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Integral transforms concerning generalized multiindex Bessel Maitland function

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Abstract
Recently several authors have done work on Bessel-Maitland function and generalized Bessel-Maitland function. Many integral transforms and fractional calculus results involving these functions have been established [5],[6],[9]-[11]. In this paper we have established the multiple integral transforms of the generalized multiindex Bessel-Maitland function, which we have defined in this article. The main results are established in terms of Fox-Wright function, then Fox-Wright function is transformed in terms of Fox-H function.

Keywords
Generalized multiindex Bessel-Maitland function, Fox-Wright function, Fox-H function, integral formulas.

AMS Subject Classification
33C10, 33D15, 33C60, 65R10.

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1. Introduction

Consider the Bessel's differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-\beta^2)y=0 \tag{1.1}$$

of order β, where β is arbitrary complex number. There are two solutions to this differential equation, which were first defined by the mathematician Daniel Bernoulli and then generalized by Friedrich Bessel.

$$J_\beta(x)=\sum_{m=0}^{\infty}\frac{(-1)^m(x/2)^{\beta+2m}}{m!\Gamma(m+\beta+1)} \tag{1.2}$$

$J_\beta(x)$ is one of the two solutions of (1.1), it is known as Bessel function of first kind [2]. Further, Edward Maitland Wright [12], introduced generalization of Bessel function known as Bessel-Maitland function as

$$J_\beta^\alpha(x)=\sum_{m=0}^{\infty}\frac{(-x)^m}{\Gamma(\alpha m+\beta+1)m!};\ \Re(\alpha)>0,\Re(\beta)>-1,x\in\mathbb{C}. \tag{1.3}$$

After that Singh et.al [8] defined generalized Bessel Maitland function as

$$J_{\beta,q}^{\alpha,\gamma}(x)=\sum_{m=0}^{\infty}\frac{(\gamma)_{qm}(-x)^m}{\Gamma(\alpha m+\beta+1)m!} \tag{1.4}$$

where $\Re(\alpha)>max\{0,q-1\}$, $\Re(\gamma)>0$, $\Re(\beta)>-1$, $x\in\mathbb{C}$ and $q\in(0,1)\cup\mathbb{N}$.
In (1.4), $(\gamma)_{qm}$ is the generalized Pochhammer symbol, which can be written in terms of gamma function as

$$(\gamma)_{qm}=\frac{\Gamma(\gamma+mq)}{\Gamma(\gamma)} \tag{1.5}$$

In a sequel of the work on multiindex Mittag-Leffler function given by Saxena and Nishimoto [13],[14]. We have defined generalized multiindex Bessel-Maitland function as

$$J_{(\beta_j),q}^{(\alpha_j),\gamma}(x)=\sum_{m=0}^{\infty}\frac{(\gamma)_{qm}(-x)^m}{\prod_{j=1}^{n}\Gamma(\alpha_j m+\beta_j+1)m!} \tag{1.6}$$

where $j=1,2,\cdots,n$; $\Re(\gamma)>0$, $\Re(\beta_j)>-1$, $\Re[\sum_{j=1}^{n}\alpha_j]>max\{0,q-1\}$, $q\in(0,1)\cup\mathbb{N}$ and $n\in\mathbb{N}$

Put $j=1,\ \alpha_1=1,\ \beta_1=\beta,\ q=0$ and replace x by $\frac{x^2}{4}$, we achieve

$$J_{\beta,0}^{1,\gamma}\left(\frac{x^2}{4}\right)=\left(\frac{2}{x}\right)J_\beta(x) \tag{1.7}$$

where $J_\beta(x)$ is well known Bessel function defined in (1.2). Equation (1.6), can also be written as

$$J_{(\beta_j),q}^{(\alpha_j),\gamma}(x)$$

$$=\frac{1}{\Gamma(\gamma)}{}_1\Psi_n\left[\begin{matrix}(\gamma,q) & & ; \\ (\beta_1+1,\alpha_1), & \cdots, & (\beta_n+1,\alpha_n);\end{matrix}\ -x\right] \tag{1.8}$$

where ${}_1\Psi_n$ is the generalized Wright hypergeometric function ${}_p\Psi_q$ [4], also called as Fox-Wright function, which is defined as

$${}_p\Psi_q\left[\begin{matrix}(a_1,A_1), & \cdots, & (a_p,A_p); \\ (b_1,B_1), & \cdots, & (b_p,B_q);\end{matrix}\ x\right]$$

$$=\sum_{k=0}^{\infty}\frac{\Gamma(a_1+kA_1)\cdots\Gamma(a_p+kA_p)x^k}{\Gamma(b_1+kB_1)\cdots\Gamma(b_q+kB_q)k!} \tag{1.9}$$

where $A_1,A_2,\cdots,A_p$ and $B_1,B_2,\cdots,B_q$ are positive real numbers such that

$$1+\sum_{j=1}^{q}B_j-\sum_{i=1}^{p}A_i>0$$

The Fox-Wright function has a relation with Fox-H function [4] defined as

$${}_p\Psi_q\left[\begin{matrix}(a_1,A_1), & \cdots, & (a_p,A_p); \\ (b_1,B_1), & \cdots, & (b_p,B_q);\end{matrix}\ x\right]$$

$$=H_{p,q+1}^{1,p}\left[-x\ \middle|\ \begin{matrix} & (1-a_1,A_1), & \cdots, & (1-a_p,A_p) \\ (0,1), & (1-b_1,B_1), & \cdots, & (1-b_p,B_q)\end{matrix}\right] \tag{1.10}$$

Here in this paper, we are going to find the integral formulas involving (1.6). For that we require the following integral formulas

$$\int_0^1 z^{\nu-1}(1-z)^{\mu-1}[az+b(1-z)]^{-\nu-\mu}dz=\frac{1}{a^\nu b^\mu}\frac{\Gamma(\nu)\Gamma(\mu)}{\Gamma(\nu+\mu)} \tag{1.11}$$

where $\Re(\mu)>0,\ \Re(\nu)>0$ and $a,\ b$ are non-zero constants and the expression $az+b(1-z)$, where $0\le z\le 1$ is non-zero.

$$\int_a^b(a-t)^{\nu-1}(t-b)^{\mu-1}dt=(a-b)^{\nu+\mu-1}\frac{\Gamma(\nu)\Gamma(\mu)}{\Gamma(\nu+\mu)} \tag{1.12}$$

where $\Re(\nu)>0,\ \Re(\mu)>0$ and $b<a$.

$$\int_0^\infty z^{\mu-1}(z+a+\sqrt{z^2+az})^{-\lambda}dz=2\lambda a^{-\lambda}\frac{a^\mu\Gamma(2\mu)\Gamma(\lambda-\mu)}{2^\mu\Gamma(1+\lambda+\mu)} \tag{1.13}$$

where $0<\Re(\mu)<\Re(\lambda)$.
The formula in (1.11), (1.12) and (1.13) are known as MacRobert [7], Erdelyi [1] and Oberhettinger [3] respectively.

2. Main Results

In this section we are going to deduce theorems on generalized multiindex Bessel-Maitland function

Theorem 2.1. *The under-mentioned result holds true under specified conditions*

$$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}[ax+b(1-x)]^{-\nu-\mu}J_{(\beta_j),q}^{(\alpha_j),\gamma}\left(\frac{xy}{ax+b(1-x)}\right)dx$$

$$=\frac{\Gamma(\mu)}{a^\nu b^\mu\Gamma(\gamma)}$$

$$\times{}_2\Psi_{n+1}\left[\begin{matrix} & & (\nu,1), & (\gamma,q); \\ (\beta_1+1,\alpha_1), & \cdots, & (\beta_n+1,\alpha_n) & (\nu+\mu,1);\end{matrix}\ \frac{-y}{a}\right] \tag{2.1}$$

$\Re(\nu),\Re(\mu),\Re(\gamma)>0,\Re(\beta_j)>-1,$
$\Re[\sum_{j=1}^{n}\alpha_j]>max\{0,q-1\}.$

Proof. Using (1.6) in left hand side of (2.1) and interchanging the order of summation and integration which is gauranteed under the convergence conditions

$$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}[ax+b(1-x)]^{-\nu-\mu}J_{(\beta_j),q}^{(\alpha_j),\gamma}\left(\frac{xy}{ax+b(1-x)}\right)dx$$

$$=\sum_{m=0}^{\infty}\frac{(\gamma)_{qm}y^m}{\prod_{j=1}^{n}\Gamma(\alpha_jm+\beta_j+1)m!}$$

$$\times\int_0^1 x^{\nu+m-1}(1-x)^{\mu-1}[ax+b(1-x)]^{-\nu-\mu-m}dx$$

Applying (1.11) and using (1.5), we achieve

$$=\frac{\Gamma(\mu)}{a^\nu b^\mu\Gamma(\gamma)}\sum_{m=0}^{\infty}\frac{\Gamma(\nu+m)\Gamma(\gamma+qm)(-y)^m}{\prod_{j=1}^{n}\Gamma(\alpha_jm+\beta_j+1)\Gamma(\nu+\mu+m)a^mm!}$$

Using (1.9), we acquire our result. □

Theorem 2.2. *The under-mentioned result holds true under specified conditions*

$$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}[ax+b(1-x)]^{-\nu-\mu}J_{(\beta_j),q}^{(\alpha_j),\gamma}\left(\frac{(1-x)y}{ax+b(1-x)}\right)dx$$

$$=\frac{\Gamma(\nu)}{a^\nu b^\mu\Gamma(\gamma)}$$

$$\times{}_2\Psi_{n+1}\left[\begin{matrix} & & (\mu,1), & (\gamma,q); \\ (\beta_1+1,\alpha_1), & \cdots, & (\beta_n+1,\alpha_n) & (\nu+\mu,1);\end{matrix}\ \frac{-y}{a}\right] \tag{2.2}$$

$\Re(\nu),\Re(\mu),\Re(\gamma)>0,\ \Re(\beta_j)>-1,$
$\Re[\sum_{j=1}^{n}\alpha_j]>max\{0,q-1\}.$

Proof. The proof is on the same lines as of Theorem 2.1 □

Theorem 2.3. *The result below is verifiable under specified conditions*

$$\int_a^b (a-x)^{\nu-1}(x-b)^{\mu-1} J_{(\beta_j),q}^{(\alpha_j),\gamma}[(a-x)y] = \frac{(a-b)^{\nu+\mu}\Gamma(\mu)}{\Gamma(\gamma)}$$

$$\times {}_2\Psi_{n+1}\left[\begin{matrix} & & (\nu,1), & (\gamma,q); \\ (\beta_1+1,\alpha_1), & \cdots, & (\beta_n+1,\alpha_n) & (\nu+\mu,1); \end{matrix} \; -(a-b)y\right] \tag{2.3}$$

$\Re(\nu), \Re(\mu), \Re(\gamma) > 0$, $\Re(\beta_j) > -1$, $\Re[\sum_{j=1}^{n} \alpha_j] > max\{0, q-1\}$ *and* $a > b$.

Proof. Using (1.6) in left hand side of (2.3) and interchanging the order of summation and integration which is gauranteed under the convergence conditions, we get

$$\int_a^b (a-x)^{\nu-1}(x-b)^{\mu-1} J_{(\beta_j),q}^{(\alpha_j),\gamma}[(a-x)y]$$

$$= \sum_{m=0}^{\infty} \frac{(\gamma)_{qm} y^m}{\prod_{j=1}^{n}\Gamma(\alpha_j m+\beta_j+1) m!} \int_a^b (a-x)^{\nu+m-1}(x-b)^{\mu-1} dx$$

Applying (1.12) and using (1.5), we achieve

$$= \frac{\Gamma(\mu)(a-b)^{\nu+\mu}}{\Gamma(\gamma)} \sum_{m=0}^{\infty} \frac{\Gamma(\nu+m)\Gamma(\gamma+qm)(-y)^m (a-b)^m}{\prod_{j=1}^{n}\Gamma(\alpha_j m+\beta_j+1)\Gamma(\nu+\mu+m) m!}$$

Using (1.9), we acquire our result. □

Theorem 2.4. *The result below is verifiable under specified conditions*

$$\int_a^b (a-x)^{\nu-1}(x-b)^{\mu-1} J_{(\beta_j),q}^{(\alpha_j),\gamma}[(x-b)y] = \frac{(a-b)^{\nu+\mu}\Gamma(\nu)}{\Gamma(\gamma)}$$

$$\times {}_2\Psi_{n+1}\left[\begin{matrix} & & (\mu,1), & (\gamma,q); \\ (\beta_1+1,\alpha_1), & \cdots, & (\beta_n+1,\alpha_n) & (\nu+\mu,1); \end{matrix} \; -(a-b)y\right] \tag{2.4}$$

$\Re(\nu), \Re(\mu), \Re(\gamma) > 0$, $\Re(\beta_j) > -1$, $\Re[\sum_{j=1}^{n} \alpha_j] > max\{0, q-1\}$ *and* $a > b$.

Proof. The proof is on the same lines as of Theorem 2.3 . □

Theorem 2.5. *The underlying result is true under specified conditions*

$$\int_0^{\infty} x^{\mu-1}(x+a+\sqrt{x^2+ax})^{-\lambda} J_{(\beta_j),q}^{(\alpha_j),\gamma}(xy)dx = \frac{\lambda 2^{1-\mu} a^{\mu-\lambda}}{\Gamma(\gamma)}$$

$$\times {}_3\Psi_{n+1}\left[\begin{matrix} & (\gamma,q), & (2\mu,2), & (\lambda-\mu,-1); \\ (\beta_1+1,\alpha_1) & \cdots & (\beta_n+1,\alpha_n) & (\lambda+\mu+1,1); \end{matrix} \; \frac{-ya}{2}\right] \tag{2.5}$$

$\Re(\beta_j) > -1$, $\Re[\sum_{j=1}^{n} \alpha_j] > max\{0, q-1\}$ *and* $0 < \Re(\mu) < \Re(\lambda)$.

Proof. Using (1.6) in left hand side of (2.5) and interchanging the order of summation and integration which is gauranteed under the convergence conditions, we get

$$\int_0^{\infty} x^{\mu-1}(x+a+\sqrt{x^2+ax})^{-\lambda} J_{(\beta_j),q}^{(\alpha_j),\gamma}(xy)dx$$

$$= \sum_{m=0}^{\infty} \frac{(\gamma)_{qm} y^m}{\prod_{j=1}^{n}\Gamma(\alpha_j m+\beta_j+1) m!} \int_0^{\infty} x^{\mu+m-1}(x+a+\sqrt{x^2+ax})^{-\lambda} dx$$

Applying (1.13) and using (1.5), we achieve

$$= \frac{\lambda 2^{1-\mu} a^{\mu-\lambda}}{\Gamma(\gamma)} \sum_{n=0}^{\infty} \frac{\Gamma(\gamma+qm)\Gamma(2\mu+2m)\Gamma(\lambda-\mu-m)(-ya)^m}{\prod_{j=1}^{n}\Gamma(\alpha_j m+\beta_j+1)\Gamma(1+\lambda+\mu+m) 2^m m!}$$

Using (1.9), we acquire our result. □

Theorem 2.6. *The underlying result is true under specified conditions*

$$\int_0^{\infty} x^{\mu-1}(x+a+\sqrt{x^2+ax})^{-\lambda} J_{(\beta_j),q}^{(\alpha_j),\gamma}\left(\frac{y}{x+a+\sqrt{x^2+ax}}\right) dx = \frac{2^{1-\mu} a^{\mu-\lambda}\Gamma(2\mu)}{\Gamma(\gamma)}$$

$$\times {}_3\Psi_{n+2}\left[\begin{matrix} & & (\gamma,q), & (\lambda+1,1), & (\lambda-\mu,-1); \\ (\beta_1+1,\alpha_1), & \cdots, & (\beta_n+1,\alpha_n), & (\lambda+\mu+1,1), & (\lambda,1); \end{matrix} \; \frac{-y}{a}\right] \tag{2.6}$$

$\Re(\beta_j) > -1$, $\Re[\sum_{j=1}^{n} \alpha_j] > max\{0, q-1\}$ *and* $0 < \Re(\mu) < \Re(\lambda)$.

Proof. The proof is on the same lines as of Theorem 2.5 . □

3. Transformation of Fox-Wright function to Fox-H function in above theorems

In this section we are going to execute the transformation of Fox-Wright function to Fox-H function in the theorems of above section with the help of (1.10)

Corollary 3.1. *Variation of Theorem 2.1*

$$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}[ax+b(1-x)]^{-\nu-\mu} J_{(\beta_j),q}^{(\alpha_j),\gamma}\left(\frac{xy}{ax+b(1-x)}\right) dx = \frac{\Gamma(\mu)}{a^{\nu} b^{\mu}\Gamma(\gamma)}$$

$$\times H_{2,n+2}^{1,2}\left[\frac{y}{a} \,\middle|\, \begin{matrix} & & & (1-\nu,1), & (1-\gamma,q) \\ (0,1), & (-\beta_1,\alpha_1), & \cdots, & (-\beta_n,\alpha_n) & (1-\nu-\mu,1) \end{matrix}\right] \tag{3.1}$$

Corollary 3.2. *Variation of Theorem 2.2*

$$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}[ax+b(1-x)]^{-\nu-\mu} J_{(\beta_j),q}^{(\alpha_j),\gamma}\left(\frac{(1-x)y}{ax+b(1-x)}\right) dx = \frac{\Gamma(\nu)}{a^{\nu} b^{\mu}\Gamma(\gamma)}$$

$$\times H_{2,n+2}^{1,2}\left[\frac{y}{a} \,\middle|\, \begin{matrix} & & & (1-\mu,1), & (1-\gamma,q) \\ (0,1), & (-\beta_1,\alpha_1), & \cdots, & (-\beta_n,\alpha_n) & (1-\nu-\mu,1) \end{matrix}\right] \tag{3.2}$$

Corollary 3.3. *Variation of Theorem 2.3*

$$\int_a^b (a-x)^{\nu-1}(x-b)^{\mu-1} J_{(\beta_j),q}^{(\alpha_j),\gamma}[(a-x)y] = \frac{(a-b)^{\nu+\mu}\Gamma(\mu)}{\Gamma(\gamma)}$$

$$\times H_{2,n+2}^{1,2}\left[(a-b)y \,\middle|\, \begin{matrix} & & & (1-\nu,1), & (1-\gamma,q); \\ (0,1), & (-\beta_1,\alpha_1), & \cdots, & (-\beta_n,\alpha_n) & (1-\nu-\mu,1); \end{matrix}\right] \tag{3.3}$$

Corollary 3.4. *Variation of Theorem 2.4*

$$\int_a^b (a-x)^{\nu-1}(x-b)^{\mu-1} J_{(\beta_j),q}^{(\alpha_j),\gamma}[(x-b)y] = \frac{(a-b)^{\nu+\mu}\Gamma(\nu)}{\Gamma(\gamma)}$$

$$\times H_{2,n+2}^{1,2}\left[(a-b)y \,\middle|\, \begin{array}{ccccc} & & & (1-\mu,1), & (1-\gamma,q); \\ (0,1), & (-\beta_1,\alpha_1), & \cdots, & (-\beta_n,\alpha_n) & (1-\nu-\mu,1); \end{array}\right] \tag{3.4}$$

Corollary 3.5. *Variation of Theorem 2.5*

$$\int_0^\infty x^{\mu-1}(x+a+\sqrt{x^2+ax})^{-\lambda} J_{(\beta_j),q}^{(\alpha_j),\gamma}(xy)dx = \frac{\lambda 2^{1-\mu}a^{\mu-\lambda}}{\Gamma(\gamma)}$$

$$\times H_{3,n+2}^{1,3}\left[\frac{ya}{2} \,\middle|\, \begin{array}{ccccc} & & (1-\gamma,q), & (1-2\mu,2), & (1-\lambda+\mu,-1) \\ (0,1), & (-\beta_1,\alpha_1), & \cdots, & (-\beta_n,\alpha_n) & (-\lambda-\mu,1) \end{array}\right] \tag{3.5}$$

Corollary 3.6. *Variation of Theorem 2.6*

$$\int_0^\infty x^{\mu-1}(x+a+\sqrt{x^2+ax})^{-\lambda} J_{(\beta_j),q}^{(\alpha_j),\gamma}\left(\frac{y}{x+a+\sqrt{x^2+ax}}\right)dx = \frac{2^{1-\mu}a^{\mu-\lambda}\Gamma(2\mu)}{\Gamma(\gamma)}$$

$$\times H_{3,n+3}^{1,3}\left[\frac{y}{a} \,\middle|\, \begin{array}{cccccc} & & & (1-\gamma,q), & (-\lambda,1), & (1-\lambda+\mu,-1) \\ (0,1), & (-\beta_1,\alpha_1), & \cdots, & (-\beta_n,\alpha_n), & (-\lambda-\mu,1) & (1-\lambda,1) \end{array}\right] \tag{3.6}$$

All the above corollaries in this section are true under the same conditions as of their theorems.

4. Special Cases

In this section we will find the particular cases of generalized multiindex Mittag-Leffler function.

Corollary 4.1. *By setting $j=1$ and putting $\alpha_1=\alpha, \beta_1=\beta$ in Theorem 2.1*

$$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}[ax+b(1-x)]^{-\nu-\mu} J_{\beta,q}^{\alpha,\gamma}\left(\frac{xy}{ax+b(1-x)}\right)dx$$

$$= \frac{\Gamma(\mu)}{a^\nu b^\mu \Gamma(\gamma)}\, {}_2\Psi_2\left[\begin{array}{cc} (\nu,1), & (\gamma,q); \\ (\beta+1,\alpha), & (\nu+\mu,1); \end{array} \frac{-y}{a}\right] \tag{4.1}$$

where $\Re(\nu),\Re(\mu),\Re(\gamma)>0$, $\Re(\beta)>-1$, $\Re[\alpha]>max\{0,q-1\}$.

Corollary 4.2. *By setting $j=1$ and putting $\alpha_1=\alpha, \beta_1=\beta$ in Theorem 2.2*

$$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}[ax+b(1-x)]^{-\nu-\mu} J_{\beta,q}^{\alpha,\gamma}\left(\frac{(1-x)y}{ax+b(1-x)}\right)dx$$

$$= \frac{\Gamma(\nu)}{a^\nu b^\mu \Gamma(\gamma)}\, {}_2\Psi_2\left[\begin{array}{cc} (\mu,1), & (\gamma,q); \\ (\beta+1,\alpha), & (\nu+\mu,1); \end{array} \frac{-y}{a}\right] \tag{4.2}$$

where $\Re(\nu),\Re(\mu),\Re(\gamma)>0, \Re(\beta)>-1$, $\Re[\alpha]>max\{0,q-1\}$.

Corollary 4.3. *By setting $j=1$ and putting $\alpha_1=\alpha, \beta_1=\beta$ in Theorem 2.3*

$$\int_a^b (a-x)^{\nu-1}(x-b)^{\mu-1} J_{\beta,q}^{\alpha,\gamma}[(a-x)y] = \frac{(a-b)^{\nu+\mu}\Gamma(\mu)}{\Gamma(\gamma)}$$

$$\times {}_2\Psi_2\left[\begin{array}{cc} (\nu,1), & (\gamma,q); \\ (\beta+1,\alpha), & (\nu+\mu,1); \end{array} -(a-b)y\right] \tag{4.3}$$

where $\Re(\nu),\Re(\mu),\Re(\gamma)>0$, $\Re(\beta)>-1$, $\Re[\alpha]>max\{0,q-1\}$ *and* $a>b$.

Corollary 4.4. *By setting $j=1$ and putting $\alpha_1=\alpha, \beta_1=\beta$ in Theorem 2.4*

$$\int_a^b (a-x)^{\nu-1}(x-b)^{\mu-1} J_{\beta,q}^{\alpha,\gamma}[(x-b)y] = \frac{(a-b)^{\nu+\mu}\Gamma(\nu)}{\Gamma(\gamma)}$$

$$\times {}_2\Psi_2\left[\begin{array}{cc} (\mu,1), & (\gamma,q); \\ (\beta+1,\alpha), & (\nu+\mu,1); \end{array} -(a-b)y\right] \tag{4.4}$$

where $\Re(\nu),\Re(\mu),\Re(\gamma)>0$, $\Re(\beta)>-1$, $\Re[\alpha]>max\{0,q-1\}$ *and* $a>b$.

Corollary 4.5. *By setting $j=1$ and putting $\alpha_1=\alpha, \beta_1=\beta$ in Theorem 2.5*

$$\int_0^\infty x^{\mu-1}(x+a+\sqrt{x^2+ax})^{-\lambda} J_{\beta,q}^{\alpha,\gamma}(xy)dx = \frac{\lambda 2^{1-\mu}a^{\mu-\lambda}}{\Gamma(\gamma)}$$

$$\times {}_3\Psi_2\left[\begin{array}{ccc} (\gamma,q), & (2\mu,2), & (\lambda-\mu,-1); \\ & (\beta+1,\alpha), & (\lambda+\mu+1,1); \end{array} \frac{-ya}{2}\right] \tag{4.5}$$

where $\Re(\beta)>-1$, $\Re[\alpha]>max\{0,q-1\}$ *and* $0<\Re(\mu)<\Re(\lambda)$.

Corollary 4.6. *By setting $j=1$ and putting $\alpha_1=\alpha, \beta_1=\beta$ in Theorem 2.6*

$$\int_0^\infty x^{\mu-1}(x+a+\sqrt{x^2+ax})^{-\lambda} J_{\beta,q}^{\alpha,\gamma}\left(\frac{y}{x+a+\sqrt{x^2+ax}}\right)dx = \frac{2^{1-\mu}a^{\mu-\lambda}\Gamma(2\mu)}{\Gamma(\gamma)}$$

$$\times {}_3\Psi_3\left[\begin{array}{ccc} (\gamma,q), & (\lambda+1,1), & (\lambda-\mu,-1) \\ (\beta+1,\alpha), & (\lambda+\mu+1,1), & (\lambda,1) \end{array} \frac{-y}{a}\right] \tag{4.6}$$

where $\Re(\beta)>-1$, $\Re[\alpha]>max\{0,q-1\}$ *and* $0<\Re(\mu)<\Re(\lambda)$.

Corollary 4.7. *By setting $j=1$ and putting $\alpha_1=\alpha$, $\beta_1=\beta$, $q=0$ Theorem 2.1*

$$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}[ax+b(1-x)]^{-\nu-\mu} J_\beta^\alpha\left(\frac{xy}{ax+b(1-x)}\right)dx$$

$$= \frac{\Gamma(\mu)}{a^{\nu}b^{\mu}} {}_1\Psi_2 \left[\begin{array}{c} (\nu,1); \\ (\beta+1,\alpha), \quad (\nu+\mu,1); \end{array} \frac{-y}{a} \right] \quad (4.7)$$

where $\Re(\alpha), \Re(\nu), \Re(\mu) > 0$, $\Re(\beta) > -1$.

Corollary 4.8. *By setting* $j = 1$ *and putting* $\alpha_1 = \alpha$, $\beta_1 = \beta$, $q = 0$ *Theorem 2.2*

$$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}[ax+b(1-x)]^{-\nu-\mu} J_{\beta}^{\alpha}\left(\frac{(1-x)y}{ax+b(1-x)}\right) dx$$

$$= \frac{\Gamma(\nu)}{a^{\nu}b^{\mu}} {}_1\Psi_2 \left[\begin{array}{c} (\mu,1); \\ (\beta+1,\alpha), \quad (\nu+\mu,1); \end{array} \frac{-y}{a} \right] \quad (4.8)$$

where $\Re(\nu), \Re(\mu), \Re(\alpha) > 0$, $\Re(\beta) > -1$.

Corollary 4.9. *By setting* $j = 1$ *and putting* $\alpha_1 = \alpha$, $\beta_1 = \beta$, $q = 0$ *Theorem 2.3*

$$\int_a^b (a-x)^{\nu-1}(x-b)^{\mu-1} J_{\beta}^{\alpha}[(a-x)y] = (a-b)^{\nu+\mu}\Gamma(\mu)$$

$$\times {}_1\Psi_2 \left[\begin{array}{c} (\nu,1); \\ (\beta+1,\alpha), \quad (\nu+\mu,1); \end{array} -(a-b)y \right] \quad (4.9)$$

where $\Re(\nu), \Re(\mu), \Re(\alpha) > 0$, $\Re(\beta) > -1$ *and* $a > b$.

Corollary 4.10. *By setting* $j = 1$ *and putting* $\alpha_1 = \alpha$, $\beta_1 = \beta$, $q = 0$ *Theorem 2.4*

$$\int_a^b (a-x)^{\nu-1}(x-b)^{\mu-1} J_{\beta}^{\alpha}[(x-b)y] = (a-b)^{\nu+\mu}\Gamma(\nu)$$

$$\times {}_1\Psi_2 \left[\begin{array}{c} (\mu,1); \\ (\beta+1,\alpha), \quad (\nu+\mu,1); \end{array} -(a-b)y \right] \quad (4.10)$$

where $\Re(\nu), \Re(\mu), \Re(\alpha) > 0$, $\Re(\beta) > -1$ *and* $a > b$.

Corollary 4.11. *By setting* $j = 1$ *and putting* $\alpha_1 = \alpha$, $\beta_1 = \beta$, $q = 0$ *Theorem 2.5*

$$\int_0^{\infty} x^{\mu-1}(x+a+\sqrt{x^2+ax})^{-\lambda} J_{\beta}^{\alpha}(xy)dx = \lambda 2^{1-\mu} a^{\mu-\lambda}$$

$$\times {}_2\Psi_2 \left[\begin{array}{cc} (2\mu,2), & (\lambda-\mu,-1); \\ (\beta+1,\alpha), & (\lambda+\mu+1,1); \end{array} \frac{-ya}{2} \right] \quad (4.11)$$

where $\Re(\beta) > -1$, $\Re(\alpha) > 0$ *and* $0 < \Re(\mu) < \Re(\lambda)$.

Corollary 4.12. *By setting* $j = 1$ *and putting* $\alpha_1 = \alpha$, $\beta_1 = \beta$, $q = 0$ *Theorem 2.6*

$$\int_0^{\infty} x^{\mu-1}(x+a+\sqrt{x^2+ax})^{-\lambda} J_{\beta}^{\alpha}\left(\frac{y}{x+a+\sqrt{x^2+ax}}\right) dx = 2^{1-\mu} a^{\mu-\lambda}\Gamma(2\mu)$$

$$\times {}_2\Psi_3 \left[\begin{array}{ccc} & (\lambda+1,1), & (\lambda-\mu,-1) \\ (\beta+1,\alpha), & (\lambda+\mu+1,1), & (\lambda,1) \end{array} \frac{-y}{a} \right] \quad (4.12)$$

where $\Re(\beta) > -1$, $\Re(\alpha) > 0$ *and* $0 < \Re(\mu) < \Re(\lambda)$.

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