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Premises on fuzzy factorizable perfect intrinsic edge-magic graphs

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Abstract
A fuzzy graph G is said to be perfect intrinsic edge-magic if it satisfies the perfect intrinsic edge-magic labelling with intrinsic super constant I_s. In this paper we introduced the order, size and strength of factorizable perfect intrinsic edge-magic graphs. Also we examined these concepts in fuzzy cycle, banner, path & star graphs.

Keywords
Intrinsic constant; perfect intrinsic edge-magic labelling; order; size; strength; factorizable and deficient graphs.

AMS Subject Classification
05C72, 05C78

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Contents

1. Introduction

Initially Fuzzy set theory was established by Zadeh L.A., [15]. Then variety of researches further productive concepts to expand fuzzy sets theory like [12] and [3]. In [2], 1987 Bhattacharya has succeeded to develop the connectivity ideas connecting fuzzy bridge and fuzzy cut nodes. A fuzzy graph contains numerous properties comparable to crisp graph appropriate to simplification of crisp graphs but it depart at several places. A crisp graph G is an order pair of vertex-set V and edge set E such that $E \subseteq V \times V$. In addition $v = |V|$ is said to order and $e = |E|$ is called size of the graph G respectively. In a crisp graph, a bijective function $\rho : V \cup E \to N$ that created a unique positive integer (To each vertex and/or edge) is called a labelling [14]. In [4], Introduced the concept of magic graph that the labels vertices and edges are natural numbers from 1 to $|V| + |E|$ such that sum of the labels of vertices and the edge between them necessity be constant in complete graph. Comprehensive the perception of magic graph with added a property that vertices constantly get smaller labels than edges and named it super edge magic labelling. Frequent other authors have explored miscellaneous types of different magic graphs [1], [11] & [5]. In [8], the topic of edge-magic labelling of graphs had its foundation in the work of Kotzig and Rosa on what they called magic valuations of graphs. These labelling are presently referred to as either edge-magic labelling or edge-magic total labelling.

Fuzzy graphs are generalization of graphs. In graphs two vertices are either related or not related to each other. Mathematically, the degree of relationship is either 0 or 1. In fuzzy graphs, the degree of correlation takes values from $[0,1]$. A fuzzy graph has facility to explain vague problems in a wide series of fields. In 1973, Kaufmann was introduced the definition of a fuzzy graph. In 1975, developed the structure of fuzzy graphs and obtained analogs of several graph theoretical concepts developed by Azriel Rosenfield [12]. In [9] & [11], NagoorGani et. al. introduced the concepts of order and size of fuzzy graphs & fuzzy labelling graphs, fuzzy magic graphs. Earlier, we have discussed the related work of fuzzy intrinsic edge-magic graphs in [6] & [7]. In this paper we have developed the concept of fuzzy factorizable perfect intrinsic edge-magic graphs. Also we discussed the some general form of order, size and strength of the respective graphs.

2. Preliminaries

Definition 2.1. *[15] A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ where for all $u, v \in V$, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.*

Definition 2.2. *A path P in a fuzzy graph is a sequence of different nodes $v_1, v_2, v_3, \ldots, v_n$ such that $\mu(v_i, v_{i+1}) > 0$; $1 \leq i \leq n$; where $n \geq 1$ is called the length of the path P. The consecutive pairs (v_i, v_{i+1}) are called the edge of the path.*

Definition 2.3. *A path P is called a cycle if $v_1 = v_n$ and $n \geq 3$ and a cycle is called a fuzzy cycle if it contains more than one weakest arc.*

Definition 2.4. *A bijection ω is a function from the set of all nodes and edges of to $[0,1]$ which assign each nodes $\sigma^\omega(a)$, $\sigma^\omega(b)$ and edge $\sigma^\omega(a,b)$ a membership value such that $\sigma^\omega(a,b) \leq \sigma^\omega(a) \wedge \sigma^\omega(b)$ for all $a, b \in V$ is called fuzzy labelling. A graph is said to be fuzzy labelling graph if it has a fuzzy labelling and it is denoted by G^ω.*

Definition 2.5. *A fuzzy labelling graph G is said to be fuzzy intrinsic labelling if $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ is bijective such that the membership values of edges and vertices are $\{z, 2z, 3z, \ldots, Nz\}$ where N is the total number of vertices and edges and let $z = 0.1$ for $N \leq 6$ & $z = 0.01$ for $N > 6$.*

Definition 2.6. *[7] A fuzzy labelling graph G is said to be fuzzy perfect intrinsic labelling if $f : \sigma \to [0,1]$ and $f : \mu \to [0,1]$ is bijective such that the membership values of edges are $\{z, 2z, 3z, \ldots, \in z\}$ and vertices are $\{(\varepsilon+1)z, (\varepsilon+2)z, \ldots(\varepsilon+v)z\}$ where $\varepsilon + v = N$ is the total number of vertices and edges and let $z = 0.1$ for $N \leq 6$ & $z = 0.01$ for $N > 6$.*

Definition 2.7. *[7] A fuzzy perfect intrinsic labelling graph is said to be a fuzzy perfect intrinsic edge-magic labelling if it has an intrinsic super constant $\lambda_s = \sigma(v_i) + \mu(v_i v_j) + \sigma(v_j)$ for all $v_i, v_j \in V$.*

Definition 2.8. *[7] A fuzzy graph G is said to be perfect intrinsic edge-magic (PIEM) if it satisfies the perfect intrinsic edge-magic labelling with intrinsic super constant λ_s.*

Definition 2.9. *The Pan graph is the graph obtained by joining a cycle graph to a singleton graph with a bridge. The 3-Pan graph is sometimes known as the Paw graph.*

Definition 2.10. *[9] Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Then the order of G is defined to be $O(G) = \sum_{v \in V} \sigma(v)$. The size of G is defined to be $S(G) = \sum_{u \neq V} \mu(u,v)$.*

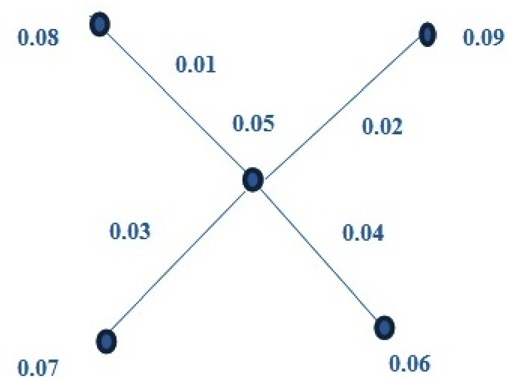
3. Fuzzy factorizable perfect intrinsic edge-magic graphs

Definition 3.1. *Let G be a fuzzy perfect intrinsic edge-magic graph. The size and order of G is denoted by α and β where*

$$\alpha = \sum_{u \neq v} \mu(u,v) \text{ and } \beta = \sum_{v \in V} \sigma(v)$$

Definition 3.2. *Let G be a FPIEM graph. If it has a composite order and size then the graph is called the factorizable point and line graph.*

Definition 3.3. *A FPIEM graph is called a factorizable graph if it has both Composite order and composite size.*



Factorizable Graph

$$\alpha = \sum_{u \neq v} \mu(u,v) = 0.10 \text{ and } \beta = \sum_{v \in V} \sigma(v) = 0.35$$

In the above graph, it has Composite order and composite size.

Definition 3.4. *Let G be a FPIEM graph. If G has a prime order and composite size or prime size and composite order then the graph is called a deficient graph.*

Definition 3.5. *Let G be a fuzzy factorizable perfect intrinsic edge-magic. Let α and β be the order and size of G.*

The strength of G is denoted by κ(kappa)and is defined as the sum of order and size of G.

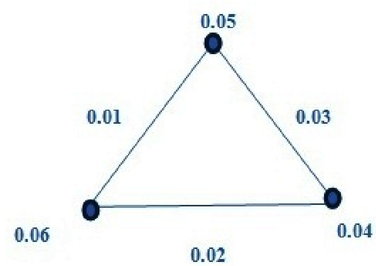
4. Size, Order & Strength of fuzzy FACPIEM graph

Theorem 4.1. *A fuzzy perfect intrinsic edge-magic cycle C_n is factorizable iff $n = 3$ with strength κ(kappa).*

Proof. Let C_n be a FPIEM graph with $'n'$ vertices. Let α & β be the size and order of G. By definition of order and size, we get

$$\alpha = \sum_{u \neq v} \mu(u,v) = (1+2+...+n)z = \Big(\frac{n(n+1)}{2}\Big)z$$
$$\beta = \sum_{v \in V} \sigma(v) = ((n+1)+(n+2)+...2n)z$$
$$= \frac{(2n)(2n+1)}{2} - \frac{n(n+1)}{2}$$
$$= n\Big(\frac{(3n+1)}{2}\Big)z$$

Let $z \to (0,1], z = 0.1$ for $N < 5$ & $z = 0.01$ for $N \geq 5$



Factorizable cycle graph

We put different values of n, we get order and size of the fuzzy cycle graph.

Put $n = 3$, we get

$$\alpha = 0.06 \ \& \ \beta = 0.15 \text{ (which are composite)} \tag{4.1}$$

At this stage, we conclude that fuzzy cycle with 3 vertices is factorizable (through definition).

Conversely, assume that the given graph is factorizable. By our assumption, C_n must have both Composite order and size. If $n = 3$ is the only possible for needed conclusion, $bcozn > 3$, which is not a PIEM.

Hence it must be $n = 3$.

The strength of G is

$$\kappa = \alpha + \beta = \Big(\frac{n(n+1)}{2}\Big)z + n\Big(\frac{(3n+1)}{2}\Big)z = [n(2n+1)]z$$

Put $n = 3$ (for our assumption) in κ, we get the strength value 0.21. □

Theorem 4.2. *Every fuzzy perfect intrinsic edge-magic path graph P_n is factorizable for all $n > 2$ with strength.*

Proof. Let P_n is a FPIEM graph with $'n'$ vertices. Let α & β be the size and order of G. By definition of order and size, we get

$$\alpha = \sum_{u \neq v} \mu(u,v) = (1+2+...+(n-1))z = \Big(\frac{n(n-1)}{2}\Big)z$$
$$\beta = \sum_{v \in V} \sigma(v) = (n+(n+1)+(n+2)+...(2n-1))z$$
$$= \frac{(2n)(2n-1)}{2} - \frac{n(n-1)}{2}$$
$$= n\Big(\frac{(3n-1)}{2}\Big)z$$

Let $z \to (0,1], z = 0.1$ for $N < 5$ & $z = 0.01$ for $N \geq 5$

We put different values of n, we get order and size of the fuzzy path graph.

For the above observation, in size of G, n or $n-1$ must be even, it gives composite size and in order of G, n or $3n-1$ is even, it also produces Composite order.

For $n = 3 \Rightarrow \alpha = 0.03$ & $\beta = 0.12$
$n = 4 \Rightarrow \alpha = 0.06$ & $\beta = 0.22$
$n = 5 \Rightarrow \alpha = 0.10$ & $\beta = 0.35, ...$ etc.,

Proceeding like this, we get α & β must be composite.

Hence G is fuzzy factorizable perfect intrinsic edge-magic path graph.

The strength of G is

$$\kappa = \alpha + \beta = \Big(\frac{n(n-1)}{2}\Big)z + n\Big(\frac{(3n-1)}{2}\Big)z = [n(2n-1)]z$$

Put $'n'$ value in κ, we get the strength values for P_n. □

Theorem 4.3. *Every fuzzy perfect intrinsic edge-magic graph is always factorizable.*

Theorem 4.4. *Every fuzzy PIEM star graph $K_{1,n}$ is factorizable for all $n > 1$.*

Proof. Let $K_{1,n}$ be a FPIEM star graph with $'n+1'$ vertices. Let α & β be the size and order of G. By definition of order and size, we get

$$\alpha = \sum_{u \neq v} \mu(u,v) = (1+2+...+n)z = \Big(\frac{n(n+1)}{2}\Big)z$$
$$\beta = \sum_{v \in V} \sigma(v) = ((n+1)+(n+2)+...(2n+1))z$$
$$= \frac{(2n)(2n+1)}{2}z - \frac{n(n+1)}{2}z$$
$$= \Big(\frac{(n+1)(3n+2)}{2}\Big)z$$

Let $z \to (0,1], z = 0.1$ for $N < 5$ & $z = 0.01$ for $N \geq 5$

We put different values of n, we get order and size of the fuzzy star graph.

From the above, in size of G, n or $n+1$ must be even, it gives composite size and in order of G, $n+1$ or $3n+2$ is even, it also produces Composite order.

For $n = 3 \Rightarrow \alpha = 0.06$ & $\beta = 0.22$
$n = 4 \Rightarrow \alpha = 0.10$ & $\beta = 0.35$
$n = 5 \Rightarrow \alpha = 0.15$ & $\beta = 0.51, ...$ etc.,

Proceeding like this, we get α & β must be composite for all $n > 1$.

Hence by definition, G is fuzzy factorizable perfect intrinsic edge-magic star graph with $'n+1'$ vertices.

The strength of G is

$$\begin{aligned}\kappa = \alpha + \beta &= \left(\frac{n(n+1)}{2}\right)z + n\left(\frac{(n+1)(3n+2)}{2}\right)z \\ &= [(n+1)(2n+1)]z\end{aligned}$$

Put $'n'$ value in κ, we get the strength values for $K_{1,n}$.

□

Theorem 4.5. *Every n-pan graph is fuzzy FACTPIEM for $n > 2$.*

Proof. Let FPIEM graph with $'n'$ vertices. Allow α & β be the size and order of G. By definition of order and size, we get

$$\begin{aligned}\alpha &= \sum_{u \neq v} \mu(u,v) = (1+2+...+(n+1))z \\ &= \left(\frac{(n+1)(n+2)}{2}\right)z \\ \beta &= \sum_{v \in V} \sigma(v) = ((n+2)+(n+3)+...(n+5))z \\ &= \frac{(n+5)(n+6)}{2} - \frac{(n+1)(n+2)}{2} \\ &= \left(\frac{(n+5)(n+6)}{2}\right)z\end{aligned}$$

Let $z \to (0,1], z = 0.1$ for $N < 5$ & $z = 0.01$ for $N \geq 5$

Here N is the total number of vertices and edges in the respective graph.

We put different values of n, we get order and size of the fuzzy n-panner graph.

For the above observation, in size of G, $n+1$ or $n+2$ must be even, it gives composite size and in order of G, $n+5$ or $n+6$ is even, it also produces composite order.

Proceeding this way, we get α & β must be composite.

Hence G is fuzzy factorizable perfect intrinsic edge-magic n-pan graph.

The strength of G is $\kappa = \alpha + \beta = \left(\frac{(n+5)(n+6)}{2}\right)z$.

Put n value in κ, we get the strength values for fuzzy FACPIEM n-pan graph. □

5. Conclusion

In this manuscript, we discussed the thought of fuzzy factorizable perfect intrinsic edge-magic graphs and given some general form of order, size and strength of the respective graphs. Also we evaluated the concern graphs like fuzzy path, cycle, star and banner graphs which are factorizable.

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