Malaya Journal of Matematik, Vol. 7, No. 4, 775-785, 2019

https://doi.org/10.26637/MJM0704/0024

Vertex edge neighborhood prime labeling of some graphs

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Abstract

The concept of vertex edge neighborhood prime labeling was introduced in [3]. We obtained some families of graphs, pendant edge attached by few graphs and m fold petal graphs are vertex edge neighborhood prime labeling.

Keywords

Neighborhood prime labeling, total neighborhood prime labeling, vertex edge neighborhood prime labeling.

AMS Subject Classification

05C78.

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Article History: Received **02** July **2019**; Accepted **14** November **2019**

Contents

1. Introduction

Throughout this paper we consider finite, simple, undirected and connected graphs, and for notations and terminology, we refer to R.Balakrishnan and K.Ranganathan [1]. $G=(V(G),E(G))$, where $V(G)$ is vertex set and $E(G)$ is edge set of the graph. $|V(G)|=p$ and $|E(G)|=q$ are the number of vertices and edges, respectively. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain conditions. See the dynamic graph labeling survey [2] by Gallian is regularly updated. A graph on n vertices admits a *prime labeling* if its vertices can be labeled with the first n natural numbers in such a way that any two adjacent vertices have relatively prime. A bijection $f:V(G)\to\{1,2,3,...,p\}$ is called neighborhood prime labeling if for each vertex $v\in V(G)$ with $deg(v)>1$, $gcd(f(u):u\in N(v))=1$. A graph which admits neighborhood prime labeling is called a *neighborhood prime graph*. This concept was introduced by Patel and Shrimali [6]. A bijection $f:V(G)\cup E(G)\to\{1,2,3,...,p+q\}$ is said to be *total neighborhood prime labeling* if it satisfies the following two conditions: **(i)** for each vertex of degree at least two, the gcd of labeling on its neighborhood vertices is one; **(ii)** for each vertex of degree at least two, the gcd of labeling on the induced edges is one. A graph which admits total neighborhood prime labeling is called *total neighborhood prime graph*. This concept was introduced by Rajeshkumar, et. al., [5]. Also, they proved that path P_n, cycle C_n, if n is even and $n\not\equiv 2 \pmod 4$ and comb are total neighborhood prime graph. Motivated by neighborhood prime graph and total neighborhood prime graph, Pandya and Shrimali [3] defined the concept of vertex edge neighborhood prime labeling.They observed that

(i) every vertex edge neighborhood prime graph is total neighborhood prime graph, but converse is not true.

(ii) the graph which is not having degree one, if it is total neighborhood prime graph, then it is vertex edge neighborhood prime graph.

Let $G=(V(G),E(G))$ be a graph, $u\in V(G)$

$$N_V(u)=\{w\in V(G)/uw \text{ is an edge}\}$$
$$N_E(u)=\{e\in E(G)/e=uv \text{ for some } v\in V(G)\}$$

Vertex edge neighborhood prime labeling is a function $f:V(G)\cup E(G)\to\{1,2,...,|V(G)\cup E(G)|\}$ with one to one correspondence and if

(i) for $u\in V(G)$ with $deg(u)=1$, $\gcd(f(w),f(uw)/w\in N_V(u))=1$.

(ii) for $u\in V(G)$ with $deg(u)\geq 2$, $\gcd(f(w)/w\in N_V(u))=1$. $\gcd(f(e)/e\in N_E(u))=1$.

A graph which admits vertex edge neighborhood prime labeling is called a *vertex edge neighborhood prime graph*. They [3] proved that path, helm, sunlet, bistar, subdivision of central edge and edges of bistar are vertex edge neighborhood prime graphs.

2. Preliminaries

We need some basic definitions as follows.
The Petersen graph $P(n,k)$ is a graph with vertex set $(u_0,u_1,...,u_{n-1},v_0,v_1,,...,v_{n-1})$ and edge set $(u_iu_{i+1},u_iv_i,v_iv_{i+2} : 0 \le i \le n-1)$ where subscripts are to be taken modulo n and $k < \frac{n}{2}$. The $k-$*polygonal book*, denoted $B_{k,n}$, is formed by n copies of a k-polygonal sharing a single edge. Each $k-$polygonal is referred to as a page of the book graph [4]. The *quadrilateral snake* Q_n is obtained from the path P_n by replacing each edge of the path by a quadrilateral C_4. An *alternative quadrilateral snake* $A(Q_n)$, where $n = 4,6,8,10,...$ from a path $u_1,u_2,u_3,...,u_n$ by joining u_i and u_{i+1} (alternately) to a new vertex v_i,w_i. That is, every alternate edge of a path is replaced by C_4. The *triangular snake* T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3. An *alternate triangular snake* $A(T_n)$, where $n = 4,6,8,10,...$ from a path $u_1,u_2,u_3,...,u_n$ by joining u_i and u_{i+1} (alternately) to a new vertex v_i. That is every alternate edge of a path is replaced by C_3. A *double triangular snake* $D(T_n)$, where $n > 1$ consists of two triangular snakes that have a common path. A *double alternate triangular snake* $DA(T_n)$, where $n = 4,6,8,10,...$ consists of two alternate triangular snake that have a common path. That is, a double alternate triangular snake is obtained from a path $u_1,u_2,u_3,...,u_n$ by joining u_i and u_{i+1} (alternately) to two new vertices v_i and w_i. For $n \ge 3$, a combination of prism graph Y_n and antiprism graph A_n is known as *convex polytope graph* R_n. It consists of the inner cycle vertices $\{u_i : 1 \le i \le n\}$, the middle cycle vertices $\{v_i : 1 \le i \le n\}$ and the outer cycle vertices $\{w_i : 1 \le i \le n\}$. $G^* = G * K_1$ is obtained by joining a single pendant edge to each vertex of G.

In section 2, we prove that $P(n,2)$, $C_n \times K_2$, T_n, barycentric cycle, R_n, $A(T_n)$, $B_{3,n}$, $B_{4,n}$, $B_{5,n}$, Q_n, $A(Q_n)$, $D(T_n)$, $DA(T_n)$, $P(n,2) * K_1$, $(C_n \times K_2) * K_1$, $T_n * K_1$, $R_n * K_1$, and barycentric cycle attached by pendant edge are vertex edge neighborhood prime labeling. In section 3, we prove that m fold petal types of graphs $P(n,2)$, $C_n \times K_2$, T_n, barycentric cycle, R_n, $A(T_n)$, sunflower are vertex edge neighborhood prime labeling.

3. Main Results

We now give vertex edge neighborhood prime labeling of some graphs.

Theorem 3.1. *The Petersen graph $P(n,2)$ where $n > 4$ is a vertex edge neighborhood prime labeling.*

Proof. Let $G = P(n,2)$ be a Petersen graph. Then
$V(G) = \{u_i, v_i : 1 \le i \le n\}$ and
$E(G) = \{x_i = u_iv_i : 1 \le i \le n\} \cup \{d_i = u_iu_{i+1} : 1 \le i \le n-1\}$
$\cup \{e_i = v_iv_{i+2} : 1 \le i \le n-2\} \cup \{d_n = u_nu_1\} \cup$
$\{e_{n-1} = v_{n-1}v_1\} \cup \{e_n = v_nv_2\}$.
Also, $|V(G)| = 2n$ and $|E(G)| = 3n$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,5n\}$ as follows.
For each $1 \le i \le n$, $f(u_i) = 2i-1, f(v_i) = 2i$,
$f(x_i) = f(u_iv_i) = p+n+i$.
$f(d_i) = f(u_iu_{i+1}) = p+2n+i$ for $1 \le i \le n-1$.
$f(d_n) = f(u_nu_1) = p+3n$.
Case 1. If n is odd,
$f(e_i) = f(v_iv_{i+2}) = p+i$ for $1 \le i \le n-2$.
$f(e_{n-1}) = f(v_1v_{n-1}) = p+n-1, f(e_n) = f(v_nv_2) = p+n$.
Case 2. If n is even,
$f(e_i) = f(v_iv_{i+2}) = p+i-1$ for $2 \le i \le n-2$.
$f(e_{n-1}) = f(v_{n-1}v_1) = p+n-2, f(e_n) = f(v_nv_2) = p+n-1, f(e_1) = f(v_1v_3) = p+n$.
In order to show that f is a vertex edge neighborhood prime labeling. Let a be any vertex of G.
For $a = v_i, 1 \le i \le n$ with $deg(a) \ge 2$.
Here, $\gcd\{f(x)/x \in N_V(v_i)\} = 1$ and
$\gcd\{f(e)/e \in N_E(v_i)\} = 1$.
For $a = u_i, 1 \le i \le n$ with $deg(a) \ge 2$. Here $u_1 \in N_V(u_n)$, $f(u_1) = 1$ and $\{f(x)/x \in N_V(u_i) : 1 \le i \le n-1\}$ contains consecutive integers.
Therefore, $\gcd\{f(x)/x \in N_V(u_i) : 1 \le i \le n\} = 1$ and $\{f(e)/e \in N_E(u_1)\}$ contains $(p+2n+1, p+3n, p+n+1)$.
Therefore, gcd $(p+2n+1, p+3n, p+n+1) = 1$ and $\{f(e)/e \in N_E(u_i)\}$ contains consecutive integers.
Therefore, gcd $\{f(e)/e \in N_E(u_i) : 1 \le i \le n\} = 1$.
Hence G is a vertex edge neighborhood prime labeling. □

Theorem 3.2. *The prism graph $C_n \times K_2$ is a vertex edge neighborhood prime labeling for all n.*

Proof. Let $G = C_n \times K_2$ be a prism graph . Then
$V(G) = \{u_i, v_i. : 1 \le i \le n\}$ and
$E(G) = \{e_i = u_iu_{i+1}, d_i = v_iv_{i+1} : 1 \le i \le n-1\} \cup$
$\{x_i = u_iv_i : 1 \le i \le n\} \cup \{e_n = u_nu_1\} \cup \{d_n = v_nv_1\}$.
Also, $|V(G)| = 2n$ and $|E(G)| = 3n$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,5n\}$ as follows.
For each $1 \le i \le n$, $f(u_i) = 2i, f(v_i) = 2i-1$,
$f(x_i) = f(u_iv_i) = p+n+i$.
For each $1 \le i \le n-1$, $f(e_i) = f(u_iu_{i+1}) = p+i$,
$f(d_i) = f(v_iv_{i+1}) = p+2n+i$.
$f(e_n) = f(u_nu_1) = p+n, f(d_n) = f(v_nv_1) = p+3n$.
In order to show that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let x be any vertex of G.
Case 1. Let $x = u_i, 1 \le i \le n$ with $deg(x) \ge 2$.
In this case $v_1 \in N_V(u_1), f(v_1) = 1$ and
$\{f(w)/w \in N_V(u_i) : 2 \le i \le n\}$ and
$\{f(e)/e \in N_E(u_i) : 1 \le i \le n\}$ also contains consecutive integers.
Case 2. If $x = v_i, 1 \le i \le n$ with $deg(x) \ge 2$, then

$v_1 \in N_V(v_n), f(v_1) = 1$ and $\{f(u)/u \in N_V(u_i) : 1 \le i \le n-1\}$ contains consecutive integers.
gcd $\{f(u)/u \in N_V(u_i) : 1 \le i \le n\} = 1$ and
$\{f(e_i)/e_i \in N_E(v_1)\}$ contains $(p+3n, p+n+1, p+2n+1)$.
gcd $(p+3n, p+n+1, p+2n+1) = 1$ and
$\{f(e_i)/e_i \in N_E(v_i) : 2 \le i \le n\}$ contains consecutive integers.
gcd $\{f(e_i)/e_i \in N_E(v_i) : 1 \le i \le n\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 3.3. *The triangular snake T_n admits vertex edge neighborhood prime labeling for all n.*

Proof. Let $G = T_n$ be a triangular snake. Then
$V(G) = \{u_i : 1 \le i \le n\} \cup \{v_i : 1 \le i \le n-1\}$ and
$E(G) = \{e_i = u_iu_{i+1}, d_i = u_iv_i, x_i = v_iu_{i+1} : 1 \le i \le n-1\}$.
Also, $|V(G)| = 2n-1$ and $|E(G)| = 3n-3$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,5n-4\}$ as follows.
$f(u_i) = 2i-1$ for $1 \le i \le n$.
For each $1 \le i \le n-1$, $f(v_i) = 2i, f(e_i) = f(u_iu_{i+1}) = p+3i-2$.
For each $1 \le i \le n-2$, $f(d_i) = f(u_iv_i) = p+3i-1, f(x_i) = f(v_iu_{i+1}) = p+3i$.
$f(d_{n-1}) = f(u_{n-1}v_{n-1}) = p+3n-3, f(x_{n-1}) = f(v_{n-1}u_n) = p+3n-4$.
We have to show that f is a vertex edge neighborhood prime labeling. We consider the following cases. Let a be any vertex of G.
Case 1. If $a = u_1, u_n$ with $deg(a) = 2$, then
gcd $\{f(x)/x \in N_V(u_1)\} =$ gcd $(f(v_1), f(u_2)) =$gcd$(2,3) = 1$,
gcd$\{f(x)/x \in N_V(u_n)\} =$ gcd$(f(u_{n-1}), f(v_{n-1})) =$ gcd$(2n-3, 2n-2) = 1$ and $\{f(e)/e \in N_E(u_1)\}$ and $\{f(e)/e \in N_E(u_n)\}$ contains consecutive numbers.
gcd$\{f(e)/e \in N_E(u_1)\} = 1$ and gcd$\{f(e)/e \in N_E(u_n)\} = 1$.
Case 2. If $a = u_i, 2 \le i \le n-1$ with $deg(a) \ge 2$, then
gcd$\{f(w)/w \in N_V(u_i)\} =gcd(2i-3, 2i-2, 2i, 2i+1) = 1$ and
gcd $\{f(e_i)/e_i \in N_E(u_i)\}$ contains consecutive numbers, so gcd$(f(e_i)/e_i \in N_E(u_i)\}$ is 1.
Case 3. If $a = v_i, 1 \le i \le n-1$ with $deg(a) = 2$, then
gcd $\{f(x)/x \in N_V(v_i)\} =gcd(2i-1, 2i+1) = 1$ and
gcd$\{f(e_i)/e_i \in N_E(v_i)\} = 1$ because $\{f(e_i)/e_i \in N_E(v_i)\}$ contains consecutive numbers.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 3.4. *The barycentric cycle graph is a vertex edge neighborhood prime labeling for all n.*

Proof. Let G be a barycentric cycle graph. Then
$V(G) = \{u_i, v_i : 1 \le i \le n\}$ and
$E(G) = \{e_i = u_iu_{i+1}, d_{2i} = v_iu_{i+1} : 1 \le i \le n-1\} \cup \{d_{2i-1} = u_iv_i : 1 \le i \le n\} \cup \{e_n = u_nu_1\} \cup \{d_{2n} = v_nu_1\}$.
Also, $|V(G)| = 2n$ and $|E(G)| = 3n$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,5n\}$ as follows.
For each $1 \le i \le n$, $f(u_i) = 2i-1, f(v_i) = 2i, f(d_{2i-1}) = f(u_iv_i) = p+n+2i-1$.
For each $1 \le i \le n-1$, $f(e_i) = f(u_iu_{i+1}) = p+i, f(d_{2i}) = f(v_iu_{i+1}) = p+n+2i$.
$f(e_n) = f(u_nu_1) = p+n, f(d_{2n}) = f(v_nu_1) = p+3n$.
We claim that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let b be any vertex of G.
Case 1. Let $b = u_i, 1 \le i \le n$ with $deg(b) \ge 2$.
In this case $\{f(x)/x \in N_V(u_i)\}$ contains consecutive integers and $\{f(e)/e \in N_E(u_i)\}$ also contains consecutive integers.
Case 2. Let $b = v_i, 1 \le i \le n$ with $deg(b) \ge 2$.
In this case $f(v_n)$ contains 1 and $\{f(x)/x \in N_V(v_i)\}$ contains $(2i-1, 2i+1), 1 \le i \le n-1$.
gcd $\{f(x)/x \in N_V(v_i) : 1 \le i \le n\} = 1$ and
$\{f(d_i)/d_i \in N_E(v_i)\}$ contains consecutive integers.
gcd$\{f(d_i)/d_i \in N_E(v_i)\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 3.5. *The convex polytope graph R_n is a vertex edge neighborhood prime labeling for all n.*

Proof. Let $G = R_n$ be a convex polytope graph. Then
$V(G) = \{u_i, v_i, w_i : 1 \le i \le n\}$ and
$E(G) = \{e_{2i-1} = u_iv_i, e'_i = v_iw_i : 1 \le i \le n\} \cup \{e_{2n} = u_1v_n\} \cup \{d'_n = w_nw_1\} \cup \{d_n = u_nu_1\} \cup \{d_i = u_iu_{i+1}, e_{2i} = u_{i+1}v_i, d'_i = w_iw_{i+1} : 1 \le i \le n-1\}$.
Also, $|V(G)| = 3n$ and $|E(G)| = 5n$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,8n\}$ as follows.
For each $1 \le i \le n$, $f(u_i) = 2n+i, f(v_i) = 2i-1, f(w_i) = 2i, f(e_{2i-1}) = f(u_iv_i) = 4n+2i-1, f(e'_i) = f(v_iw_i) = 7n+i$.
For each $1 \le i \le n-1$, $f(d_i) = f(u_iu_{i+1}) = 3n+i, f(e_{2i}) = f(u_{i+1}v_i) = 4n+2i, f(d'_i) = f(w_iw_{i+1}) = 6n+i$.
$f(d_n) = f(u_nu_1) = 4n, f(e_{2n}) = f(u_1v_n) = 6n, f(d'_n) = f(w_nw_1) = 7n$.
We claim that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let a be any vertex of G.
Case 1. If $a = u_i$, $1 \le i \le n$ with $deg(u_i) \ge 2$, then
$v_1 \in N_V(u_1), f(v_1) = 1$ and $\{f(x)/x \in N_V(u_i)\}$ contains $(2i-3, 2i-1, 2n+i-1, 2n+i+1), 2 \le i \le n-1$.
gcd$(2i-3, 2i-1, 2n+i-1, 2n+i+1) = 1$ and
$\{f(x)/x \in N_V(u_n)\}$ contains $(2i-3, 2i-1, 2n+1, 3n-1)$.
gcd$(2i-3, 2i-1, 2n+1, 3n-1) = 1$.
gcd$\{f(x)/x \in N_V(u_i) : 1 \le i \le n\}$ and $\{f(e_i)/e_i \in N_E(u_i)\}$ contains consecutive integers.
gcd$\{f(e_i)/e_i \in N_E(u_i)\} = 1$.
Case 2. If $x = v_i, 1 \le i \le n$ with $deg(v_i) \ge 2$, then
$v_1 \in N_V(v_n), f(v_1) = 1$ and $\{f(w_i)/w_i \in N_V(v_i) : 1 \le i \le n-1\}$ contains consecutive integers.
gcd$\{f(w_i)/w_i \in N_V(v_i) : 1 \le i \le n\} = 1$
and $\{f(e_i)/e_i \in N_E(v_i)\}$ contains consecutive integers.
gcd$\{f(e_i)/e_i \in N_E(v_i)\} = 1$.
Case 3. If $x = w_i, 1 \le i \le n$ with $deg(w_i) \ge 2$, then
$v_1 \in N_V(w_1), f(v_1) = 1$ and $\{f(x)/x \in N_V(w_i) : 2 \le i \le n\}$

contains consecutive integers.
$\gcd\{f(x)/x \in N_V(w_i) : 1 \leq i \leq n\} = 1$ and
$\{f(d_i)/d_i \in N_E(w_i)\}$
contains consecutive integers.
$\gcd\{f(d_i)/d_i \in N_E(w_i)\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 3.6. *The alternate triangular snake $A(Tn)$ admits a vertex edge neighborhood prime labeling for all* $n = 4,6,8,10,...$

Proof. Let $G = A(T_n)$ be an alternate triangular snake. Then
$V(G) = \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq \frac{n}{2} - 1\}$ and
$E(G) = \{e_i = u_iu_{i+1} : 1 \leq i \leq n-1\} \cup$
$\{d_{2i-1} = u_{2i}v_i, d_{2i} = v_iu_{2i+1} : 1 \leq i \leq \frac{n}{2} - 1\}$.
Also, $|V(G)| = \frac{3n}{2} - 1$ and $|E(G)| = 2n-3$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,..., \frac{7n}{2} - 4\}$ as follows.
$f(v_i) = 3i$ for $1 \leq i \leq \frac{n}{2} - 2$.
$f(u_{2i}) = 3i-2$ for $1 \leq i \leq \frac{n}{2}$.
$f(v_{\frac{n}{2}-1}) = \frac{3n}{2} - 1, f(u_1) = \frac{3n}{2}$, $f(e_{n-1}) = f(u_{n-1}u_n) = \frac{3n}{2} - 3$.
For each $1 \leq i \leq \frac{n}{2} - 1$, $f(u_{2i+1}) = 3i-1, f(e_{2i-1}) = f(u_{2i-1}u_{2i}) = \frac{3n}{2} + 4i - 3, f(e_{2i}) = f(u_{2i}u_{2i+1}) = \frac{3n}{2} + 4i - 2, f(d_{2i-1}) = f(u_{2ivi}) = \frac{3n}{2} + 4i, f(d_{2i}) = f(v_iu_{2i+1}) = \frac{3n}{2} + 4i - 1$.
In order to show that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let x be any vertex of G.
Case 1. Let $x = u_i, 2 \leq i \leq n-1$ with $deg(u_i) \geq 2$.
In this case $\{f(w)/w \in N_V(u_i)\}$ and $\{f(e_i)/e_i \in N_E(u_i)\}$ contains consecutive integers.
Case 2. Let $x = u_1, u_n$ with $deg(x) = 1$.
In both the case, $f(u_2) = 1$ and $f(u_{n-1}), f(e_{n-1})$ are consecutive integers.
$\gcd\{f(w), f(xw)/w \in N_V(x)\} = 1$.
Case 3. If $x = v_i, 1 \leq i \leq \frac{n}{2} - 1$ with $deg(x) = 2$, then $\{f(w)/w \in N_V(v_i)\} = (3i-2, 3i-1)$ which is two consecutive numbers.
$\gcd(3i-2, 3i-1) = 1$ and $\gcd\{f(d_i)/d_i \in N_E(v_i)\} = 1$ because $\{f(d_i)/d_i \in N_E(v_i)\}$ contains consecutive numbers.
Hence G is a vertex edge neighborhood prime labeling for all $n = 4,6,8,10,...$ □

Theorem 3.7. *The triangular Book $B_{3,n}$, where $n > 1$ is a vertex edge neighborhood prime labeling.*

Proof. Let $G = B_{3,n}$ be a triangular Book. Then
$V(G) = \{u_i : i = 1,2\} \cup \{v_i : 1 \leq i \leq n\}$ and
$E(G) = \{e' = u_1u_2\} \cup \{e_i = u_1v_i, d_i = u_2v_i : 1 \leq i \leq n\}$.
Also, $|V(G)| = n+2$ and $|E(G)| = 2n+1$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,3n+3\}$ as follows.
$f(u_1) = 1, f(v_1) = 2, f(u_2) = 3, f(e') = f(u_1u_2) = p+1$.
$f(v_i) = i+2$ for $2 \leq i \leq n$.
For each $1 \leq i \leq \lceil \frac{n}{2} \rceil$, $f(e_{2i-1}) = f(u_1v_{2i-1}) = p+4i-2$, $f(d_{2i-1}) = f(u_2v_{2i-1}) = p+4i-1$.
For each $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$, $f(e_{2i}) = f(u_1v_{2i}) = p+4i+1, f(d_{2i}) = f(u_2v_{2i}) = p+4i$.
By considering following cases we prove f is vertex edge neighborhood prime labeling. Let x be any vertex of G.
Case 1. Let $x = u_1, u_2$ with $deg(x) \geq 2$.
In this case $\{f(w) : w \in N_V(x)\}$ contains consecutive integers and $\{f(e) : e \in N_E(x)\}$ also contains consecutive integers.
Case 2. If $x = v_i$, where $1 \leq i \leq n$ with $deg(x) = 2$, then $\{f(w) : w \in N_V(v_i)\} = (f(u_1), f(u_2)) = (1,3)$.
$\gcd\{f(w) : w \in N_V(v_i)\} = 1$ and $\{f(d) : d \in N_E(v_i)\}$ contains consecutive integers.
$\gcd\{f(d) : d \in N_E(v_i)\} = 1$.
Hence G is a vertex edge neighborhood prime labeling. □

Theorem 3.8. *The rectangular book $B_{4,n}$ is a vertex edge neighborhood prime labeling for all n.*

Proof. Let $G = B_{4,n}$ be a rectangular book. Then
$V(G) = \{u_i, v_i : 1 \leq i \leq n+1\}$ and
$E(G) = \{e_i = u_1u_{i+1}, x_i = v_1v_{i+1} : 1 \leq i \leq n\} \cup$
$\{d_i = u_iv_i : 1 \leq i \leq n+1\}$.
Also, $|V(G)| = 2n+2$ and $|E(G)| = 3n+1$.
Define a bijective function $f : V(G) \cup E(G) \longrightarrow \{1,2,3,...,5n+3\}$ as follows.
For each $1 \leq i \leq n+1$, $f(u_i) = 2i-1, f(v_i) = 2i$.
$f(e_1) = f(u_1u_2) = p+1, f(d_2) = f(u_2v_2) = p+2$,
$f(x_1) = f(v_1v_2) = p+3, f(d_1) = f(u_1v_1) = p+4$.
For each $1 \leq i \leq n-1, f(e_{i+1}) = f(u_1u_{i+2}) = p+3i+2$,
$f(d_{i+2}) = f(u_{i+2}v_{i+2}) = p+3i+3, f(x_{i+1}) = f(v_1v_{i+2}) = p+3i+4$.
We claim that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let a be any vertex of G.
Case 1. If $a = u_i$, where $1 \leq i \leq n+1$ with $deg(a) \geq 2$, then $\gcd\{f(w) : w \in N_v(u_i)\} = 1$ because $u_1 \in N_V(u_i)$, $2 \leq i \leq n+1, f(u_1) = 1$ and $f(u_1)$ contains consecutive integers.
$\gcd\{f(d_i) : d_i \in N_E(u_i)\} = 1$ because $\{f(d_i) : d_i \in N_E(u_i)\}$ contains consecutive numbers.
Case 2. If $a = v_i$ where $1 \leq i \leq n+1$ with $deg(a) \geq 2$, then $f(v_1)$ contains 1 and $\{f(x) : x \in N_V(v_i)\} = (f(v_1), f(u_i)) = (2, 2i-1), 2 \leq i \leq n+1$.
$\gcd(2, 2i-1) = 1$.
$\gcd\{f(x) : x \in N_V(v_i)\} = 1$ and $\gcd\{f(e) : e \in N_E(v_i)\} = 1$ because $\{f(e) : e \in N_E(v_i)\}$ contains consecutive numbers.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 3.9. *The pentagonal book $B_{5,n}$ where $n > 1$ is a vertex edge neighborhood prime labeling.*

Proof. Let $G = B_{5,n}$ be a pentagonal book. Then $V(G) = \{u_i : i = 1,2\} \cup \{v_i, w_i, x_i : 1 \leq i \leq n\}$ and $E(G) = \{a = u_1u_2\} \cup \{e_i = u_iv_i, d_i = v_iw_i, d'_i = w_ix_i, e'_i = u_2x_i : 1 \leq i \leq n\}$.
Also, $|V(G)| = 3n+2$ and $|E(G)| = 4n+1$.
Define a bijective function $f : V(G) \cup E(G) \longrightarrow \{1,2,3,...,7n+3\}$ as follows.
$f(u_1) = 3, f(v_1) = 2, f(u_2) = 1, f(a) = f(u_1u_2) = p+1$.

$$f(w_i) = \begin{cases} 3i+1; & i \text{ is odd} \\ 3i+2; & i \text{ is even} \end{cases}$$

$$f(x_i) = \begin{cases} 3i+2; & i \text{ is odd} \\ 3i+1; & i \text{ is even} \end{cases}$$

$f(v_i) = 3i$ for $2 \leq i \leq n$.
For each $1 \leq i \leq \lceil \frac{n}{2} \rceil, f(e_{2i-1}) = f(u_1v_{2i-1}) = p+8i-6, f(d_{2i-1}) = f(v_{2i-1}w_{2i-1}) = p+8i-5, f(d'_{2i-1}) = f(w_{2i-1}x_{2i-1}) = p+8i-4, f(e'_{2i-1}) = f(u_2x_{2i-1}) = p+8i-3$.
For each $1 \leq i \leq \lfloor \frac{n}{2} \rfloor, f(e_{2i}) = f(u_1v_{2i}) = p+8i+1, f(d_{2i}) = f(v_{2i}w_{2i}) = p+8i, f(d'_{2i}) = f(w_{2i}x_{2i}) = p+8i-1, f(e'_{2i}) = f(u_2x_{2i}) = p+8i-2$.
We claim that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let a be any vertex of G.
Case 1. If $a = x_i, 1 \leq i \leq n$ with $deg(a) = 2$, then $\gcd\{f(w)/w \in N_V(x_i)\} = 1$ because $u_2 \in N_V(x_i), f(u_2) = 1$ and $\gcd\{f(d_i)/d_i \in N_E(x_i)\} = 1$ because $\{f(d_i)/d_i \in N_E(x_i)\}$ contains consecutive numbers.
Case 2. If $a = u_1$ with $deg(a) \geq 2$, then $\gcd\{f(x)/x \in N_V(u_1)\} = 1$ because $u_2 \in N_V(u_1), f(u_2) = 1$ and $\{f(d_i)/d_i \in N_E(u_1)\}$ contains consecutive numbers.
Case 3. If $a = u_2$ with $deg(a) \geq 2$, then $\gcd\ \{f(w)/w \in N_V(u_2)\} = \gcd\ (3, f(x_i)) = 1$ and $\gcd\{f(e)/e \in N_E(u_2)\}$ contains consecutive numbers.
Case 4. If $a = v_i, 1 \leq i \leq n$ with $deg(a) = 2$, then $\gcd\{f(x)/x \in N_V(v_i)\} = \gcd(3, f(w_i)) = 1$ and $\{f(d_i)/d_i \in N_E(v_i)\}$ contains consecutive numbers.
Case 5. If $a = w_i, 1 \leq i \leq n$ with $deg(a) = 2$, then $\gcd\{f(x)/x \in N_V(w_i)\} = \gcd(f(v_i), f(x_i)) = 1$ and $\{f(e)/e \in N_E(w_i)\}$ contains consecutive integers.
Hence G is a vertex edge neighborhood prime labeling. □

Theorem 3.10. *The quadrilateral snake Q_n admits vertex edge neighborhood prime labeling for all n.*

Proof. Let $G = Q_n$ be a quadrilateral snake. Then $V(G) = \{u_i : 1 \leq i \leq n\} \cup \{v_i, w_i : 1 \leq i \leq n-1\}$ and $E(G) = \{e_i = u_iu_{i+1}, d_i = u_iv_i, y_i = v_iw_i : 1 \leq i \leq n-1\} \cup \{x_i = w_iu_{i+1} : 1 \leq i \leq n-1\}$.
Also, $|V(G)| = 3n-2$ and $|E(G)| = 4n-4$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,7n-6\}$ as follows.
$f(u_i) = 3i-2$ for $1 \leq i \leq n$.
For each $1 \leq i \leq n-1, f(v_i) = 3i, f(w_i) = 3i-1, f(e_i) = f(u_iu_{i+1}) = p+4i-3, f(y_i) = f(v_iw_i) = p+4i-1$.
For each $1 \leq i \leq n-2, f(d_i) = f(u_iv_i) = p+4i-2, f(x_i) = f(w_iu_{i+1}) = p+4i$.
$f(d_{n-1}) = f(u_{n-1}v_{n-1}) = p+4n-4, f(x_{n-1}) = f(w_{n-1}u_n) = p+4n-6$.
By considering following cases we prove f is vertex edge neighborhood prime labeling. Let a be any vertex of G.
Case 1. $a = u_i$, $1 \leq i \leq n$, $deg(a) \geq 2$.
In this case $\{f(w)/w \in N_V(u_i)\}$ contains consecutive integers and $\{f(e)/e \in N_E(u_i)\}$ also contains consecutive integers.
Case 2. $a = v_i$, $1 \leq i \leq n-1$, $deg(a) = 2$. In this case $\{f(x)/x \in N_V(v_i)\}$ contains consecutive integers and $\{f(d)/d \in N_E(v_i)\}$ also contains consecutive integers.
Case 3. $a = w_i$, $1 \leq i \leq n-1$, $deg(a) = 2$.
Here, $\gcd\{f(x)/x \in N_V(w_i)\} = 1$ and $\gcd\{f(e)/e \in N_E(w_i)\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 3.11. *The alternate quadrilateral snake $A(Qn)$ admits a vertex edge neighborhood prime labeling for all n.*

Proof. Let $G = A(Q_n)$ be an alternate quadrilateral snake. Then $V(G) = \{u_i : 1 \leq i \leq n\} \cup \left\{v_i, w_i : 1 \leq i \leq \frac{n}{2} - 1\right\}$ and $E(G) = \{e_i = u_iu_{i+1}; 1 \leq i \leq n-1\} \cup \left\{d_{2i-1} = u_{2i}v_i, d_{2i} = u_{2i+1}w_i, e'_i = v_iw_i : 1 \leq i \leq \frac{n}{2} - 1\right\}$.
Also, $|V(G)| = 2n-2$ and $|E(G)| = \frac{5n}{2} - 4$.
Define a bijective function $f : V(G) \cup E(G) \to \left\{1,2,3,..., \frac{9n}{2} - 6\right\}$ as follows.
$f(u_1) = 2n-1, f(u_n) = 2n-2, f(e_{n-1}) = f(u_{n-1}u_n) = 2n-3$.
For each $1 \leq i \leq \frac{n}{2} - 1, f(u_{2i}) = 4i-3, f(u_{2i+1}) = 4i, f(v_i) = 4i-1, f(w_i) = 4i-2, f(e_{2i-1}) = f(u_{2i-1}u_{2i}) = 2n+5i-5, f(e_{2i}) = f(u_{2i}u_{2i+1}) = 2n+5i-4, f(d_{2i}) = f(u_{2i+1}w_i) = 2n+5i-3, f(e'_i) = f(v_iw_i) = 2n+5i-2, f(d_{2i-1}) = f(u_{2i}v_i) = 2n+5i-1$.
We claim that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let a be any vertex of G.
Case 1. If $a = u_1, u_n$ with $deg(a) = 1$, then $\gcd\{f(x), f(xa)/x \in N_V(a)\} = 1$ because $u_2 \in N_V(u_1), f(u_2) = 1$ and $f(u_{n-1}), f(e_{n-1}) = f(u_nu_{n-1})$ are consecutive integers.
Case 2. If $a = u_i, 2 \leq i \leq n-1$ with $deg(a) \geq 2$.
In this case $\{f(w)/w \in N_V(u_i)\}$ contains consecutive integers and $\{f(e)/e \in N_E(u_i)\}$ also contains consecutive integers.
Case 3. If $a = v_i, w_i, 1 \leq i \leq \frac{n}{2} - 1$ with $deg(a) = 2$, then $\gcd\{f(x)/x \in N_V(a)\} = 1$ and $\gcd\{f(d_i)/d_i \in N_E(a)\} = 1$ because $\{f(x)/x \in N_V(a)\}$ and $\{f(d_i)/d_i \in N_E(a)\}$ contains consecutive numbers.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 3.12. *A double triangular snake $D(T_n)$, where $n > 1$ is a vertex edge neighborhood prime labeling.*

Proof. Let $G = D(T_n)$ be a double triangular snake. Then
$V(G) = \{u_i : 1 \leq i \leq n\} \cup \{v_i, w_i : 1 \leq i \leq n-1\}$ and
$E(G) = \{e_i = u_iu_{i+1}, d_{2i-1} = u_iv_i : 1 \leq i \leq n-1\} \cup$
$\{d_{2i} = v_iu_{i+1}, x_{2i-1} = u_iw_i, x_{2i} = w_iu_{i+1} : 1 \leq i \leq n-1\}$.
Also, $|V(G)| = 3n-2$ and $|E(G)| = 5n-5$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,8n-7\}$ as follows.
$f(u_i) = 2i-1$ for $1 \leq i \leq n$.
For each $1 \leq i \leq n-1, f(v_i) = 2i, f(w_i) = 2n-1+i, f(e_i) = f(u_iu_{i+1}) = 3n+5i-2, f(d_{2i-1}) = f(u_iv_i) = 3n+5i-6, f(d_{2i}) = f(v_iu_{i+1}) = 3n+5i-5, f(x_{2i-1}) = f(u_iw_i) = 3n+5i-3, f(x_{2i}) = f(w_iu_{i+1}) = 3n+5i-4$.
We prove f is a vertex edge neighborhood prime labeling, by considering the following cases. Let a be any vertex of G.
Case 1. $a = v_i, w_i$, where $1 \leq i \leq n-1$. Here $deg(a) = 2$.
In both the cases $\{f(w)/w \in N_V(a)\} = (2i-1, 2i+1)$ which are consecutive integers.
gcd $\{f(w)/w \in N_V(a)\} = 1$ and $\{f(e)/e \in N_E(a)\}$ contains consecutive integers.
gcd $\{f(e)/e \in N_E(a)\} = 1$.
Case 2: $a = u_i$, where $1 \leq i \leq n$ with $deg(a) \geq 2$.
In this case $\gcd\{f(x)/x \in N_V(u_i)\} = 1$ and
$\gcd\{f(d_i)/d_i \in N_E(u_i)\} = 1$ because both the sets are consecutive integers.
Hence G is a vertex edge neighborhood prime labeling for all $n > 1$. □

Theorem 3.13. *The double alternate triangular snake $DA(T_n)$, where $n = 4,6,8,10,...$ admits a vertex edge neighborhood prime labeling.*

Proof. Let $G = DA(T_n)$ be a double alternate triangular snake. Then
$V(G) = \{u_i : 1 \leq i \leq n\} \cup \left\{v_i, w_i : 1 \leq i \leq \frac{n}{2}-1\right\}$ and
$E(G) = \{e_i = u_iu_{i+1} : 1 \leq i \leq n-1\} \cup$
$\left\{d_{2i-1} = u_{2i}v_i : 1 \leq i \leq \frac{n}{2}-1\right\} \cup$
$\left\{d_{2i} = v_iu_{2i+1}, x_{2i-1} = u_{2i}w_i, x_{2i} = w_iu_{2i+1} : 1 \leq i \leq \frac{n}{2}-1\right\}$.
Also, $|V(G)| = 2n-2$ and $|E(G)| = 3n-5$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,5n-7\}$ as follows.
$f(u_1) = 3, f(u_2) = 1, f(e_{n-1}) = f(u_{n-1}u_n) = 2n-2$.
$f(u_i) = 2i-1$ for $3 \leq i \leq n$.
For each $1 \leq i \leq \frac{n}{2}-1, f(v_i) = 2i, f(w_i) = n+2i-2, f(e_{2i-1}) = f(u_{2i-1}u_{2i}) = 2n+6i-6, f(x_{2i-1}) = f(u_{2i}w_i) = 2n+6i-5, f(x_{2i}) = f(u_{2i+1}w_i) = 2n+6i-4, f(d_{2i}) = f(u_{2i+1}v_i) = 2n+6i-3, f(d_{2i-1}) = f(u_{2i}v_i) = 2n+6i-2, f(e_{2i}) = f(u_{2i}u_{2i+1}) = 2n+6i-1$.
For proving f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let a be any vertex of G.
Case 1. $a = u_1, u_n$, here $deg(u_1) = deg(u_n) = 1$.
In this case $f(u_2) = 1$ and $f(u_{n-1}), f(e_{n-1})$ are consecutive integers.
gcd $\{f(w), f(aw)/w \in N_V(a)\} = 1$.
For $u_i \neq u_1, u_n$, where $deg(u_i) \geq 2$.
In this case $f(u_2), f(u_3)$ contains consecutive integers.
$N_V(u_i) = (2i-3, 2i+1), 4 \leq i \leq n-1$.
$\gcd(2i-3, 2i+1) = 1$ and $\{f(e)/e \in N_E(u_i)\}$ contains consecutive integers.
$\gcd\{f(x)/x \in N_V(u_i)\} = 1$ and $\gcd\{f(e)/e \in N_E(u_i)\} = 1$.
Case 2. For $a = v_1, w_1$ with $deg(a) = 2$.
Here $u_2 \in N_V(a), f(u_2) = 1$.
For $a = v_i, w_i$, where $2 \leq i \leq \frac{n}{2}-1$. Here $deg(a) = 2$.
Then $\{f(w)/w \in N_V(a)\} = (4i-1, 4i+1)$.
$\gcd(4i-1, 4i+1) = 1$.
$\gcd\{f(w)/w \in N_V(a)\} = 1$ and $\{f(e)/e \in N_E(a)\}$ contains consecutive integers.
$\gcd\{f(e)/e \in N_E(a)\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all $n = 4,6,8,10,...$ □

Theorem 3.14. *The graph $P(n,2) * K_1$, where $n > 4$ is a vertex edge neighborhood prime labeling.*

Proof. Let $G = P(n,2) * K_1$ be a graph. Then
$V(G) = \{u_i, v_i, w_i. : 1 \leq i \leq n\}$ and
$E(G) = \{x_i = u_iv_i, y_i = v_iw_i : 1 \leq i \leq n\} \cup \{e_{n-1} = u_{n-1}u_1\} \cup \{d_i = v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{e_i = u_iu_{i+2} : 1 \leq i \leq n-2\} \cup \{e_n = u_nu_2\} \cup \{d_n = v_nv_1\}$.
Also, $|V(G)| = 3n$ and $|E(G)| = 4n$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,7n\}$ as follows.
For each $1 \leq i \leq n$, $f(w_i) = 2i, f(v_i) = 2n+2i-1$,
$f(y_i) = f(v_iw_i) = 2n+2i, f(x_i) = f(u_iv_i) = 5n+i$.
For each $1 \leq i \leq n-1, f(u_{n+1-i}) = 2i+1, f(d_i) = f(v_iv_{i+1}) = 4n+1+i$.
$f(u_1) = 1, f(d_n) = f(v_nv_1) = 4n+1$.
Case 1. n is odd,
$f(e_i) = f(u_iu_{i+2}) = 6n+i$ for $1 \leq i \leq n-2$.
$f(e_{n-1}) = f(u_{n-1}u_1) = 7n-1, f(e_n) = f(u_nu_2) = 7n$.
Case 2. n is even,
$f(e_i) = f(u_iu_{i+2}) = 6n+i-1$ for $2 \leq i \leq n-2$.
$f(e_{n-1}) = f(u_{n-1}u_1) = 7n-2, f(e_n) = f(u_nu_2) = 7n-1, f(e_1) = f(u_1u_3) = 7n$.
We claim that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let x be any vertex of G.
For $x = w_i, 1 \leq i \leq n$ with $deg(x) = 1$.
In this case $f(v_i), f(v_iw_i)$ are consecutive integers.
$\gcd\{f(v_i), f(v_iw_i)/v_i \in N_V(w_i)\} = 1$.
For $x = v_i, 1 \leq i \leq n$ with $deg(x) \geq 2$, then $\{f(x)/x \in N_V(v_i)\}$ contains $(2i, 2n+3-2i, 2n+2i-3, 2n+2i+1)$,
$2 \leq i \leq n-1$.
$\gcd(2i, 2n+3-2i, 2n+2i-3, 2n+2i+1) = 1$ and $f(v_1), f(v_n)$ contains consecutive integers.
$\gcd\{f(x)/x \in N_V(v_i) : 1 \leq i \leq n\} = 1$ and $\{f(e)/e \in N_E(v_i)\}$ contains consecutive integers.
$\gcd\{f(e)/e \in N_E(v_i)\} = 1$
For $x = u_i, 1 \leq i \leq n$ with $deg(x) \geq 2$.
$\gcd\{f(x)/x \in N_V(u_i)\} = 1$, $\gcd\{f(e)/e \in N_E(u_i)\} = 1$.
Hence G is a vertex edge neighborhood prime labeling. □

Theorem 3.15. *The graph $(C_n \times K_2) * K_1$ is a vertex edge* *neighborhood prime labeling for all n.*

Proof. Let $G = (C_n \times K_2) * K_1$ be a graph. Then
$V(G) = \{u_i, v_i, w_i : 1 \leq i \leq n\}$ and
$E(G) = \{x_i = u_i v_i, y_i = v_i w_i : 1 \leq i \leq n\} \cup \{e_n = u_n u_1\}$
$\cup \{e_i = u_i u_{i+1}, d_i = v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{d_n = v_n v_1\}$.
Also, $|V(G)| = 3n$ and $|E(G)| = 4n$.
Define a bijective function $f : V(G) \cup E(G) \to \{1, 2, 3, ..., 7n\}$ as follows.
For each $1 \leq i \leq n$, $f(u_i) = 2i-1, f(w_i) = 2i, f(v_i) = 2n + 2i - 1, f(y_i) = f(v_i w_i) = 2n + 2i, f(x_i) = f(u_i v_i) = 5n + i$.
For each $1 \leq i \leq n-1$, $f(d_i) = f(v_i v_{i+1}) = 4n + 1 + i$,
$f(e_i) = f(u_i u_{i+1}) = 6n + 1 + i$.
$f(e_n) = f(u_n u_1) = 6n + 1, f(d_n) = f(v_n v_1) = 4n + 1$.
We claim that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let b be any vertex of G.
Case 1. If $b = w_i$, $1 \leq i \leq n$ with $deg(b) = 1$, then
$\{f(v_i), f(v_i w_i)/v_i \in N_V(w_i)\}$ contains $(2n + 2i - 1, 2n + 2i)$ which is two consecutive integers.
$\gcd\{f(v_i), f(v_i w_i)/v_i \in N_V(w_i)\} = 1$.
Case 2. Let $b = v_i, 1 \leq i \leq n$ with $deg(b) \geq 2$.
In this case $\{f(x)/x \in N_V(v_i)\}$ and $\{f(e)/e \in N_E(v_i)\}$ contains consecutive integers.
Case 3. Let $b = u_i, 1 \leq i \leq n$ with $deg(b) \geq 2$.
In this case $u_1 \in N_V(u_n), f(u_1) = 1$ and $f(u_1)$ contains $(2n + 2i - 1, 3, 2n - 1)$ and $\{f(x)/x \in N_V(u_i)\}$ contains $(2n + 2i - 1, 2i + 1, 2i - 3), 2 \leq i \leq n - 1$.
$\gcd\{f(x)/x \in N_V(u_i) : 1 \leq i \leq n\} = 1$ and
$\{f(d_i)/d_i \in N_E(u_i)\}$ contains consecutive integers.
$\gcd\{f(d_i)/d_i \in N_E(u_i) : 1 \leq i \leq n\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 3.16. *The graph $T_n * K_1$ is a vertex edge neighborhood prime labeling for all $n > 3$.*

Proof. Let $G = T_n * K_1$ be a graph. Then
$V(G) = \{u_i : 1 \leq i \leq n\} \cup \{v_i, w_i : 1 \leq i \leq n-3\}$ and
$E(G) = \{e_i = u_i u_{i+1} : 1 \leq i \leq n-1\} \cup$
$\{x_{2i-1} = u_{i+1} v_i, x_{2i} = v_i u_{i+2}, d_i = v_i w_i : 1 \leq i \leq n-3\}$.
Also, $|V(G)| = 3n - 6$ and $|E(G)| = 4n - 10$.
Define a bijective function $f : V(G) \cup E(G) \to \{1, 2, 3, ..., 7n - 16\}$ as follows.
$f(u_{i+1}) = 2i - 1$ for $1 \leq i \leq n-1$.
For each $1 \leq i \leq n-3, f(v_i) = 2n + 2i - 3, f(d_i) = f(v_i w_i) = 2n + 2i - 4, f(e_{i+1}) = f(u_{i+1} u_{i+2}) = 4n + 3i - 9, f(x_{2i-1}) = f(u_{i+1} v_i) = 4n + 3i - 7, f(x_{2i}) = f(v_i u_{i+2}) = 4n + 3i - 8$.
$f(w_i) = 2i$ for $2 \leq i \leq n-3$.
$f(w_1) = 4n - 8, f(u_1) = 2, f(e_1) = f(u_1 u_2) = 4n - 7$,
$f(e_{n-1}) = f(u_{n-1} u_n) = 2n - 4$.
In order to show that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let x be any vertex of G.
Case 1. Let $x = w_i$, $1 \leq i \leq n-3$ with $deg(x) = 1$.
In this case $\{f(v_i), f(d_i) = f(v_i w_i)/v_i \in N_V(w_i)\} = (2n + 2i - 3, 2n + 2i - 4)$ which are consecutive integers.
$\gcd\{f(v_i), f(v_i w_i)/v_i \in N_V(w_i)\} = 1$.
Case 2. Let $x = v_i$, $1 \leq i \leq n-3$. Here $deg(x) \geq 2$.
In this case $\{f(d)/d \in N_V(v_i)\}$ and $\{f(e)/e \in N_E(v_i)\}$ contains consecutive integers.
Case 3. For $x = u_1, u_n$. Here $deg(u_1) = deg(u_n) = 1$.
In both the cases $f(u_2) = 1$ and $f(u_{n-1}), f(u_{n-1} u_n)$ are consecutive integers.
$\gcd\{f(d), f(xd)/d \in N_V(x)\} = 1$.
For $u_i \neq u_1, u_n$ with $deg(u_i) \geq 2$. $N_V(u_i) = (2n - 5 + 2i, 2n - 7 + 2i, 2i - 5, 2i - 1), 3 \leq i \leq n-2$ and $f(u_2)$ contains consecutive integers. $f(u_{n-1})$ contains $(2n - 3, 2n - 7, 4n - 9)$.
gcd $(2n - 3, 2n - 7, 4n - 9) = 1$.
$\gcd\{f(d)/d \in N_V(u_i) : 2 \leq i \leq n-1\}$ and
$\{f(e)/e \in N_E(u_i) : 2 \leq i \leq n-1\}$
contains consecutive integers.
$\gcd\{f(e)/e \in N_E(u_i)\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all $n > 3$. □

Theorem 3.17. *The graph $R_n * K_1$ is a vertex edge neighborhood prime labeling for all n.*

Proof. Let $G = R_n * K_1$ be a graph. Then
$V(G) = \{u_i, v_i, w_i, x_i : 1 \leq i \leq n\}$ and
$E(G) = \{d_{2i-1} = u_i v_i, e'_i = w_i x_i : 1 \leq i \leq n\} \cup$
$\{d_{2i} = u_{i+1} v_i, e_i = w_i w_{i+1}, c'_i = v_i v_{i+1} : 1 \leq i \leq n-1\} \cup$
$\{d_{2n} = u_1 v_n\} \cup \{c_i = u_i u_{i+1} : 1 \leq i \leq n-1\} \cup$
$\{c_n = u_n u_1\} \cup \{e_n = w_n w_1\} \cup \{c'_n = v_n v_1\}$.
Also, $|V(G)| = 4n$ and $|E(G)| = 7n$.
Define a bijective function $f : V(G) \cup E(G) \to \{1, 2, 3, ..., 11n\}$ as follows.
For each $1 \leq i \leq n, f(u_i) = 2i, f(v_i) = 2i - 1, f(w_i) = 2n + 2i - 1, f(x_i) = 4n + i, f(e'_i) = 2n + 2i, f(d_{2i-1}) = f(u_i v_i) = 6n + 2i - 1, f(d'_i) = f(v_i w_i) = 10n + i$.
For each $1 \leq i \leq n-1, f(c_i) = f(u_i u_{i+1}) = 5n + i$,
$f(d_{2i}) = f(u_{i+1} v_i) = 6n + 2i, f(c'_i) = f(v_i v_{i+1}) = 8n + i, f(e_i) = f(w_i w_{i+1}) = 9n + i$.
$f(c_n) = f(u_n u_1) = 6n, f(d_{2n}) = f(u_1 v_n) = 8n, f(c'_n) = f(v_n v_1) = 9n, f(e_n) = f(w_n w_1) = 10n$.
By considering following cases we prove f is a vertex edge neighborhood prime labeling. Let a be any vertex of G.
Case 1. Let $a = u_i$, $1 \leq i \leq n$ with $deg(u_i) \geq 2$.
In this case $\{f(w)/w \in N_V(u_i)\}$ contains consecutive integers and $\{f(e)/e \in N_E(u_i)\}$ also contains consecutive integers.
Case 2. If $a = v_i$, $1 \leq i \leq n$ with $deg(v_i) \geq 2$, then
$v_1 \in N_V(v_n), f(v_1) = 1$ and $\{f(w)/w \in N_V(v_i) : 1 \leq i \leq n-1\}$ contains consecutive integers.
$\gcd\{f(w)/w \in N_V(v_i) : 1 \leq i \leq n\} = 1$ and $\{f(e)/e \in N_E(v_i)\}$ also contains consecutive integers.
$\gcd\{f(e)/e \in N_E(v_i)\} = 1$.
Case 3. If $a = w_i, 1 \leq i \leq n$ with $deg(w_i) \geq 2$, then
$v_1 \in N_V(w_1), f(v_1) = 1$ and $\{f(x)/x \in N_V(w_i) : 2 \leq i \leq n-1\}$ contains $(f(v_i), f(x_i), f(w_{i-1}), f(w_{i+1})) = (2i - 1, 4n + i, 2n + 2i - 3, 2n + 2i + 1)$.
$\gcd(2i - 1, 4n + i, 2n + 2i - 3, 2n + 2i + 1) = 1$ and

$\{f(x)/x \in N_V(w_n)\}$ contains $(f(v_n), f(x_n), f(w_{n-1}), f(w_1))$
$= (2i-1, 4n+i, 2n+2i-3, 2n+1)$.
$\gcd(2i-1, 4n+i, 2n+2i-3, 2n+1) = 1$.
$\gcd\{f(x)/x \in N_V(w_i) : 1 \le i \le n\} = 1$ and
$\{f(d_i)/d_i \in N_E(w_i)\}$ contains consecutive integers.
$\gcd\{f(d_i)/d_i \in N_E(w_i)\} = 1$.
Case 4. Let $a = x_i$, $1 \le i \le n$ with $deg(x_i) = 1$.
In this case $f(e'_i) = f(w_i x_i)$ and $f(w_i)$ contains consecutive integers.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 3.18. *The barycentric cycle attached by pendant edge of a graph is a vertex edge neighborhood prime labeling for all n.*

Proof. Let G be a barycentric cycle attached by pendant edge of a graph. Then
$V(G) = \{u_i, v_i, w_i : 1 \le i \le n\}$ and
$E(G) = \{e_i = u_i u_{i+1}, d_{2i} = v_i u_{i+1} : 1 \le i \le n-1\} \cup \{e_n = u_n u_1\} \cup \{d_{2i-1} = u_i v_i, x_i = v_i w_i : 1 \le i \le n\} \cup \{d_{2n} = v_n u_1\}$.
Also, $|V(G)| = 3n$ and $|E(G)| = 4n$.
Define a bijective function $f : V(G) \cup E(G) \to \{1, 2, 3, ..., 7n\}$ as follows.
For each $1 \le i \le n$, $f(u_i) = 2i-1, f(w_i) = 2i, f(v_i) = 2n+2i-1, f(x_i) = f(v_i w_i) = 2n+2i, f(d_{2i-1}) = f(u_i v_i) = 4n+2i-1$.
For each $1 \le i \le n-1, f(d_{2i}) = f(v_i u_{i+1}) = 4n+2i, f(e_i) = f(u_i u_{i+1}) = 6n+i$.
$f(d_{2n}) = f(v_n u_1) = 6n, f(e_n) = f(u_n u_1) = 7n$.
For proving f is a vertex edge neighborhood prime labeling, we consider the following cases. Let a be any vertex of G.
Case 1. If $a = w_i$, $1 \le i \le n$ with $deg(a) = 1$, then
$\{f(x_i), f(w_i x_i)/x_i \in N_V(w_i)\}$
$= (2n+2i-1, 2n+2i), 1 \le i \le n$ which is two consecutive integers.
$\gcd(2n+2i-1, 2n+2i) = 1$.
Case 2. Let $a = v_i$, $1 \le i \le n$ with $deg(a) \ge 2$.
In this case $\{f(x)/x \in N_V(v_i)\}$ contains consecutive integers and $\{f(e)/e \in N_E(v_i)\}$ also contains consecutive integers.
Case 3. Let $a = u_i$, $1 \le i \le n$ with $deg(a) \ge 2$.
In this case $f(u_n)$
contains 1 and $\{f(x)/x \in N_V(u_1)\} = \{2n+2i-1, 3, 4n-1, 2n-1\}$ and $\{f(x)/x \in N_V(u_i)\}$ contains $(2i-3, 2i+1, 2n+2i-3, 2n+2i-1), 2 \le i \le n-1$.
$\gcd\{f(x)/x \in N_V(u_i)\} = 1$ and $\{f(e)/e \in N_E(u_i)\}$ contains consecutive integers.
$\gcd\{f(e)/e \in N_E(u_i)\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all n. □

4. m fold pedal graphs.

In this section, vertex edge neighborhood prime labeling of m fold petal types of graphs.

Theorem 4.1. *The m fold petal Petersen graph $P(n,2)$, where $n \ge 5$ is a vertex edge neighborhood prime labeling.*

Proof. Let $G = P(n,2)$ be a m fold petal Petersen graph. Then
$V(G) = \{u_j, v_j : 1 \le j \le n\} \cup \{c_{i,j} : 1 \le i \le m, 1 \le j \le n\}$ and
$E(G) = \{e_j = u_j u_{j+2} : 1 \le j \le n-2\} \cup \{e_{n-1} = u_{n-1} u_1\} \cup \{d_j = u_j v_j : 1 \le j \le n\} \cup \left\{e'_j = v_j v_{j+1} : 1 \le j \le n-1\right\} \cup \{e_n = u_n u_2\} \cup \{e_{i,2j-1} = v_j c_{i,j} : 1 \le i \le m, 1 \le j \le n\} \cup \{e_{i,2n} = v_1 c_{i,n} : 1 \le i \le m\} \cup \{e'_n = v_n v_1\} \cup \{e_{i,2j} = v_{j+1} c_{i,j} : 1 \le i \le m, 1 \le j \le n-1\}$.
Also, $|V(G)| = n(m+2)$ and $|E(G)| = n(2m+3)$.
Define a bijective function $f : V(G) \cup E(G) \to \{1, 2, 3, ..., n(3m+5)\}$ as follows.
For each $1 \le j \le n$, $f(v_j) = 2j-1, f(u_j) = 2j, f(d_j) = f(u_j v_j) = p+n+j$.
For each $1 \le i \le m$ and $1 \le j \le n$, $f(c_{i,j}) = (i+1)n+j$,
$f(e_{i,2j-1}) = f(v_j c_{i,j}) = p+(2i+1)n+2j-1, f(e_{i,2j}) = f(v_{j+1} c_{i,j}) = p+(2i+1)n+2j$.
$f(e'_j) = f(v_j v_{j+1}) = p+2n+j$ for $1 \le j \le n-1$.
$f(e'_n) = f(v_n v_1) = p+3n, f(e_{i,2n}) = f(v_1 c_{i,n}) = p+n(2i+3)$.
Case 1. n is odd,
$f(e_j) = f(u_j u_{j+2}) = p+j$ for $1 \le j \le n-2$.
$f(e_{n-1}) = f(u_{n-1} u_1) = p+n-1, f(e_n) = f(u_n u_2) = p+n$.
Case 2. n is even,
$f(e_j) = f(u_j u_{j+2}) = p+j-1$ for $2 \le j \le n-2$.
$f(e_{n-1}) = f(u_{n-1} u_1) = p+n-2, f(e_n) = f(u_n u_2) = p+n-1, f(e_1) = f(u_1 u_3) = p+n$.
We claim that f is a vertex edge neighborhood prime labeling.
Let x be any vertex of G.
For $x = u_i, 1 \le i \le n$ with $deg(x) \ge 2$.
Here, $\gcd\{f(w)/w \in N_V(x)\} = 1$ and $\gcd\{f(e_i)/e_i \in N_E(x)\} = 1$.
For $x = v_i, 1 \le i \le n$ with $deg(v_i) \ge 2$.
Here, $v_1 \in N_V(v_n), f(v_1) = 1$
and $\{f(w_i)/w_i \in N_V(v_i) : 1 \le i \le n-1\}$ contains consecutive integers and $\{f(d_i)/d_i \in N_E(v_i)\}$ also contains consecutive integers.
$\gcd\{f(w_i)/w_i \in N_V(v_i) : 1 \le i \le n\} = 1$ and
$\gcd\{f(d_i)/d_i \in N_E(v_i)\} = 1$.
For $x = c_{i,j}$, where $1 \le i \le m$ and $1 \le j \le n$ with $deg(x) = 2$.
$v_1 \in N_V(c_{i,n}), f(v_1) = 1$ and $\{f(w_i)/w_i \in N_V(c_{i,j})\}$ contains $(2i-1, 2i+1)$, where $1 \le i \le m$ and $1 \le j \le n-1$.
$\gcd(2i-1, 2i+1) = 1$.
$\gcd\{f(w_i)/w_i \in N_V(c_{i,j})\}$ and $\{f(e_i)/e_i \in N_E(c_{i,j})\}$ contains consecutive integers.
$\gcd\{f(e_i)/e_i \in N_E(c_{i,j})\} = 1$.
Hence G is a vertex edge neighborhood prime labeling. □

Theorem 4.2. *The m fold petal prism is a vertex edge neighborhood prime labeling for all n.*

Proof. Let $G = C_n \times K_2$ be a m fold petal prism. Then
$V(G) = \{u_j, v_j : 1 \le j \le n\} \cup \{c_{i,j} : 1 \le i \le m, 1 \le j \le n\}$ and
$E(G) = \{d_j = u_j u_{j+1}, e_j = v_j v_{j+1} : 1 \le j \le n-1\} \cup$

$\{e_n = v_nv_1\} \cup \{e_{i,j} = u_jc_{i,j} : 1 \leq i \leq m, 1 \leq j \leq n\} \cup \left\{e'_{i,j} = u_{j+1}c_{i,j} : 1 \leq i \leq m, 1 \leq j \leq n-1\right\} \cup \{d_n = u_nu_1\} \cup \left\{e'_{i,n} = u_1c_{i,n} : 1 \leq i \leq m\right\} \cup \left\{e'_j = u_jv_j : 1 \leq j \leq n\right\}$.
Also, $|V(G)| = n(m+2)$ and $|E(G)| = (2m+3)n$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,(3m+5)n\}$ as follows.
For each $1 \leq j \leq n$, $f(u_j) = 2j-1, f(v_j) = 2j, f(e'_j) = f(u_jv_j) = p+n+j, f(d_j) = f(u_ju_{j+1}) = p+2n+j$.
For each $1 \leq i \leq m$ and $1 \leq j \leq n$, $f(c_{i,j}) = (i+1)n+j$,
$f(e_{i,j}) = f(u_jc_{i,j}) = p+(2i+1)n+2j-1$.
$f(e_n) = f(v_nv_1) = p+n$.
$f(e_j) = f(v_jv_{j+1}) = p+j$ for $1 \leq j \leq n-1$.
$f(e'_{i,j}) = f(u_{j+1}c_{i,j}) = p+(2i+1)n+2j$ for $1 \leq i \leq m$ and $1 \leq j \leq n-1$.
$f(e'_{i,n}) = f(u_1c_{i,n}) = p+(2i+3)n$ for $1 \leq i \leq m$.
We claim that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let x be any vertex of G.
Case 1. If $x = u_i, 1 \leq i \leq n$ with $deg(u_i) \geq 2$, then $\{f(w)/w \in N_V(u_i)\}$ and $\{f(e)/e \in N_E(u_i)\}$ contains consecutive integers.
Case 2. If $x = v_i, 1 \leq i \leq n$ with $deg(v_i) \geq 2$.
In this case $\gcd\{f(w_i)/w_i \in N_V(v_i)\} = 1$ because $u_1 \in N_V(v_1)$, $f(u_1) = 1$ and $\{f(w_i)/w_i \in N_V(u_i) : 2 \leq i \leq n\}$ contains consecutive integers and $\gcd\{f(e_i)/e_i \in N_E(v_i)\} = 1$ because $\{f(e_i)/e_i \in N_E(v_i)\}$ contains consecutive integers.
Case3. If $x = c_{i,j}$, where $1 \leq i \leq m$ and $1 \leq j \leq n$ with $deg(x) = 2$, then $\{f(w_i)/w_i \in N_V(x)\}$ and $\{f(d_i)/d_i \in N_E(x)\}$ contains consecutive integers.
$\gcd\{f(w_i)/w_i \in N_V(x)\} = 1$ and $\gcd\{f(d_i)/d_i \in N_E(x)\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 4.3. *The m fold petal triangular snake is a vertex edge neighborhood prime labeling for all n.*

Proof. Let $G = T_n$ be a m fold petal triangular snake. Then $V(G) = \{u_j : 1 \leq j \leq n\} \cup \{c_{i,j} : 1 \leq i \leq m, 1 \leq j \leq n-1\}$ and
$E(G) = \{e_j = u_ju_{j+1} : 1 \leq j \leq n-1\} \cup \left\{e_{i,j} = u_jc_{i,j}, e'_{i,j} = u_{j+1}c_{i,j} : 1 \leq i \leq m, 1 \leq j \leq n-1\right\}$
Also, $|V(G)| = n+m(n-1)$ and $|E(G)| = (n-1)(2m+1)$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,n(3m+2)-(3m+1)\}$ as follows.
$f(u_j) = 2j-1$ for $1 \leq j \leq n$.
For each $1 \leq j \leq n-1$ and $i = 1$, $f(c_{i,j}) = 2j$,
$f(e_j) = f(u_ju_{j+1}) = p+3j-2$,
$f(e_{i,n-1}) = f(u_{n-1}c_{i,n-1}) = p+3n-4$,
$f(e'_{i,n-1}) = f(u_nc_{i,n-1}) = p+3n-3$.
For each $1 \leq j \leq n-2$ and $i = 1$, $f(e_{i,j}) = f(u_jc_{i,j}) = p+3j-1, f(e'_{i,j}) = f(u_{j+1}c_{i,j}) = p+3j$.
For each $1 \leq i \leq m$ and $1 \leq j \leq n-1$, $f(c_{i,j}) = i(n-1)+1+j, f(e_{i,j}) = f(u_ic_{i,j}) = p+(2i-1)(n-2)+2(i-1)+2j, f(e'_{i,j}) = f(u_{j+1}c_{i,j}) = p+(2i-1)(n-2)+(2i-1)+2j$.
We have to prove f is a vertex edge neighborhood prime labeling. We consider the following cases. Let x be any vertex of G.
Case 1. Let $x = u_i, 1 \leq i \leq n$ with $deg(x) \geq 2$.
In this case $\{f(x)/x \in N_V(u_i)\}$ and $\{f(e)/e \in N_E(u_i)\}$ contains consecutive integers.
Case 2. Let $x = c_{i,j}$, where $1 \leq i \leq m$ and $1 \leq j \leq n-1$. Here, $deg(x) = 2$. Then $\{f(w)/w \in N_V(c_{i,j})\}$ contains $(2i-1, 2i+1), 1 \leq i \leq n-1$.
$\gcd(2i-1, 2i+1) = 1$.
$\gcd\{f(w)/w \in N_V(c_{i,j})\} = 1$ and $\{f(d_i)/d_i \in N_E(c_{i,j})\}$ contains consecutive integers.
$\gcd\{f(d_i)/d_i \in N_E(c_{i,j})\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 4.4. *The m fold petal barycentric cycle graph is a vertex edge neighborhood prime labeling for all n.*

Proof. Let G be a m fold petal barycentric cycle graph. Then $V(G) = \{u_j : 1 \leq j \leq 2n\} \cup \{c_{i,j} : 1 \leq i \leq m, 1 \leq j \leq 2n\}$ and $E(G) = \{e_{i,2j-1} = u_jc_{i,j} : 1 \leq i \leq m, 1 \leq j \leq 2n\} \cup \{d_j = u_{2j-1}u_{2j+1} : 1 \leq j \leq n-1\} \cup \{d_n = u_{n-1}u_1\} \cup \{e_{2n} = u_nu_1\} \cup \{e_j = u_ju_{j+1} : 1 \leq j \leq 2n-1\} \cup \{e_{i,4n} = u_1c_{i,2n} : 1 \leq i \leq m\} \cup \{e_{i,2j} = u_{j+1}c_{i,j} : 1 \leq i \leq m, 1 \leq j \leq 2n-1\}$.
Also, $|V(G)| = 2n(m+1)$ and $|E(G)| = (3+4m)n$.
Define a bijective function $f : V(G) \cup E(G) \to \{1,2,3,...,(6m+5)n\}$ as follows.
For each $1 \leq j \leq n$, $f(u_{2j-1}) = 2j-1, f(u_{2j}) = 2j$.
For each $1 \leq i \leq m$ and $1 \leq j \leq 2n$, $f(c_{i,j}) = 2in+j$,
$f(e_{i,2j-1}) = f(u_jc_{i,j}) = p+(4i-1)n+2j-1$.
$f(d_j) = f(u_{2j-1}u_{2j+1}) = p+j$ for $1 \leq j \leq n-1$.
$f(e_j) = f(u_ju_{j+1}) = p+n+j$ for $1 \leq j \leq 2n-1$.
$f(d_n) = f(u_{n-1}u_1) = p+n, f(e_{2n}) = f(u_nu_1) = p+3n$.
$f(e_{i,2j}) = f(u_{j+1}c_{i,j}) = p+(4i-1)n+2j$ for $1 \leq i \leq m$ and $1 \leq j \leq 2n-1$.
$f(e_{i,4n}) = f(u_1c_{i,2n}) = p+(4i+3)n$ for $1 \leq i \leq m$.
We claim that f is a vertex edge neighborhood prime labeling, for that we consider the following cases. Let x be any vertex of G.
Case 1. Let $x = u_i, 1 \leq i \leq 2n$ with $deg(u_i) \geq 2$.
In this case $\{f(w)/w \in N_V(u_i)\}$ and $\{f(e)/e \in N_E(u_i)\}$ contains consecutive integers.
Case 2. If $x = c_{i,j}$, where $1 \leq i \leq m$ and $1 \leq j \leq 2n$ with $deg(x) = 2$, then $\{f(w)/w \in N_V(x)\}$ contains $(j, j+1), 1 \leq i \leq m$ and $1 \leq j \leq 2n-1$.
$\gcd(j, j+1) = 1$ and $u_1 \in N_V(c_{i,2n}), f(u_1) = 1$.
$\gcd\{f(w)/w \in N_V(x)\} = 1$ and $\{f(e_i)/e_i \in N_E(x)\}$ contains consecutive integers.
$\gcd\{f(e_i)/e_i \in N_E(x)\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 4.5. *The m fold petal convex polytope graph R_n is* *a vertex edge neighborhood prime labeling for all n.*

Proof. Let $G = R_n$ be a m fold petal convex polytope graph. Then
$V(G) = \{u_j, v_j, w_j : 1 \le j \le n\} \cup \{x_{i,j} : 1 \le i \le m, 1 \le j \le n\}$
and
$E(G) = \left\{e_{2j-1} = u_jv_j, d''_j = v_jw_j : 1 \le j \le n\right\} \cup \{d_n = u_1u_n\}$
$\cup \{e_{2n} = u_1v_n\} \cup \{d'_n = v_1v_n\} \cup \{d_j = u_ju_{j+1} : 1 \le j \le n-1\}$
$\cup \left\{e_{2j} = u_{j+1}v_j, d'_j = v_jv_{j+1}, e'_j = w_jw_{j+1} : 1 \le j \le n-1\right\}$
$\cup \{e'_n = w_1w_n\} \cup \{e_{i,2j-1} = w_jx_{i,j} : 1 \le i \le m, 1 \le j \le n\}$
$\cup \{e_{i,2j} = w_{j+1}x_{i,j} : 1 \le i \le m, 1 \le j \le n-1\} \cup$
$\{e_{i,2n} = w_1x_{i,n} : 1 \le i \le m\}$.
Also, $|V(G)| = n(m+3)$ and $|E(G)| = 2n(m+3)$.
Define a bijective function $f : V(G) \cup E(G) \to$
$\{1,2,3,...,3n(m+3)\}$ as follows.
For each $1 \le j \le n$, $f(u_j) = 2j, f(v_j) = 2n+j, f(w_j) =$
$2j-1, f(e_{2j-1}) = f(u_jv_j) = p+n+2j-1, f(d''_j) =$
$f(v_jw_j) = p+5n+j$.
For each $1 \le j \le n-1$, $f(d_j) = f(u_ju_{j+1}) = p+j, f(e_{2j}) =$
$f(u_{j+1}v_j) = p+n+2j, f(d'_j) = f(v_jv_{j+1}) = p+3n+j,$
$f(e'_j) = f(w_jw_{j+1}) = p+4n+j$.
For each $1 \le i \le m$ and $1 \le j \le n$, $f(x_{i,j}) = (i+2)n+j,$
$f(e_{i,2j-1}) = p+(2i+4)n+2j-1$.
$f(d_n) = f(u_1u_n) = p+n, f(e_{2n}) = f(u_1v_n) = p+3n,$
$f(d'_n) = f(v_1v_n) = p+4n, f(e'_n) = f(w_1w_n) = p+5n$.
$f(e_{i,2j}) = p+(2i+4)n+2j$ for $1 \le i \le m$ and $1 \le j \le n-1$.
$f(e_{i,2n}) = p+2n(i+3)$ for $1 \le i \le m$.
By considering following cases we prove f is a vertex edge neighborhood prime labeling. Let a be any vertex of G.
Case 1. Let $b = u_i, v_i$, where $1 \le i \le n$ with $deg(b) \ge 2$.
In this case $\{f(w)/w \in N_V(b)\}$ contains consecutive integers and $\{f(e)/e \in N_E(b)\}$ also contains consecutive integers.
Case 2. Let $b = w_i, 1 \le i \le n$ with $deg(w_i) = 1$. Here,
$\gcd\{f(x)/x \in N_V(w_i)\} = 1$ and $\gcd\{f(e)/e \in N_E(w_i)\} = 1$.
Case 3. If $b = x_{i,j}$, where $1 \le i \le m$ and $1 \le j \le n$ with $deg(b) = 2$, then $w_1 \in N_V(x_{i,n}), f(w_1) = 1$ and
$\{f(w)/w \in N_V(b)\}$ contains $(f(w_i), f(w_{i+1})) = (2j-1, 2j+1)$, $1 \le i \le m$ and $1 \le j \le n-1$.
$\gcd(2j-1, 2j+1) = 1$.
$\gcd\{f(w)/w \in N_V(x_{i,j}) : 1 \le i \le m, 1 \le j \le n\} = 1$ and
$\gcd\{f(d_i)/d_i \in N_E(x_{i,j})\} = 1$ because $\{f(d_i)/d_i \in N_E(x_{i,j})\}$ contains consecutive integers.
Hence G is a vertex edge neighborhood prime labeling for all n. □

Theorem 4.6. *The m fold petal alternate triangular snake admits a vertex edge neighborhood prime labeling for all* $n = 4, 6, 8, 10, ...$

Proof. Let G be a m fold petal alternate triangular snake. Then
$V(G) = \{u_j : 1 \le j \le n\} \cup \{c_{i,j} : 1 \le i \le m, 1 \le j \le (\frac{n}{2}) - 1\}$
and
$E(G) = \{e_j = u_ju_{j+1} : 1 \le j \le n-1\} \cup$
$\{e_{i,2j-1} = u_{2j}c_{i,j} : 1 \le i \le m, 1 \le j \le \frac{n}{2} - 1\} \cup$
$\{e_{i,2j} = u_{2j+1}c_{i,j} : 1 \le i \le m, 1 \le j \le \frac{n}{2} - 1\}$.
Also, $|V(G)| = n + m(\frac{n}{2} - 1)$ and $|E(G)| = (n-1) + 2m(\frac{n}{2} - 1)$.
Define a bijective function $f : V(G) \cup E(G) \to$
$\{1,2,3,...,2n-1+3m(\frac{n}{2}-1)\}$ as follows.
For each $1 \le j \le \frac{n}{2} - 1$, $f(u_{2j}) = 3j-2, f(u_{2j+1}) =$
$3j-1, f(e_{2j-1}) = f(u_{2j-1}u_{2j}) = p+4j-2, f(e_{2j}) =$
$f(u_{2j}u_{2j+1}) = p+4j-1, f(e_{1,2j-1}) = f(u_{2j}c_{1,j}) =$
$p+4j+1, f(e_{1,2j}) = f(u_{2j+1}c_{1,j}) = p+4j$.
$f(u_n) = \frac{3n}{2} - 2, f(c_{1,\frac{n}{2}-1}) = \frac{3n}{2} - 1, f(u_1) = \frac{3n}{2}$,
$f(e_{n-1}) = f(u_{n-1}u_n) = \frac{3n}{2} - 3$.
$f(c_{1,j}) = 3j$ for $1 \le j \le \frac{n}{2} - 2$.
For each $2 \le i \le m$ and $1 \le j \le \frac{n}{2} - 1$, $f(c_{i,j}) = \frac{(i+1)n}{2}$
$+(i-2)+j, f(e_{i,2j-1}) = f(u_{2j}c_{i,j}) = p+i(n-2)+$
$2j, f(e_{i,2j}) = f(u_{2j+1}c_{i,j}) = p+i(n-2)+1+2j$.
We claim that f is a vertex edge neighborhood prime labeling, for that we consider the following cases.
Let a be any vertex of G.
Case 1. For $a = u_1, u_n$. Here $deg(a) = 1$.
In both the case, $f(u_2) = 1$ and $f(u_{n-1}), f(e_{n-1})$ are consecutive integers.
$\gcd\{f(w), f(aw)/w \in N_V(a)\} = 1$.
For $u_i \ne u_1, u_n, deg(u_i) \ge 2$, then $\{f(x)/x \in N_V(u_i)\}$ except u_{n-2} contains consecutive integers and $\{f(x)/x \in N_V(u_{n-2})\}$ contains $(\frac{3n}{2} - 7, \frac{3n}{2} - 4, \frac{3n}{2} - 1)$.
$\gcd\{\frac{3n}{2} - 7, \frac{3n}{2} - 4, \frac{3n}{2} - 1\} = 1$.
$\gcd\{f((x)/x \in N_V(u_i) : 2 \le i \le n-1\} = 1$ and
$\{f(e)/e \in N_E(u_i)\}$ contains consecutive integers.
$\gcd\{f(e)/e \in N_E(u_i) : 2 \le i \le n-1\} = 1$.
Case 2. Let $a = c_{i,j}$ where $1 \le i \le m$ and
$1 \le j \le \frac{n}{2} - 1$ with $deg(a) = 2$. Here, $\gcd\{f(x)/x \in N_V(a)\} =$
1 and $\gcd\{f(d_i)/d_i \in N_E(a)\} = 1$ because both
$\{f(x)/x \in N_V(a)\}$ and $\{f(d_i)/d_i \in N_E(a)\}$ contains consecutive integers.
Hence G is a vertex edge neighborhood prime labeling. □

Theorem 4.7. *The m fold petal sunflower is a vertex edge neighborhood prime labeling for all* $n \ge 3$.

Proof. Let G be a m fold petal sunflower. Then
$V(G) = \{u_j : 1 \le j \le n\} \cup \{c_{i,j} : 1 \le i \le m, 1 \le j \le n\}$ and
$E(G) = \{e_j = u_ju_{j+1} : 1 \le j \le n-1\} \cup \{e_n = u_nu_1\} \cup$
$\{e_{i,2j-1} = u_jc_{i,j}, e_{i,2j} = u_{j+1}c_{i,j} : 1 \le i \le m, 1 \le j \le n\}$.
Also, $|V(G)| = (m+1)n$ and $|E(G)| = (2m+1)n$.
Define a bijective function $f : V(G) \cup E(G) \to$
$\{1,2,3,...,n(3m+2)\}$ as follows.
For each $1 \le j \le n$, $f(u_j) = 2j-1, f(c_{1,j}) = 2j$.
$f(c_{i,j}) = in+j$ for $1 \le i \le m$ and $2 \le j \le n$.
$f(e_j) = f(u_ju_{j+1}) = p+j$ for $1 \le j \le n-1$.
$f(e_n) = f(u_nu_1) = p+n$.
For each $1 \le i \le m$ and $1 \le j \le n$, $f(e_{i,2j-1}) = f(u_jc_{i,j})$
$= p+(2i-1)n-1+2j, f(e_{i,2j}) = f(u_{j+1}c_{i,j}) =$
$p+(2i-1)n+2j$.
By considering following cases we prove f is a vertex edge neighborhood prime labeling. Let b be any vertex of G.
Case 1. If $b = u_i, 1 \le i \le n$ with $deg(u_i) \ge 2$, then
$u_1 \in N_V(u_n), f(u_1) = 1$ and $\{f(x)/x \in N_V(u_i) : 1 \le i \le n-1\}$

contains consecutive integers.
$\gcd\{f(x)/x \in N_V(u_i) : 1 \leq i \leq n\} = 1$ and
$\{f(e_i)/e_i \in N_E(u_i)\}$ also contains consecutive integers.
$\gcd\{f(e_i)/e_i \in N_E(u_i)\} = 1$.
Case 2. If $b = c_{i,j}$, where $1 \leq i \leq m$ and $1 \leq j \leq n$ with $deg(b) = 2$, then $u_1 \in N_V(c_{i,n}), f(u_1) = 1$ and
$\left\{f(x)/x \in N_V(c_{i,j})\right\}$ contains $(2i-1, 2i+1), 1 \leq i \leq m$ and $1 \leq j \leq n-1$.
$\gcd(2i-1, 2i+1) = 1$.
$\gcd\left\{f(x)/x \in N_V(c_{i,j}) : 1 \leq i \leq m, 1 \leq j \leq n\right\} = 1$ and
$\left\{f(d_i)/d_i \in N_E(c_{i,j})\right\}$ contains consecutive integers.
$\gcd\left\{f(d_i)/d_i \in N_E(c_{i,j})\right\} = 1$.
Hence G is a vertex edge neighborhood prime labeling for all $n \geq 3$. □

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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