(\hat{a},\hat{b})})$ between interval valued fuzzy point to an interval valued fuzzy set with respect to intervals $\hat{a},\hat{b}$ and combining both the concepts we define $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy KU-ideals in KU-algebras and investigated some of their related properties. Some characterizations of these generalized interval valued fuzzy KU-ideal are derived.

Keywords

KU-algebra, Fuzzy ideal , $(\in,\in\vee q)$-fuzzy ideal, $(a,b;\in_a,\in_a\vee q_{(a,b)})$-fuzzy ideal, $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal, Homomorphism.

AMS Subject Classification

06F35, 03E72, 03G25.

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Article History: Received **24** July **2019**; Accepted **04** November **2019**

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1. Introduction

The concept of fuzzy sets was first initiated by Zadeh([27]) in 1965. Since then these ideas have been applied to other algebraic structures such as group, semi-group, ring, vector spaces etc. Imai and Iseki ([8]) introduced BCK-algebra as a generalization of notion of the concept of set theoretic difference and propositional calculus and in the same year Iseki ([9]) introduced the notion of BCI-algebra which is a generalization of BCK-algebra. Xi([23]) applied the concept of fuzzy set to BCK-algebra. and discussed some properties and also introduced fuzzy subalgebra and fuzzy ideals in BCK-algebra. The class of BCK-algebra is a proper subclass of the class of BCI-algebras. Since then, a great deal of literature has been produced on the theory of BCK/BCI-algebras. In particular, emphasis seems to have been put on the ideal theory of BCK/BCI-algebras. Prabpayak and Leerawat ([16]) introduced a new algebraic structure of type BCK/BCI, which is called KU-algebra. They gave the concept of homomorphisms of KU-algebras and investigated some related properties in ([17]). The study of KU-algebras in fuzzy context was first initiated by Mostafa et al. ([18]). They also introduced the notion of fuzzy (n-fold) KU-ideals of KU-algebras ([21].They also studied KU-algebras in terms of interval-valued fuzzy sets in ([19]).Muhiuddin ([14]) applied the bipolar-valued fuzzy set theory to KU-algebras, and introduced the notions of bipolar fuzzy KU-subalgebras and bipolar fuzzy KU-ideals in KU-algebras. He considered the specifications of a bipolar fuzzy KU-subalgebra, a bipolar fuzzy KU-ideal in KU-algebras and discussed the relations between a bipolar fuzzy KU-subalgebra and a bipolar fuzzy KU-ideal and provided conditions for a bipolar fuzzy KU-subalgebra to be a bipolar fuzzy KU-ideal. Gulistan et al. ([22]) studied (α,β)-fuzzy KU-ideals in KU-algebras and discussed some special properties. Akram et al.([1]) introduced the notion of $(\tilde{\theta},\tilde{\delta})$-interval-valued fuzzy KU-ideals of KU-algebras and obtained some related properties.

As a generalization of fuzzy set interval-valued fuzzy set were proposed by Zadeh ([28]) as a natural extension of fuzzy sets. Interval-valued fuzzy subsets have many applications in several areas. Biswas ([4]) defined interval valued fuzzy subgroups i.e., interval valued fuzzy subgroups of Rosenfeld's nature, and investigated some elementary properties. The concept of interval-valued fuzzy sets have been studied in various algebraic structures, see ([7, 11, 19, 25, 26, 30]).

The concept of fuzzy point introduced by Ming and Ming in [12] and also they introduced the idea of relation "belongs to" and "quasi coincident with" between fuzzy point and a fuzzy set. Bhakat and Das [2, 3] used the combined relation of "belongs to" and "quasi coincident with" between fuzzy point and a fuzzy set to introduce the concept of $(\in, \in \vee q)$-fuzzy subgroup, $(\in, \in \vee q)$-fuzzy subring and $(\in, \in \vee q)$-level subset. Zhan, Jun and Davvaz [29] introduced $(\in, \in \vee q)$-fuzzy ideals in *BCI*-algebra in 2009. Lee et al[11] introduced interval-valued $(\in, \in \vee q_k)$-fuzzy ideals of rings and Ma et al. [30] studied interval valued fuzzy $(p-, q-, a-)$ideals of BCI-algebras and $(\in, \in \vee q)$-interval-valued fuzzy (p-,q-,a-) ideals of BCI-algebras with some related properties. Dutta et al. [7] investigated interval-valued fuzzy prime and semiprime ideals of a hyper semiring.

The notion of an $(a, b; \in_a, \in_a \vee q_{(a,b)})$-fuzzy subalgebra / subgroups introduced by Das in [5, 6]. It is found that $(a, b; \in_a, \in_a \vee q_{(a,b)})$-fuzzy structure is the generalisation of $(\in, \in \vee q)$-fuzzy structure. Motivated by this, combining both the notion of interval-valued fuzzy point and $(a, b; \in_a, \in_a \vee q_{(a,b)})$-fuzzy structure we introduce a new notion of $(\hat{a}, \hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy structure of KU-algebra and establish some related properties.

2. Preliminaries

In this section, we will recall some concepts related to KU-algebra, fuzzy point, interval-valued fuzzy point and interval-valued fuzzy sets.

Definition 2.1. *A KU-algebra is a non-empty set X with a constant* 0 *and a binary operation* $*$ *satisfying the following axioms:*

(i) $(x*y)*[(y*z)*(x*z)] = 0,$

(ii) $x*0 = 0,$

(iii) $0*x = x\, \forall\, x, y, z \in X.$

(iv) $x*y = 0 = y*x \Rightarrow x = y\, \forall\, x, y, z \in X.$

For brevity, we also call X a BG-algebra. We can define a partial ordering " $\le$ *" on X by* $x \le y$ *iff* $y*x = 0$

Definition 2.2. *A non-empty subset S of a KU-algebra X is called a subalgebra of X if* $x*y \in S$ *for all* $x, y \in S$.

Definition 2.3. *A nonempty subset I of a KU-algebra X is called a KU-ideal of X if*

(i) $0 \in I,$

(ii) $x*(y*z) \in I,\ y \in I \Rightarrow x*z \in I,\ \forall\, x, y, z \in X.$

Definition 2.4. *A fuzzy set* μ *in X is called a fuzzy KU-ideal of X if it satisfies the following conditions:*

(i) $\mu(0) \ge \mu(x),$

(ii) $\mu(x*z) \ge \min\{\mu(x*(y*z)), \mu(y)\}\ \forall\, x, y, z \in X.$

Definition 2.5. *A fuzzy set* μ *of the form*

$$\mu(y) = \begin{cases} t, & if \quad y = x, \quad t \in (0,1] \\ 0, & if \quad y \ne x \end{cases}$$

is called a fuzzy point with support x and value t and is denoted by x_t.

Definition 2.6. *A fuzzy point* x_t *is said to belong to (respectively be quasi coincident with) a fuzzy set* μ *written as* $x_t \in \mu$ *(respectively* $x_t q\mu$*) if* $\mu(x) \ge t$ *(respectively* $\mu(x) + t > 1$*). If* $x_t \in \mu$ *or* $x_t q\mu$*, then we write* $x_t \in \vee q\mu$*. (Note* $\overline{\in \vee q}$ *means* $\in \vee q$ *does not hold).*

Definition 2.7. *A fuzzy subset* μ *of a KU-algebra X is said to be an* $(\in, \in \vee q)$*-fuzzy ideal of X if*

$$(i) x_t \in \mu \Rightarrow 0_t \in \vee q\mu.$$
$$(ii)(x*(y*z))_t, y_s \in \mu \Rightarrow (x*z)_{m(t,s)} \in \vee q\mu.$$

Definition 2.8. *A fuzzy subset* μ *of a KU-algebra X is said to be an* (α, β)*-fuzzy ideal of X, if*

$$(i) x_t \alpha\mu \Rightarrow 0_t \beta\mu.$$
$$(ii)(x*(y*z))_t, y_s \alpha\mu \Rightarrow x*z_{m(t,s)} \beta\mu\ \forall\, x, y \in X,$$

where $m(t,s) = \min\{t, s\}$ *and* $\alpha, \beta \in \{\in, q, \in \vee q, \in \wedge q\}$ *and* $\alpha \ne \in \wedge q$.

Definition 2.9. *Let* λ *be a fuzzy set in X and* $0 \le t < 1$*, then t-cut set of fuzzy set* λ *are given by* $\lambda_t = \{x \in G | \lambda(x) \ge t\}$

The notion of interval-valued fuzzy set was introduced by L.A.Zadeh[13]. To consider the notion of interval-valued fuzzy sets, we need the following definitions. An interval number on [0, 1], denoted by $\hat{a}$, is defined as the closed sub interval of [0, 1], where $\hat{a} = [\underline{a}, \overline{a}]$,satisfying $0 \le \underline{a} \le \overline{a} \le 1$. Let D[0, 1] denote the set of all such interval numbers on [0, 1] and also denote the interval numbers [0, 0] and [1, 1] by $\hat{0}$ and $\hat{1}$ respectively.
Let $\hat{a}_1 = [\underline{a_1}, \overline{a_1}]$ and $\hat{a}_2 = [\underline{a_2}, \overline{a_2}] \in D[0, 1]$. Define on D[0, 1] the relations $\le, =, <, +, .$ by

1. $\hat{a}_1 \le \hat{a}_2 \Leftrightarrow \underline{a_1} \le \underline{a_2}$ and $\overline{a_1} \le \overline{a_2}$
2. $\hat{a}_1 = \hat{a}_2 \Leftrightarrow \underline{a_1} = \underline{a_2}$ and $\overline{a_1} = \overline{a_2}$

3. $\hat{a}_1 < \hat{a}_2 \Leftrightarrow \underline{a_1} < \underline{a_2}$ and $\overline{a_1} < \overline{a_2}$
4. $\hat{a}_1 + \hat{a}_2 \Leftrightarrow [\underline{a_1} + \underline{a_2},\ \overline{a_1} + \overline{a_2}]$
5. $\hat{a}_1.\hat{a}_2 \Leftrightarrow [min(\underline{a_1}\underline{a_2}, \underline{a_1}\overline{a_2}, \overline{a_1}\underline{a_2}, \overline{a_1}\overline{a_2}),$ $max(\underline{a_1}\underline{a_2}, \underline{a_1}\overline{a_2}, \overline{a_1}\underline{a_2}, \overline{a_1}\overline{a_2})] = [\underline{a_1}\underline{a_2}, \overline{a_1}\overline{a_2}]$
6. $k\hat{a} = [k\underline{a}, k\overline{a}]$ where $0 \leq k \leq 1$

Now consider two intervals $\hat{a}_1 = [\underline{a_1}, \overline{a_1}], \hat{a}_2 = [\underline{a_2}, \overline{a_2}] \in D[0,\ 1]$ then we define refine minimum rmin as $rmin(\hat{a}_1, \hat{a}_2)$ $= [min(\underline{a_1}, \underline{a_2}), min(\overline{a_1}, \overline{a_2})]$ and refine maximum as rmax $rmax(\hat{a}_1, \hat{a}_2) = [max(\underline{a_1}, \underline{a_2}), max(\overline{a_1}, \overline{a_2})]$ generally if $\hat{a}_i = [\underline{a_1}, \overline{a_i}], \hat{b}_i = [\underline{b_1}, \overline{b_i}] \in D[0,\ 1]$ for i= 1,2,3,...then we define $rmax(\hat{a}_i, \hat{b}_i) = [max(\underline{a_i}, \underline{b_i}), max(\overline{a_i}, \overline{b_i})]$ and $rmin(\hat{a}_i, \hat{b}_i)$ $= [min(\underline{a_i}, \underline{b_i}), min(\overline{a_i}, \overline{b_i})]$ and $rinf_i(\hat{a}_i) = [\wedge_i \underline{a_i}, \wedge_i \overline{a_i}]$ and $rsup_i(\hat{a}_i) = [\vee_i \underline{a_i}, \vee_i \overline{a_i}]$.
$(D[0,\ 1], \preceq)$ is a complete lattice with $\wedge = rmin$, $\vee = rmax, \hat{0} = [0\,0]$ and $\hat{1} = [1\,1]$ being the least and the greatest element respectively.

Definition 2.10. *An interval-valued fuzzy set defined on a non empty set X as an objects having the form* $\hat{\mu} = \{x, [\underline{\mu}(x), \overline{\mu}(x)]\}$, $\forall x \in X$ *where* $\underline{\mu}$ *and* $\overline{\mu}$ *are two fuzzy sets in X such that* $\underline{\mu}(x) \leq \overline{\mu}(x)$ *for all* $x \in X$. *Let* $\hat{\mu}(x) = [\underline{\mu}(x), \overline{\mu}(x)], \forall x \in X$.*Then* $\hat{\mu}(x) \in D[0\ 1], \forall x \in X$.
If $\hat{\mu}$ *and* $\hat{\nu}$ *be two interval-valued fuzzy sets in X, then we define*

- $\hat{\mu} \subset \hat{\nu} \Leftrightarrow$ *for all* $x \in X, \quad \underline{\mu}(x) \leq \underline{\nu}(x)$ *and* $\overline{\mu}(x) \leq \overline{\nu}(x)$.
- $\hat{\mu} = \hat{\nu} \Leftrightarrow$ *for all* $x \in X, \quad \underline{\mu}(x) = \underline{\nu}(x)$ *and* $\overline{\mu}(x) = \overline{\nu}(x)$.
- $(\hat{\mu} \cup \hat{\nu})(x) = \hat{\mu}(x) \vee \hat{\nu}(x)$ $= [max\{\underline{\mu}(x), \underline{\nu}(x)\}, max\{\overline{\mu}(x), \overline{\nu}(x)\}]$.
- $(\hat{\mu} \cap \hat{\nu})(x) = \hat{\mu}(x) \wedge \hat{\nu}(x)$ $= [min\{\underline{\mu}(x), \underline{\nu}(x)\}, min\{\overline{\mu}(x), \overline{\nu}(x)\}]$.
- $(\hat{\mu} \times \hat{\nu})(x,y) = \hat{\mu}(x) \wedge \hat{\nu}(y)$ $= [min\{\underline{\mu}(x), \underline{\nu}(y)\}, min\{\overline{\mu}(x), \overline{\nu}(y)\}]$.
- $\hat{\mu}^c(x) = [1 - \overline{\mu}(x), 1 - \underline{\mu}(x)]$.

Definition 2.11. *Let* $\hat{\mu}$ *be an interval-valued fuzzy set in X. Then for every* $[0,\ 0] < \hat{t} \leq [1,\ 1]$, *the crisp set* $\hat{\mu}_t = \{x \in X \mid \hat{\mu}(x) \geq \hat{t}\}$ *is called the level subset of* $\hat{\mu}$.

Definition 2.12. *An interval-valued fuzzy set* $\hat{\mu}$ *in KU-algebra X is called an interval-valued fuzzy KU-subalgebra of X if* $\hat{\mu}(x*y) \geq rmin\{\hat{\mu}(x), \hat{\mu}(y)\}$ *for all* $x, y \in X$.

Definition 2.13. *A interval-valued fuzzy set* $\hat{\mu}$ *in X is called an interval-valued fuzzy KU-ideal of X if it satisfies the following conditions:*

(i) $\hat{\mu}(0) \geq \hat{\mu}(x)$,

(ii) $\hat{\mu}(x*z) \geq rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y)\}\ \forall\, x,y,z \in X$.

Definition 2.14. *A interval-valued fuzzy subset* $\hat{\mu}$ *of a KU-algebra X is said to be an* $(\in, \in \vee q)$*-interval-valued fuzzy ideal of X if*

$$(i) x_{\hat{t}} \in \hat{\mu} \Rightarrow 0_{\hat{t}} \in \vee q \hat{\mu}.$$
$$(ii) (x*(y*z))_{\hat{t}}, y_{\hat{t}} \in \hat{\mu} \Rightarrow (x*z)_{rmin(\hat{t},\hat{s})} \in \vee q \hat{\mu}.$$

3. $(\hat{a}, \hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideals of KU-algebra

Now onwards, let X denote a KU-algebra and $\hat{a}, \hat{b} \in D[0,1]$ such that $\hat{0} < \hat{a} < \hat{b} \leq \hat{1}$ also let $\hat{c} = rmin(2\hat{b}, \hat{1}), \hat{d} = rmin(2\hat{b}, \hat{1} + \hat{a})$ and $\hat{k} = \hat{d}/2$.

Definition 3.1. *Ma et al [30] extended the notion of belongingness and quasi-coincidence of a fuzzy point with a fuzzy set and defined the notions of belongingness and quasi-coincidence of an interval-valued fuzzy point with an interval-valued fuzzy set. For any interval-valued fuzzy set* $\hat{\mu} = \{x, [\underline{\mu}(x), \overline{\mu}(x)]\}$ *and* $\hat{t} = [\underline{t}, \overline{t}]$, *we define* $\hat{\mu} + \hat{t} = [\underline{\mu}(x) + \underline{t}, \overline{\mu}(x) + \overline{t}]$ *for all* $x \in X$. *In particular if* $\underline{\mu}(x) + \underline{t} > 1$, *we write as* $\hat{\mu} + \hat{t} > [1,1] = \hat{1}$
Let $x \in X$ *and* $\hat{t} \in D[0,1]$, *an interval-valued fuzzy set* $\hat{\mu}$ *of a KU-algebra X of the form*

$$\hat{\mu}(y) = \begin{cases} \hat{t} \neq [0,0], & if \quad y = x, \quad \hat{t} \in D(0,1] \\ \hat{0} = [0,0], & if \quad y \neq x \end{cases}$$

is said to be an interval-valued fuzzy point with support x and interval-valued value $\hat{t}$ *and is denoted by* $x_{\hat{t}}$.

Let $\hat{\mu}$ *be an interval-valued fuzzy set in X. An interval-valued fuzzy point* $x_{\hat{t}}$ *is said to belongs to* $\hat{\mu}$ *w.r.t* $\hat{a}$ *denoted by* $x_{\hat{t}} \in_{\hat{a}} \hat{\mu}$(*resp.coincident with* $\hat{\mu}$ *w.r.t* $(\hat{a}, \hat{b})$ *denoted by* $x_{\hat{t}} q_{(\hat{a},\hat{b})} \mu$) *if* $\hat{\mu}(x) \geq rmax(\hat{a}, \hat{t})$ (*respt.* $\hat{\mu}(x) + \hat{t} > \hat{d}$ *i.e.,* $\hat{\mu}(x) + \hat{t} > rmin\{2\hat{b}, \hat{1} + \hat{a}\} = \hat{d}$.) *If* $x_{\hat{t}} \in_{\hat{a}} \hat{\mu}$ *or* $x_{\hat{t}} q_{(\hat{a},\hat{b})} \hat{\mu}$, *then we write* $x_{\hat{t}} \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})} \hat{\mu}$.

Definition 3.2. *An interval-valued fuzzy subset* $\hat{\mu}$ *of a KU-algebra X is said to be an interval valued* $(\hat{a}, \hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$*-fuzzy ideal of X if*

(i) $x_{\hat{t}} \in_{\hat{a}} \hat{\mu} \Rightarrow 0_{\hat{t}} \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})} \hat{\mu}$.

(ii) $(x*(y*z))_{\hat{t}}, y_{\hat{s}} \in_{\hat{a}} \hat{\mu} \Rightarrow (x*z)_{rmin(\hat{t},\hat{s})} \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})} \hat{\mu}$. $\forall x, y \in X, \forall \hat{s}, \hat{t} \in (\hat{a}, \hat{1}]$

Remark 3.3. *When* $\hat{a} = \hat{0}, \hat{b} = \hat{1}$, *then* $\hat{d} = \hat{1}, \hat{k} = \frac{1}{2}$. *then* $(\hat{a}, \hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$*-interval valued fuzzy ideal becomes an* $(\in, \in \vee q)$*-interval valued fuzzy ideal.*

Example 3.4. *Consider KU-algebra* $X = \{0,1,2,3,4\}$ *with the following cayley table.*

Table 1. Example of $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-fuzzy ideal.

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	3
3	0	0	2	0	2
4	0	0	0	0	0

Define a map $\hat{\mu}: X \to D[0,1]$ *by* $\hat{\mu}(0) = [0.6,0.9], \hat{\mu}(1) = [0.5,0.6], \hat{\mu}(2) = [0.4,0.5], \hat{\mu}(3) = [0.2,0.3], \hat{\mu}(3) = [0.2,0.3]$. *Let* $\hat{a} = [0.3,0.4], \hat{b} = [0.6,0.8]$ *then* $\hat{d} = rmin\{[1.2,1.6], [1.3,1.4]\} = [1.2,1.4], \hat{k} = \hat{d}/2 = [0.6,0.7]$ *then by routine calculations it can be verified that* $\hat{\mu}$ *is an* $([0.3,0.4],[0.6,0.8]; \in_{[0.3,0.4]}, \in_{[0.3,0.4]} \vee q_{([0.3,0.4],[0.6,0.8])})$*-interval valued fuzzy ideal X.*

Theorem 3.5. *An interval-valued fuzzy subset* $\hat{\mu}$ *of a KU-algebra X is an* $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$*-interval-valued fuzzy ideal of X if*

(i) $\hat{\mu}(0) \geq rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$,

(ii) $\hat{\mu}(x*z) \geq rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$.

Proof. Suppose $\hat{\mu}$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval-valued fuzzy ideal of X.
(i) Assume that (i) is not valid, then there exists some $x \in X$ such that $\hat{\mu}(0) < min\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$. choose an interval $\hat{t}$ such that

$$\hat{\mu}(0) < rmax(\hat{a},\hat{t}) < rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \quad (3.1)$$

$\Rightarrow \hat{\mu}(x) > rmax(\hat{a},\hat{t})$
$\Rightarrow x_{\hat{t}} \in_{\hat{a}} \hat{\mu}$
$\Rightarrow 0_{\hat{t}} \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow 0_{\hat{t}} \in_{\hat{a}} \hat{\mu}$ or $0_{\hat{t}} q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow \hat{\mu}(0) \geq rmax(\hat{a},\hat{t})$ or $\hat{\mu}(0) + \hat{t} > rmin(2\hat{b}, \hat{1}+\hat{a})$
$\Rightarrow \hat{\mu}(0) \geq rmax(\hat{a},\hat{t})$ or $rmin(2\hat{b}, \hat{1}+\hat{a}) < \hat{\mu}(0) + \hat{t}$
$< rmax(\hat{a},\hat{t}) + \hat{t} = 2rmax(\hat{a},\hat{t})$ by (3.1)
$\Rightarrow \hat{\mu}(0) \geq rmax(\hat{a},\hat{t})$ or $rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2}) < rmax(\hat{a},\hat{t})$ which contradicts (3.1)
Hence (i) is valid.
(ii) Assume that (ii) is not valid then there exists some $x,y,z \in X$ such that $\hat{\mu}(x*z) < min\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$ choose an interval number $\hat{t}$ such that

$$\hat{\mu}(x*z) < rmax(\hat{a},\hat{t}) < rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \quad (3.2)$$

$\Rightarrow \hat{\mu}(y) > rmax(\hat{a},\hat{t})$ and $\hat{\mu}(x*(y*z)) > rmax(\hat{a},\hat{t})$
$\Rightarrow (x*(y*z))_{\hat{t}}, y_{\hat{t}} \in_{\hat{a}} \hat{\mu}$
$\Rightarrow (x*z)_{\hat{t}} \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow (x*z)_{\hat{t}} \in_{\hat{a}} \hat{\mu}$ or $(x*z)_{\hat{t}} q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow \hat{\mu}(x*z) \geq rmax(\hat{a},\hat{t})$ or $\hat{\mu}(x*z) + \hat{t} > rmin(2\hat{b}, \hat{1}+\hat{a})$
$\Rightarrow \hat{\mu}(x*z) \geq rmax(\hat{a},\hat{t})$ or $rmin(2\hat{b}, \hat{1}+\hat{a}) < \hat{\mu}(x*z) + \hat{t}$
$< rmax(\hat{a},\hat{t}) + \hat{t} = 2rmax(\hat{a},\hat{t})$ by (3.2)
$\Rightarrow \hat{\mu}((x*z)) \geq rmax(\hat{a},\hat{t})$ or $rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2}) < rmax(\hat{a},\hat{t})$ which contradicts (3.2)
Hence (ii) is valid.

□

Remark 3.6. *Every interval valued fuzzy KU-ideal is an* $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$*-interval valued fuzzy ideal but the converse is not true as shown in following Example.*

Example 3.7. *Consider KU-algebra* $X = \{0,1,2,3,4\}$ *with the following cayley table.*

Table 2. Illustration of converse of Remark 3.6.

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	3
3	0	0	2	0	2
4	0	0	0	0	0

Define a map $\hat{\mu}: X \to D[0,1]$ *by* $\hat{\mu}(0) = [0.3,0.4], \hat{\mu}(1) = [0.5,0.6], \hat{\mu}(2) = [0.4,0.5], \hat{\mu}(3) = \hat{\mu}(4) = [0.4,0.5]$. *Let* $\hat{a} = [0.1,0.2]\hat{b} = [0.3,0.4]$ *then* $\hat{d} = rmin\{[0.6,0.8],[1.1,1.2]\} = [0.6,0.8], \hat{k} = \hat{d}/2 = [0.3,0.4]$ *then* $\hat{\mu}$ *is an* $([0.1,0.2],[0.3,0.4]; \in_{[0.1,0.2]}, \in_{[0.1,0.2]} \vee q_{([0.3,0.4],[0.3,0.4])})$*-interval valued fuzzy ideal X by Theorem 3.5, however it is not an interval valued fuzzy ideal of X, since* $\hat{\mu}(0) = [0.3,0.4] \not\geq \hat{\mu}(1) = [0.5,0.6]$.

'

Theorem 3.8. *If* $\hat{\lambda}$ *and* $\hat{\mu}$ *be two* $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$*-interval valued fuzzy ideals of X, then* $\hat{\lambda} \cap \mu$ *is also an* $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$*-interval valued fuzzy ideal of X.*

Proof. Here $\hat{\lambda}, \hat{\mu}$ both are $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideals of X.Therefore

$$\hat{\mu}(0) \geq rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \quad (3.3)$$

$$\hat{\mu}(x*z) \geq rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \quad (3.4)$$

$$\hat{\lambda}(0) \geq rmin\{\hat{\lambda}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \quad (3.5)$$

$$\hat{\lambda}((x*z)) \geq rmin\{\hat{\lambda}(x*(y*z)), \hat{\lambda}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \quad (3.6)$$

Now

$$\begin{aligned}(\hat{\lambda}\cap\hat{\mu})(0) &= rmin\{\hat{\lambda}(\hat{0}),\hat{\mu}(\hat{0})\}\\ &\geq rmin\{rmin\{\hat{\lambda}(x),rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\},\\ &\quad rmin\{\hat{\mu}(x),rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\}\\ &= rmin\{rmin(\hat{\lambda}(x),\hat{\mu}(x),rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\\ &= rmin\{(\hat{\lambda}\cap\hat{\mu})(x),rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\end{aligned}$$

$$\begin{aligned}&(\hat{\lambda}\cap\hat{\mu})(x*z)\\ =\ & rmin\{\hat{\lambda}(x*z),\hat{\mu}(x*z)\}\\ \geq\ & rmin\{rmin\{\hat{\lambda}(x*(y*z)),\hat{\lambda}(y),rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\},\\ & rmin\{\hat{\mu}(x*(y*z)),\hat{\mu}(y),\hat{\mu}(rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\}\\ =\ & rmin\{rmin(\hat{\lambda}(x*(y*z)),\hat{\mu}(x*(y*z))),\\ & rmin(\hat{\lambda}(y),\hat{\mu}(y)),rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\\ =\ & rmin\{(\hat{\lambda}\cap\hat{\mu})(x*(y*z),(\hat{\lambda}\cap\hat{\mu})(y),rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\end{aligned}$$

Hence $(\hat{\lambda}\cap\hat{\mu})$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. □

Theorem 3.9. *If $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}})$-interval valued fuzzy ideal of X, then $\hat{\mu}$ is an interval valued fuzzy ideal of X.*

Proof. Let $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}})$-interval valued fuzzy ideal of X, To prove $\hat{\mu}$ is an interval valued fuzzy ideal of X. Let $x\in X$ such that $rmax(\hat{a},\hat{t})=\hat{\mu}(x)$ then $\hat{\mu}(x)\geq rmax(\hat{a},\hat{t})$
i.e $x_{\hat{t}}\in_{\hat{a}}\hat{\mu},\Rightarrow 0_{\hat{t}}\in_a\hat{\mu}$ [Since $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}})$-interval valued fuzzy ideal]
$\Rightarrow\hat{\mu}(0)\geq rmax(\hat{a},\hat{t})=\hat{\mu}(x)$
$\Rightarrow\hat{\mu}(0)\geq\hat{\mu}(x)$
Again let $x,y,z\in X$ such that $rmax(\hat{a},\hat{t})=\hat{\mu}(x*(y*z))$, $rmax(\hat{a},\hat{s})=\hat{\mu}(y)$ then $\hat{\mu}(x*(y*z))\geq rmax(\hat{a},\hat{t}),\hat{\mu}(y)\geq rmax(\hat{a},\hat{s})$ i.e., $(x*(y*z))_{\hat{t}},y_{\hat{s}}\in_a\hat{\mu},\Rightarrow(x*z)_{rmin(\hat{t},\hat{s})}\in_a\hat{\mu}$
[Since $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}})$-interval valued fuzzy ideal]

$$\begin{aligned}\Rightarrow\hat{\mu}(x*z) &\geq rmax\{\hat{a},rmin(\hat{t},\hat{s})\}\\ &= rmin\{rmax(\hat{a},\hat{t}),rmax(\hat{a},\hat{s})\}\\ \Rightarrow\hat{\mu}(x*z) &\geq rmin\{\hat{\mu}(x*(y*z)),\hat{\mu}(y)\}\end{aligned}$$

Hence $\hat{\mu}$ is an interval valued fuzzy ideal of X. □

Theorem 3.10. *If $\hat{\mu}$ is a $(\hat{a},\hat{b};q_{(\hat{a},\hat{b})},q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X, then it is also an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}})$-interval valued fuzzy ideal of X.*

Proof. Let $\hat{\mu}$ be a $(\hat{a},\hat{b};q_{(\hat{a},\hat{b})},q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. Let $x\in X$ such that $x_{\hat{t}}\in_{\hat{a}}\hat{\mu}$ then $\hat{\mu}(x)\geq rmax(\hat{a},\hat{t})$
$\Rightarrow\hat{\mu}(x)+\hat{\delta}>rmax(\hat{a},\hat{t})$
$\Rightarrow\hat{\mu}(x)+\hat{\delta}-rmax(\hat{a},\hat{t})+rmin(2\hat{b},\hat{1}+\hat{a})>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow x_{(\hat{\delta}-rmax(\hat{a},\hat{t})+rmin(2\hat{b},\hat{1}+\hat{a})}q_{(\hat{a},\hat{b}))}\hat{\mu}$
$\Rightarrow(0)_{(\hat{\delta}-rmax(\hat{a},\hat{t})+rmin(2\hat{b},\hat{1}+\hat{a})}q_{(\hat{a},\hat{b})}\hat{\mu}$
[Since $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal.]
$\Rightarrow\hat{\mu}(0)+\hat{\delta}-rmax(\hat{a},\hat{t})+rmin(2\hat{b},\hat{1}+\hat{a})>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow\hat{\mu}(0)+\hat{\delta}>rmax(\hat{a},\hat{t})$
$\Rightarrow\hat{\mu}(0)\geq rmax(\hat{a},\hat{t})$
$\Rightarrow 0_{\hat{t}}\in_a\hat{\mu}$
Therefore $x_{\hat{t}}\in_{\hat{a}}\hat{\mu}\Rightarrow 0_{\hat{t}}\in_{\hat{a}}\hat{\mu}$.
Again let $x,y,z\in X$ such that $(x*(y*z))_{\hat{t}},y_{\hat{s}}\in_{\hat{a}}\hat{\mu}$
$\Rightarrow\hat{\mu}(x*(y*z))\geq rmax(\hat{a},\hat{t}),\hat{\mu}(y)\geq rmax(\hat{a},\hat{s})$
$\Rightarrow\hat{\mu}(x*(y*z))+\hat{\delta}>rmax(\hat{a},\hat{t}),\hat{\mu}(y)+\delta>rmax(\hat{a},\hat{s})$
$\Rightarrow\hat{\mu}(x*(y*z))+\hat{\delta}-rmax(\hat{a},\hat{t})+rmin(2\hat{b},\hat{1}+\hat{a})$
$>rmin(2\hat{b},\hat{1}+\hat{a})$, and
$\hat{\mu}(y)+\hat{\delta}-rmax(\hat{a},\hat{s})+rmin(2\hat{b},\hat{1}+\hat{a})>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow(x*(y*z))_{(\hat{\delta}-rmax(\hat{a},\hat{t})+rmin(2\hat{b},\hat{1}+\hat{a})}q_{(\hat{a},\hat{b})}\hat{\mu}$ and
$(y)_{(\hat{\delta}-rmax(\hat{a},\hat{s})+rmin(2\hat{b},\hat{1}+\hat{a}))}q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow(x*z)_{\{rmin(\hat{\delta}-rmax(\hat{a},\hat{t})+min(2b,1+a),\hat{\delta}-rmax(\hat{a},\hat{s})+rmin(2\hat{b},\hat{1}+\hat{a})\}}q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow\hat{\mu}(x*z)+rmin\{\hat{\delta}-rmax(\hat{a},\hat{t})+rmin(2\hat{b},\hat{1}+\hat{a}),$
$\hat{\delta}-rmax(\hat{a},\hat{s})+rmin(2\hat{b},\hat{1}+\hat{a})\}$
$>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow\hat{\mu}(x*z)+\hat{\delta}-rmax\{rmax(\hat{a},\hat{t}),rmax(\hat{a},\hat{s})\}$
$+rmin(2\hat{b},\hat{1}+\hat{a})>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow\mu(x*z)+\hat{\delta}>rmax\{rmax(\hat{a},\hat{t}),rmax(\hat{a},\hat{s})$
$\geq rmin\{rmax(\hat{a},\hat{t}),rmax(\hat{a},\hat{s})\}\}$
$\Rightarrow\hat{\mu}(x*z)\geq rmin\{rmax(\hat{a},\hat{t}),rmax(\hat{a},\hat{s})\}$
$\Rightarrow\hat{\mu}(x*z)\geq rmin\{rmax(\hat{a},\hat{t},\hat{s})\}=rmax(\hat{a},rmin(\hat{t},\hat{s}))$
$\Rightarrow(x*z)_{rmin(\hat{t},\hat{s})}\in_a\hat{\mu}$
i.e $(x*(y*z))_{\hat{t}},y_{\hat{s}}\in_{\hat{a}}\mu,\Rightarrow(x*z)_{rmin(\hat{t},\hat{s})}\in_a\hat{\mu}$
Hence $\hat{\mu}$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}})$-interval valued fuzzy ideal of X. □

Theorem 3.11. *An interval valued fuzzy subset $\hat{\mu}$ of KU-algebra X is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X.*
(i) If $\hat{\mu}(x)<rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\ \forall x\in X$, then $\hat{\mu}$ is also an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}})$- interval valued fuzzy ideal of X.
(ii) If $\hat{\mu}(x)\geq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$ for some $x\in X$, then $\hat{\mu}(0)\geq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$

Proof. (i) Let $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X and $\hat{\mu}(x)<rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\ \forall x\in X$
Let $x_{\hat{t}}\in_{\hat{a}}\hat{\mu}\Rightarrow\hat{\mu}(x)\geq rmax(\hat{a},\hat{t})$
$\Rightarrow rmax(\hat{a},\hat{t})\leq\hat{\mu}(x)<rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$ and
also $\hat{\mu}(0)<rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$
$\Rightarrow\hat{\mu}(0)+rmax(\hat{a},\hat{t})<rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})+rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$

$= rmin(2\hat{b}, \hat{1}+\hat{a})$
$\Rightarrow \hat{\mu}(0)+\hat{t} < rmin(2\hat{b}, \hat{1}+\hat{a}) \Rightarrow \hat{\mu}(0)+\hat{t} \not> rmin(2\hat{b}, \hat{1}+\hat{a})$
$\Rightarrow 0_{\hat{t}}\overline{q}_{(a,b)}\hat{\mu}$ therefore $x_t \in_a \hat{\mu} \Rightarrow 0_t\overline{q}_{(\hat{a},\hat{b})}\hat{\mu}$
Since $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X, therefore we must have $x_t \in_{\hat{t}} \hat{\mu} \Rightarrow 0_t \in_a \hat{\mu}$
Again let $(x*(y*z))_{\hat{t}}, y_{\hat{t}} \in_{\hat{a}} \hat{\mu}$
$\Rightarrow rmax(\hat{a},\hat{t}) \leq \hat{\mu}(x*(y*z)) < rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$ and $rmax(\hat{a},\hat{s}) \leq \hat{\mu}(y) < rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$
$\Rightarrow rmin\{rmax(\hat{a},\hat{t}), rmax(\hat{a},\hat{s})\} < rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$
$\Rightarrow rmax(\hat{a}, rmin(\hat{t},\hat{s})) < rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$ and also
$\hat{\mu}(x*z)+rmax(\hat{a}, rmin(\hat{t},\hat{s})) < rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})+rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2}) = rmin(2\hat{b}, \hat{1}+\hat{a})$
$\Rightarrow \hat{\mu}(x*z)+rmin(\hat{t},\hat{s}) < rmin(2\hat{b}, \hat{1}+\hat{a})$
Since $\hat{\mu}$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-fuzzy ideal of X
i.e., either $\hat{\mu}(x*z) \geq rmax(\hat{a}, rmin(\hat{t},\hat{s}))$
or $\hat{\mu}(x*z)+rmin(\hat{t},\hat{s}) > rmin(2\hat{b}, \hat{1}+\hat{a})$
So we must have $\hat{\mu}(x*z) \geq rmax(\hat{a}, rmin(\hat{t},\hat{s}))$
i.e., $(x*z)_{rmin(\hat{t},\hat{s})} \in_{\hat{a}} \hat{\mu}$
Hence $\hat{\mu}$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}})$-interval valued fuzzy ideal of X.
(ii) we have $\hat{\mu}(x) \geq rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$ for some $x \in X$, then

$$\begin{aligned}\hat{\mu}(0) &= \hat{\mu}(x*x))\\ &\geq rmin\{\hat{\mu}(x*(x*x)), \hat{\mu}(x), \frac{\hat{1}+\hat{a}}{2})\\ &= rmin\{\hat{\mu}(0), \hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\\ &\geq rmin\{\hat{\mu}(0), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2}), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\\ &= rmin\{\hat{\mu}(0), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\end{aligned}$$

$$\begin{aligned}\Rightarrow \hat{\mu}(0) &\geq rmin\{\hat{\mu}(0), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\\ \Rightarrow \hat{\mu}(0) &\geq rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\end{aligned}$$

□

Theorem 3.12. *An interval valued fuzzy set $\hat{\mu}$ in X is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X if and only if the level set $\hat{\mu}_{\hat{t}} = \{x \in X|\hat{\mu}(x) \geq \hat{t}\}$ is an ideal of X for all $\hat{t} \in D(\hat{0}, rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})]$ and $\hat{\mu}_{\hat{t}} \neq \phi$*

Proof. Assume that $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X and $\hat{t} \in D(\hat{0}, \frac{\hat{1}+\hat{a}}{2})$. Let $x \in X$ such that $x \in \hat{\mu}_{\hat{t}}$, therefore $\hat{\mu}(x) \geq \hat{t}$
Now by the Theorem 3.5
$\hat{\mu}(0) \geq rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \geq rmin\{\hat{t}, rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} = \hat{t}$
$\Rightarrow \hat{\mu}(0) \geq \hat{t} \Rightarrow 0 \in \hat{\mu}_{\hat{t}}$
Again let $x,y,z \in X$ such that $x*(y*z), y \in \hat{\mu}_{\hat{t}}$. Therefore $\hat{\mu}(x*(y*z)) \geq \hat{t}, \hat{\mu}(y) \geq \hat{t}$
Now by the Theorem 3.5
$\hat{\mu}(x*z) \geq rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$
$\geq rmin\{\hat{t}, \hat{t}, rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} = \hat{t} \Rightarrow \hat{\mu}(x*z) \geq \hat{t} \Rightarrow (x*z) \in \hat{\mu}_{\hat{t}}$
Therefore $x*(y*z), y \in \hat{\mu}_{\hat{t}} \Rightarrow (x*z) \in \hat{\mu}_{\hat{t}}$. Therefore $\hat{\mu}_{\hat{t}}$ is a ideal of X.
Conversely,
Suppose that $\hat{\mu}$ be an interval valued fuzzy set in X and $\hat{\mu}_{\hat{t}} = \{x \in X|\hat{\mu}(x) \geq \hat{t}\}$ is an ideal of X for all $\hat{t} \in D(\hat{0}, rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})]$.
To prove $\hat{\mu}$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. Suppose $\hat{\mu}$ is not an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. Then there exists some $x,y,z \in X$ such that at least one of $\hat{\mu}(0) < rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$ and $\hat{\mu}(x*z) < rmin\{\hat{\mu}(x*(y*z), \hat{\mu}(y), min(b, \frac{1+a}{2})\}$ hold. Suppose $\hat{\mu}(0) < rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$ holds. Choose an interval number $\hat{t} \in D(\hat{0}, rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})]$. such that

$$\hat{\mu}(0) < \hat{t} < rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \quad (3.7)$$

$\Rightarrow \hat{\mu}(x) > \hat{t}$
$\Rightarrow x \in \hat{\mu}_{\hat{t}} \Rightarrow 0 \in \hat{\mu}_{\hat{t}}$ [Since $\hat{\mu}_{\hat{t}}$ is an ideal
$\Rightarrow \hat{\mu}(0) \geq \hat{t}$
which contradicts (3.7).
Therefore we must have $\hat{\mu}(0) \geq rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$
Again if $\hat{\mu}(x*z) \leq rmin\{\hat{\mu}(x*(y*z), \mu(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$ holds. Again choose an interval number
$\hat{t} \in D(\hat{0}, rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})]$, such that

$$\hat{\mu}(x*z) < \hat{t} < rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \quad (3.8)$$

$\Rightarrow (x*(y*z)), y \in \hat{\mu}_{\hat{t}}$. Since $\hat{\mu}_{\hat{t}}$ is an ideal of X, it follows that $x*z \in \hat{\mu}_{\hat{t}}$ so that $\hat{\mu}(x*z) \geq \hat{t}$ which contradicts (3.8).
Hence we must have
$\hat{\mu}(x*z) \geq rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$.
Consequently $\hat{\mu}$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. □

Theorem 3.13. *If $\hat{\mu}$ is a non zero $(\hat{a},\hat{b};q_{(\hat{a},\hat{b})},\in_{\hat{a}})$ (or $(\hat{a},\hat{b};q_{(\hat{a},\hat{b})},q_{(\hat{a},\hat{b})})$)-interval valued fuzzy ideal of X. Then the set*

$$X_0 = \{x \in X|\hat{\mu}(x) > \hat{0}\}$$

is an ideal of X.

Proof. First part for $(\hat{a},\hat{b};q_{(\hat{a},\hat{b})},\in_{\hat{a}})$-interval valued fuzzy ideal.
Let $x \in X_0$. Then $\hat{\mu}(x) > \hat{0}$ Therefore $\hat{\mu}(x)+rmin(2\hat{b}, \hat{1}+\hat{a}) > rmin(2\hat{b}, \hat{1}+\hat{a})$ that is $x_{rmin(2\hat{b},\hat{1}+\hat{a})}q_{(\hat{a},\hat{b})}\hat{\mu}$ Since $\hat{\mu}$ is a non zero $(\hat{a},\hat{b};q_{(\hat{a},\hat{b})},\in_{\hat{a}})$-interval valued fuzzy ideal of X. Therefore $0_{rmin(2\hat{b},\hat{1}+\hat{a})}q_{(\hat{a},\hat{b})}\hat{\mu}$ which implies $\hat{\mu}(0)+rmin(2\hat{b}, \hat{1}+\hat{a}) > rmin(2\hat{b}, \hat{1}+\hat{a})$ i.e., $\hat{\mu}(0) > \hat{0}$ i.e.,$0 \in X_0$.

Again let $(x*(y*z)), y \in X_0$. Then $\hat{\mu}(x*(y*z)) > \hat{0}, \hat{\mu}(y) > \hat{0}$ Therefore $\hat{\mu}(x*(y*z))+rmin(2\hat{b}, \hat{1}+\hat{a}) > rmin(2\hat{b}, \hat{1}+$

$\hat{a})$ and $\hat{\mu}(y)+rmin(2\hat{b},\hat{1}+\hat{a}) > rmin(2\hat{b},\hat{1}+\hat{a})$that is $(x*(y*z))_{rmin(2\hat{b},\hat{1}+\hat{a})}q_{(\hat{a},\hat{b})}\hat{\mu}, y_{rmin(2\hat{b},\hat{1}+\hat{a})}q_{(\hat{a},\hat{b})}\hat{\mu}$. Since $\hat{\mu}$ is a non zero $(\hat{a},\hat{b};q_{(\hat{a},\hat{b})},\in_{\hat{a}})$-interval valued fuzzy ideal of X. Therefore $(x*z)_{rmin(2\hat{b},\hat{1}+\hat{a})}q_{(\hat{a},\hat{b})}\hat{\mu}$ implies $\hat{\mu}(x*z)+rmin(2\hat{b},\hat{1}+\hat{a}) > rmin(2\hat{b},\hat{1}+\hat{a})$ i.e., $\hat{\mu}(x*z) > \hat{0}$ i.e.,$(x*z)\in X_0$. Hence X_0 is an ideal of X.

Similarly second part can be prove for $(\hat{a},\hat{b};q_{(\hat{a},\hat{b})},q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal.

□

Definition 3.14. *Let $\hat{\mu}$ be a fuzzy set in KU-algebra X and $\hat{t}\in D(0\ 1]$,let*

$$\begin{aligned}\hat{\mu}_{\hat{t}} &= \{x\in X|x_{\hat{t}}\in_{\hat{a}}\hat{\mu}\} = \{x\in X|\hat{\mu}(x)\geq rmax(\hat{a},\hat{t})\}\\ <\hat{\mu}>_{\hat{t}} &= \{x\in X|x_{\hat{t}}q_{(\hat{a},\hat{b})}\hat{\mu}\}\\ &= \{x\in X|\hat{\mu}(x)+\hat{t} > rmin(2\hat{b},\hat{1}+\hat{a})\}\end{aligned}$$

$[\hat{\mu}]_{\hat{t}} = \{x\in X|x_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}\hat{\mu}\}$
$= \{x\in X|\hat{\mu}(x)\geq rmax(\hat{a},\hat{t})\, or\, \mu(x)+\hat{t} > rmin(2\hat{b},\hat{1}+\hat{a})\}$

Here $\hat{\mu}_{\hat{t}}$ is called $\in_a$ level set of $\hat{\mu}$, $<\hat{\mu}>_{\hat{t}}$ is called $q_{(\hat{a},\hat{b})}$ level set of $\hat{\mu}$ and $\overline{[\hat{\mu}]}_v$ is called $(\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-level set of $\hat{\mu}$ Clearly $[\hat{\mu}]_{\hat{t}} = <\hat{\mu}>_{\hat{t}}\cup\hat{\mu}_{\hat{t}}$

Theorem 3.15. *An interval valued fuzzy set $\hat{\mu}$ in X is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X if and only if the level set $[\hat{\mu}]_{\hat{t}}$ is an ideal of X for all $\hat{t}\in D(\hat{0},\hat{1}]$. We called $[\hat{\mu}]_{\hat{t}}$ as $\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}$ levels ideals of $\hat{\mu}$.*

Proof. Assume that $\hat{\mu}$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. To prove $[\hat{\mu}]_{\hat{t}}$ is an ideal of X. Let $x\in[\hat{\mu}]_{\hat{t}}$ for $\hat{t}\in D(0\ 1]$ then $x_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}\hat{\mu}$ Therefore we have $\hat{\mu}(x)\geq rmax(\hat{a},\hat{t})$ or $\hat{\mu}(x)+\hat{t} > rmin(2\hat{b},\hat{1}+\hat{a})$. Since $\hat{\mu}$ in X is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. Therefore

$$\hat{\mu}(0)\geq rmin\{\hat{\mu}(x), rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\},\forall \quad x,y\in X \quad (3.9)$$

Now we have the following cases:
CaseI. Let $\hat{\mu}(x)\geq rmax(\hat{a},\hat{t})$ now if $\hat{t}\leq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$ also since $\hat{a}<\hat{b}$ therefore $rmax(\hat{a},\hat{t}) < rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$. Hence by eqn 3.9

$$\begin{aligned}\hat{\mu}(0) &\geq rmin\{\hat{\mu}(x), rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmin\{rmax(\hat{a},\hat{t}), rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmax(\hat{a},\hat{t})\end{aligned}$$

Which implies $0_{\hat{t}}\in_{\hat{a}}\hat{\mu}$.
Again if $\hat{t} > rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$ also since $\hat{a}<\hat{b}$ therefore $rmax(\hat{a},\hat{t})\geq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$. Hence by eqn 3.9

$$\begin{aligned}\hat{\mu}(0) &\geq rmin\{\hat{\mu}(x), rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmin\{rmax(\hat{a},\hat{t}), rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\end{aligned}$$

Which implies $\hat{\mu}(0)\geq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$. Therefore
$\hat{\mu}(0)+rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\geq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})+rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$
$= rmin(2\hat{b},\hat{1}+\hat{a})$.
Therefore $0_{\hat{t}}q_{(\hat{a},\hat{b})}\hat{\mu}$.
Hence from above $0_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}\hat{\mu}$. i.e., $0\in[\hat{\mu}]_{\hat{t}}$.
CaseII. Let $\hat{\mu}(x)+\hat{t} > rmin(2\hat{b},\hat{1}+\hat{a})$ and if $\hat{t}\leq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$ also since $\hat{a}<\hat{b}$ therefore $rmax(\hat{a},\hat{t}) < rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$. Hence by eqn 3.9

$$\begin{aligned}\hat{\mu}(0) &\geq rmin\{\hat{\mu}(x), rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmin\{rmin(2\hat{b},\hat{1}+\hat{a})-\hat{t}, rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\\ &\geq rmax(\hat{a},\hat{t})\end{aligned}$$

Therefore $0_{\hat{t}}\in_{\hat{a}}\hat{\mu}$.
Again if $\hat{t} > rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$ also since $\hat{a}<\hat{b}$ therefore $rmax(\hat{a},\hat{t})\geq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$. Hence by eqn 3.9

$$\begin{aligned}\hat{\mu}(0) &\geq rmin\{\hat{\mu}(x), rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmin\{rmin(2\hat{b},\hat{1}+\hat{a})-\hat{t}, rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmin(2\hat{b},\hat{1}+\hat{a})-\hat{t}\\ \hat{\mu}(0)+\hat{t} &\geq rmin(2\hat{b},\hat{1}+\hat{a})\end{aligned}$$

Therefore $0_{\hat{t}}q_{(\hat{a},\hat{b})}\hat{\mu}$.
Hence from above $0_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}\hat{\mu}$. i.e., $0\in[\hat{\mu}]_{\hat{t}}$.
Again let $x*(y*z), y\in[\hat{\mu}]_{\hat{t}}$ for $\hat{t}\in D(\hat{0},\hat{1}]$ then $(x*(y*z))_{\hat{t}},(y)_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}\hat{\mu}$ then $\hat{\mu}(x*(y*z))\geq rmax(\hat{a},\hat{t})$ or $\hat{\mu}(x*(y*z))+\hat{t} > rmin(2\hat{b},\hat{1}+\hat{a})$ and $\hat{\mu}(y)\geq rmax(\hat{a},\hat{t})$ or $\hat{\mu}(y)+\hat{t} > rmin(2\hat{b},\hat{1}+\hat{a})$. Since $\hat{\mu}$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. Therefore

$$\hat{\mu}(x*z)\geq rmin\{\hat{\mu}(x*(y*z)),\quad \hat{\mu}(y),\quad rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})\},\quad \forall \quad x,y\in X \quad (3.10)$$

CaseI. Let $\hat{\mu}(x*(y*z))\geq rmax(\hat{a},\hat{t})$ and $\hat{\mu}(y)\geq rmax(\hat{a},\hat{t})$ if $\hat{t}\leq rmin(\hat{b},\frac{\hat{1}+\hat{a}}{2})$ also since $\hat{a}<\hat{b}$ therefore $rmax(\hat{a},\hat{t}) <$

$rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$. Hence by eqn 3.10

$$\begin{aligned}\hat{\mu}(x*z) &\geq rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmin\{rmax(\hat{a},\hat{t}), rmax(\hat{a},\hat{t}), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmax(\hat{a},\hat{t})\end{aligned}$$

Therefore $(x*z)_{\hat{t}} \in_{\hat{a}} \hat{\mu}$

Again if $\hat{t} > rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$ also since $\hat{a} < \hat{b}$ therefore $rmax(\hat{a},\hat{t}) \geq rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$. Hence by 3.10

$$\begin{aligned}\hat{\mu}(x*z) &\geq rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmin\{rmax(\hat{a},\hat{t}), rmax(\hat{a},\hat{t}), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\\ &\geq rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\end{aligned}$$

Therefore $\hat{\mu}(x*z) \geq rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$

$\hat{\mu}(x*z) + rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2}) \geq rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2}) + rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$

$= rmin(2\hat{b}, \hat{1}+\hat{a})$.

Therefore $(x*z)_{\hat{t}} q_{(\hat{a},\hat{b})} \hat{\mu}$.

Hence from above $(x*z)_{\hat{t}} \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})} \hat{\mu}$ i.e., $x*z \in [\hat{\mu}]_{\hat{t}}$.

CaseII. Let $\hat{\mu}(x*(y*z)) \geq rmax(\hat{a},\hat{t})$ and $\hat{\mu}(y) + \hat{t} > rmin(2\hat{b}, \hat{1}+\hat{a})$ Assume $\hat{t} \leq rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$ also since $\hat{a} < \hat{b}$ therefore $rmax(\hat{a},\hat{t}) < rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$. Hence by eq 3.10

$$\begin{aligned}&\hat{\mu}(x*z)\\ \geq\ & rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\\ \geq\ & rmin\{rmax(\hat{a},\hat{t}), rmin(2\hat{b}, \hat{1}+\hat{a}) - \hat{t}, rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\\ \geq\ & rmax(\hat{a},\hat{t})\end{aligned}$$

Therefore $(x*z)_{\hat{t}} \in_{\hat{a}} \hat{\mu}$

Again if $\hat{t} > rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$ also since $\hat{a} < \hat{b}$ therefore $rmax(\hat{a},\hat{t}) \geq rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$. Hence by 3.10

$$\begin{aligned}&\hat{\mu}(x*z)\\ \geq\ & rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\\ \geq\ & rmin\{rmax(\hat{a},\hat{t}), rmin(2\hat{b}, \hat{1}+\hat{a}) - \hat{t}, rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}\\ \geq\ & rmin(2\hat{b}, \hat{1}+\hat{a}) - \hat{t}\end{aligned}$$

$$\therefore \hat{\mu}(x*z) + \hat{t} \geq rmin(2\hat{b}, \hat{1}+\hat{a})$$

Therefore $(x*z)_{\hat{t}} q_{(\hat{a},\hat{b})} \hat{\mu}$

Hence from above $(x*z)_{\hat{t}} \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})} \hat{\mu}$. i.e., $x*z \in [\hat{\mu}]_{\hat{t}}$

CaseIII. Let $\hat{\mu}(x*(y*z)) + \hat{t} > rmin(2\hat{b}, \hat{1}+\hat{a})$ and $\hat{\mu}(y) \geq rmax(\hat{a},\hat{t})$

Similar to Case II.

CaseIV. Let $\hat{\mu}(x*(y*z)) + \hat{t} > rmin(2\hat{b}, \hat{1}+\hat{a})$ and $\hat{\mu}(y) + \hat{t} > rmin(2\hat{b}, \hat{1}+\hat{a})$

Similar to Case II.

Conversely, let $\hat{\mu}$ be an interval valued fuzzy set in X and $\hat{t} \in D(0\ 1]$ such that $[\hat{\mu}]_{\hat{t}}$ is an ideal of X. To prove $\hat{\mu}$ is an$(\hat{a},\hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X.If $\hat{\mu}$ is not an$(\hat{a},\hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X, then there exists $x,y,z \in X$ such that at least one of $\hat{\mu}(0) < rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$ and $\hat{\mu}(x*z) < rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$ hold.

Suppose $\hat{\mu}(0) < rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$ hold. Then choose an interval $\hat{t}$ such that

$$\hat{\mu}(0) < rmax(\hat{a},\hat{t}) < rmin\{\hat{\mu}(x), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \quad (3.11)$$

Which implies $\hat{\mu}(x) > rmax(\hat{a},\hat{t})$ i.e., $x_{\hat{t}} \in_{\hat{a}} \hat{\mu}$. Since $\hat{\mu}$ is an$(\hat{a},\hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. Therefore we must have $0_{\hat{t}} \in_{\hat{a}} \hat{\mu}$ i.e., $\hat{\mu}(0) > rmax(\hat{a},\hat{t})$ which contradicts eqn 3.11. Again if $\hat{\mu}(x*z) < rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\}$ hold. Then choose an interval $\hat{t}$ such that

$$\begin{aligned}&\hat{\mu}(x*z) < rmax(\hat{a},\hat{t})\\ &< rmin\{\hat{\mu}(x*(y*z)), \hat{\mu}(y), rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})\} \quad (3.12)\end{aligned}$$

Which implies $\hat{\mu}(x*(y*z)) > rmax(\hat{a},\hat{t}), \hat{\mu}(y) > rmax(\hat{a},\hat{t})$ i.e., $(x*(y*z))_{\hat{t}} \in_{\hat{a}} \hat{\mu}$, $x_{\hat{t}} \in_{\hat{a}} \hat{\mu}$ since $\hat{\mu}$ is an$(\hat{a},\hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. Therefore we must have $(x*z)_{\hat{t}} \in_{\hat{a}} \hat{\mu}$ i.e., $\hat{\mu}(x*z) > rmax(\hat{a},\hat{t})$ which contradicts 3.12. Hence $\hat{\mu}$ is an$(\hat{a},\hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X. □

Theorem 3.16. *Let X ,Y be two KU-algebras. Then their cartesian product $X \times Y = \{(x,y) \mid x \in X, y \in Y\}$ is also a KU-algebra under the binary operation $*$ defined in $X \times Y$ by $(x,y)*(p,q) = (x*p, y*q)$ for all $(x,y),(p,q) \in X \times Y$.*

Proof. Straightforward.

□

Definition 3.17. *Let $\hat{\lambda}$ and $\hat{\mu}$ be two $(\hat{a},\hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideals of KU-algebra X. Then their cartesian product $\hat{\lambda} \times \hat{\mu}$ is defined by*

$$(\hat{\lambda} \times \hat{\mu})(x,y) = rmin\{\hat{\lambda}(x), \hat{\mu}(x)\}$$

where $(\hat{\lambda} \times \hat{\mu}) : X \times X \to D[0,1] \quad \forall x,y \in X$.

Theorem 3.18. *Let $\hat{\lambda}$ and $\hat{\mu}$ be two $(\hat{a},\hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideals of a KU-algebra X. Then $\hat{\lambda} \times \hat{\mu}$*

is also an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of $X\times X$.

Proof. Similar to Theorem 3.8 □

Definition 3.19. *Let X and X' be two KU-algebras. Then a mapping $f:X\to X'$ is said to be homomorphism if $f(x*y)=f(x)*f(y)\ \forall x,y\in X$.*

Theorem 3.20. *Let X and X' be two KU-algebras and $f:X\to X'$ be a homomorphism. Then $f(0)=0'$, where $0\in X$ and $0'\in X'$.*

Proof. We have $f(0)=f(x*x)=f(x)*f(x)=0'$ □

Theorem 3.21. *Let X and X' be two KU-algebras and $f:X\to X'$ be a homomorphism. If $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X', then $f^{-1}(\hat{\mu})$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X.*

Proof. $f^{-1}(\hat{\mu})$ is defined as $f^{-1}(\hat{\mu})(x)=\hat{\mu}(f(x))\forall x\in X$.
Let $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X' and $x\in X$ such that $x_{\hat{t}}\in_{\hat{a}} f^{-1}(\hat{\mu})$ then $f^{-1}(\hat{\mu})(x)\geq rmax(\hat{a},\hat{t}),\hat{\mu}f(x)\geq rmax(\hat{a},\hat{t})$
$\Rightarrow (f(x)_{\hat{t}}\in_{\hat{a}}\hat{\mu}$
$\Rightarrow 0'_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}\hat{\mu}$ [Since $\hat{\mu}$ be an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X']
$\Rightarrow 0'_{\hat{t}}\in_{\hat{a}}\hat{\mu}$ or $0'_{\hat{t}}q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow \hat{\mu}(0')\geq rmax(\hat{a},\hat{t})$ or $\hat{\mu}(0')+\hat{t}\geq rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow \hat{\mu}(f(0))\geq rmax(\hat{a},\hat{t})$ or $\hat{\mu}(f(0))+\hat{t}\geq rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow f^{-1}(\hat{\mu})(0)\geq rmax(\hat{a},\hat{t})$
or $\quad f^{-1}(\hat{\mu})(0)+\hat{t}\geq rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow 0_{\hat{t}}\in_a f^{-1}(\hat{\mu})$ or $0_{\hat{t}}q_{(\hat{a},\hat{b})}f^{-1}(\hat{\mu})$
$\Rightarrow 0_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}f^{-1}(\hat{\mu})$
Therefore $x_{\hat{t}}\in_{\hat{a}} f^{-1}(\hat{\mu})\Rightarrow 0_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}f^{-1}(\hat{\mu})$
Again let $x,y,z\in X$ such that $(x*y(*z))_{\hat{t}},y_{\hat{t}}\in_{\hat{a}} f^{-1}(\hat{\mu})$
then $f^{-1}(\hat{\mu})(x*y(*z))\geq rmax(\hat{a},\hat{t})$ and $f^{-1}(\hat{\mu})(y)$
$\geq rmax(\hat{a},\hat{s})$ $\hat{\mu}f(x*y(*z))\geq rmax(\hat{a},\hat{t})$ and
$\hat{\mu}f(y)\geq rmax(\hat{a},\hat{s})$
$\Rightarrow [f(x*(y*z))]_{\hat{t}}\in_{\hat{a}}\hat{\mu}$ and $[f(y]_{\hat{s}}\in_{\hat{a}}\hat{\mu}$
$\Rightarrow [f(x)*(f(y)*f(z))]_{\hat{t}}\in_{\hat{a}}\hat{\mu}$ and $[f(y)]_{\hat{t}}\in_a\hat{\mu}$
$\Rightarrow [f(x)*f(z)]_{rmin(\hat{t},\hat{s})}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow [f(x)*f(z)]_{rmin(\hat{t},\hat{s})}\in_{\hat{a}}\hat{\mu}$ or $[f(x)*f(z)]_{rmin(\hat{t},\hat{s})}q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow \hat{\mu}(f(x*z))\geq rmax(\hat{a},rmin(\hat{t},\hat{s}))$
or $\qquad \hat{\mu}f(x*z)+rmin(\hat{t},\hat{s})\geq rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow f^{-1}(\hat{\mu})(x*z)\geq rmax(\hat{a},rmin(\hat{t},\hat{s}))$ or
$f^{-1}(\hat{\mu})(x*z)+rmin(\hat{t},\hat{s})\geq rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow (x*z)_{rmin(\hat{t},\hat{s})}\in_{\hat{a}} f^{-1}(\hat{\mu})$ or $(x*z)_{rmin(\hat{t},\hat{s})}q_{(\hat{a},\hat{b})}f^{-1}(\hat{\mu})$
$\Rightarrow (x*z)_{rmin(\hat{t},\hat{s})}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}f^{-1}(\hat{\mu})$
Therefore $(x*(y*z))_{\hat{t}},y_{\hat{s}}\in_{\hat{a}} f^{-1}(\hat{\mu})$
$\Rightarrow (x*z)_{rmin(\hat{t},\hat{s})}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}f^{-1}(\hat{\mu})$
Hence the proof □

Theorem 3.22. *Let X and X' be two KU-algebras and $f:X\to X'$ be an onto homomorphism. If $\hat{\mu}$ be an interval valued fuzzy subset of X' such that $f^{-1}(\hat{\mu})$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X, then $\hat{\mu}$ is also an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X'.*

Proof. Let $x'\in X'$ such that $x'_t\in_{\hat{a}}\hat{\mu}$ where $\hat{t}\in D[0\,1]$
then $\hat{\mu}(x')\geq rmax(\hat{a},\hat{t})$ since f is onto so there exists $x\in X$ such that $f(x)=x'$. Now $\hat{\mu}(x')\geq rmax(\hat{a},\hat{t})$
$\Rightarrow\ \hat{\mu}(f(x))\geq rmax(\hat{a},\hat{t})$
$\Rightarrow f^{-1}(\hat{\mu})(x)\geq rmax(\hat{a},\hat{t})$
$\Rightarrow x_{\hat{t}}\in_{\hat{a}} f^{-1}(\hat{\mu})$
$\Rightarrow 0_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}f^{-1}(\hat{\mu})$
[since $f^{-1}(\hat{\mu})$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X]
$\Rightarrow 0_{\hat{t}}\in_{\hat{a}} f^{-1}(\hat{\mu})$ or $0_t q_{(\hat{a},\hat{b})}f^{-1}(\hat{\mu})$
$\Rightarrow f^{-1}(\hat{\mu})(0)\geq rmax(\hat{a},\hat{t})$
or $f^{-1}(\hat{\mu})(0)+\hat{t}>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow \hat{\mu}(f(0))\geq rmax(\hat{a},\hat{t})$ or $\hat{\mu}(f(0))+\hat{t}>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow \hat{\mu}(0')\geq rmax(\hat{a},\hat{t})$ or $\hat{\mu}(0')+\hat{t}>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow 0'_{\hat{t}}\in_{\hat{a}}\hat{\mu}$ or $0'_{\hat{t}}q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow 0'_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}\hat{\mu}$
Therefore $x'_{\hat{t}}\in_{\hat{a}}\hat{\mu}\Rightarrow 0'_{\hat{t}}\in_a\vee q_{(\hat{a},\hat{b})}\hat{\mu}$
Again let $x',y',z'\in X'$ such that $(x'*y'(*z'))_{\hat{t}},y'_{\hat{s}}\in_{\hat{a}}\hat{\mu}$ where $\hat{t},\hat{s}\in D[0\,1]$
then $\mu(x'*(y'*z'))\geq rmax(\hat{a},\hat{t}),\hat{\mu}(y')\geq rmax(\hat{a},\hat{t})$ since f is onto so there exists $x,y\in X$
such that $f(x)=x',f(y)=y',f(z)=z'$ also f is homomorphism so
$f(x*(y*z))=f(x)*(f(y)*f(z)=x'*(y'*z')$
Now $\hat{\mu}(x'*(y'*z'))\geq rmax(\hat{a},\hat{t})$ and $\hat{\mu}(y')\geq rmax(\hat{a},\hat{t})$
$\Rightarrow \hat{\mu}(f(x)*(f(y)*f(z))\geq rmax(\hat{a},\hat{t})$ and $\mu(f(y))\geq rmax(\hat{a},\hat{t})$
$\Rightarrow \hat{\mu}(f(x*(y*z)))\geq rmax(\hat{a},\hat{t})$ and $\mu(f(y))\geq rmax(\hat{a},\hat{s})$
[Since f is homomorphism.]
$\Rightarrow f^{-1}(\hat{\mu})(x*(y*z))\geq rmax(\hat{a},\hat{t})$ and $f^{-1}(\hat{\mu})(y)\geq rmax(\hat{a},\hat{t})$
$\Rightarrow (x*(y*z))_{\hat{t}}\in_{\hat{a}} f^{-1}(\hat{\mu})$ and $y_{\hat{s}}\in_{\hat{a}} f^{-1}(\hat{\mu})$
$\Rightarrow x_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}f^{-1}(\hat{\mu})$
[since $f^{-1}(\hat{\mu})$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X]
$\Rightarrow (x*z)_{\hat{t}}\in_{\hat{a}} f^{-1}(\hat{\mu})$ or $(x*z)_{\hat{t}}q_{(\hat{a},\hat{b})}f^{-1}(\mu)$
$\Rightarrow f^{-1}(\mu)(x*z)\geq rmax(\hat{a},\hat{t})$
or $\qquad f^{-1}(\mu)(x*z)+\hat{t}>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow \hat{\mu}(f(x*z))\geq rmax(\hat{a},\hat{t})$
or $\qquad \hat{\mu}(f(x*z))+\hat{t}>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow \hat{\mu}(f(x)*f(z)))\geq rmax(\hat{a},\hat{t})$
or $\qquad \hat{\mu}(f(x)*f(z)))+\hat{t}>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow \hat{\mu}(x'*z')\geq rmax(\hat{a},\hat{t})$ or $\hat{\mu}(x'*z')+\hat{t}>rmin(2\hat{b},\hat{1}+\hat{a})$
$\Rightarrow (x'*z')_{\hat{a},\hat{b}}\in_{\hat{a}}\hat{\mu}$ or $(x'*z')_{\hat{t}}q_{(\hat{a},\hat{b})}\hat{\mu}$
$\Rightarrow (x'*z')_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}\mu$
Therefore $(x'*(y'*z'))_{\hat{t}},y'_{\hat{s}}\in_{\hat{a}}\hat{\mu}\Rightarrow (x'*z')_{\hat{t}}\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})}\hat{\mu}$.
Hence $\hat{\mu}$ is an $(\hat{a},\hat{b};\in_{\hat{a}},\in_{\hat{a}}\vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X'. □

Theorem 3.23. *Let I be an ideal of X and let $\hat{\mu}$ be an interval valued fuzzy set of X such that*
(i) $\hat{\mu}(x) = \hat{0}$, for all $x \in X \setminus I$,
(ii) $\hat{\mu}(x) \geq rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$, for all $x \in I$.
Then $\hat{\mu}$ is an $(q_{(\hat{a},\hat{b})}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of X.

Proof. Let $x \in X$ and $\hat{t} \in D(0, rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})]$ be such that $x_{\hat{t}} q_{(\hat{a},\hat{b})} \hat{\mu}$. Then we get $\hat{\mu}(x) + \hat{t} > rmin\{2\hat{b}, \hat{1}+\hat{a}\} = \hat{d}$. Since I is an ideal therefore $0 \in I$, i. e., $\hat{\mu}(0) \geq rmin\{2\hat{b}, \hat{1}+\hat{a}\}$. Now if $rmin\{2\hat{b}, \hat{1}+\hat{a}\} \geq \hat{t}$ then $\hat{\mu}(0) \geq rmin\{2\hat{b}, \hat{1}+\hat{a}\} \geq \hat{t}$ which implies $\hat{\mu}(0) \geq rmax\{\hat{a}, \hat{t}\}$ i.e., $0_{\hat{t}} \in \hat{\mu}$. If $\hat{t} > rmin\{2\hat{b}, \hat{1}+\hat{a}\}$ then $\mu(0) + \hat{t} > 2rmin\{2\hat{b}, \hat{1}+\hat{a}\}$ and so $0_{\hat{t}} q_{(\hat{a},\hat{b})} \hat{\mu}$. Hence $0_{\hat{t}} \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})}$.
Again let $x, y, z \in X$ and $\hat{t}, \hat{s} \in D(0\, rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})]$ be such that $(x*(y*z))_{\hat{t}} q_{(\hat{a},\hat{b})} \hat{\mu}$ and$x_{\hat{s}} q_{(\hat{a},\hat{b})} \hat{\mu}$. Then we get that $\hat{\mu}(x*(y*z)) + \hat{t} > rmin\{2\hat{b}, \hat{1}+\hat{a}\}$ and $\hat{\mu}(y) + \hat{s} > rmin\{2\hat{b}, \hat{1}+\hat{a}\}$. We can conclude that $x*z \in X$, since in otherwise $x*z \in X \setminus I$, and therefore $\hat{t} > rmin\{2\hat{b}, \hat{1}+\hat{a}\}$ *or* $\hat{s} > rmin\{2\hat{b}, \hat{1}+\hat{a}\}$ which is a contradiction. If $rmin(\hat{t}, \hat{s}) > rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$, then $\hat{\mu}(x*z) + rmin(\hat{t}, \hat{s}) > rmin\{2\hat{b}, \hat{1}+\hat{a}\}$ and so $(x*z)_{rmin(\hat{t},\hat{s})} q_{(\hat{a},\hat{b})} \hat{\mu}$. If $rmin(\hat{t}, \hat{s}) \leq rmin(\hat{b}, \frac{\hat{1}+\hat{a}}{2})$, then $\hat{\mu}(x*z) \geq rmin(\hat{t}, \hat{s})$ i.e., $\hat{\mu}(x*z) \geq rmax\{\hat{a}, rmin(\hat{t}, \hat{s})\}$ and thus $(x*z)_{rmin(\hat{t},\hat{s})} \in_{\hat{a}} \hat{\mu}$.. Hence $(x*z)_{rmin(\hat{t},\hat{s})} \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})}$. □

4. Conclusion

In this paper, we have introduced $(\hat{a}, \hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal of KU-algebra and discussed some related properties. $(\hat{a}, \hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal is the Generalised form ideals in KU-algebra, since but putting $\hat{a} = \hat{0}, \hat{b} = \hat{1}$, we get $\hat{d} = \hat{1}, \hat{k} = \frac{1}{2}$. then $(\hat{a}, \hat{b}; \in_{\hat{a}}, \in_{\hat{a}} \vee q_{(\hat{a},\hat{b})})$-interval valued fuzzy ideal becomes an $(\in, \in \vee q)$-interval valued fuzzy ideal. Further interval valued fuzzy ideal i.e., $(\in, \in)$-interval valued fuzzy ideal is a particular case of $(\in, \in \vee q)$-interval valued fuzzy ideal and also fuzzy ideal is a particular case of interval valued fuzzy ideal. It is our hope that this work would other foundations for further study of the theory of BCK/BCI-algebras.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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