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# On semi-topological rings

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**Abstract**

We introduce and study the semi-topological rings. Some examples of semi-topological rings are provided. We investigate some permanence properties of semi-topological rings. Along with other results, it is proved that translation of an open (resp. closed) set in a semi-topological ring is semi-open (resp. semi-closed), that multiplication of an open (resp. closed) set in a semi-topological ring by an invertible element of the ring is semi-open (resp. semi-closed). We also prove that any ring homomorphism between a semi-topological ring and a topological ring which is continuous at zero is semi-continuous everywhere.

**Keywords**

Semi-open sets, semi-closed sets, semi-continuous mappings, semi-topological rings.

**AMS Subject Classification**

16W80, 16W99.

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## Contents



## 1. Introduction

As we are familiar with some classical kinds of topological spaces like topological vector spaces, topological rings and topological groups because of their vast applications in almost all branches of mathematics, investigation and exploration of new facts about these spaces become a regular, interesting and useful activity. Because of their nice and unique properties, these spaces earn great importance in all advanced branches of mathematics like mathematical analysis, functional analysis, fixed point theory, algebra, complex analysis, variational inequalities, etc. After these advents of the interplay between algebraic and topological structures, many researchers and mathematicians have been working in these fields of study. In this day and age, several generalizations and similar structures of these spaces are appeared and these subjects are getting interesting for study day after day.

This paper presents the innovation of semi-topological rings, bringing together the areas of topology and ring theory. This innovation is derived from the study of the well-known class of topological rings. A topological ring is a ring endowed with a topology which turns out the ring operations continuous. The work on topological rings is quite active since 1930s and till this age, field of topological rings has been extensively developed. Kaplansky [3–5], Warner [10] and many more have done classical work on topological rings. Recently, Salih [9] introduced the irresolute topological rings. In [8], we introduced the notion of $\alpha-$irresolute topological rings which is basically independent of topological rings as well as irresolute topological rings.

This paper is organized as follows: Section 1 and 2 provide the background and basic topological concepts that are required for the creation of semi-topological rings. In Section 3, we give the definition of a semi-topological ring, elaborate this concept through some examples and briefly interpret how semi-topological rings are a generalization of topological rings. Section 4 is devoted to some permanence properties of semi-topological rings. Finally, references are given.

## 2. Preliminaries

Throughout the present paper, $X$ denotes a topological space on which no separation axioms are assumed. For $A \subseteq X$, $Cl(A)$ and $Int(A)$ denote the closure of $A$ and the interior of $A$

respectively. The notations $\varepsilon$ and $\delta$ denote negligibly small positive numbers.

In 1963, N. Levine [6] introduced the concept of semi-open sets in topological spaces. He defines a set $S$ in a topological space $X$ to be semi-open if there exists an open set $U$ in $X$ such that $U \subseteq S \subseteq Cl(U)$; or equivalently, a subset $S$ of $X$ is semi-open if $S \subseteq Cl(Int(S))$. The complement of a semi-open set is said to be semi-closed; or equivalently, a set $S$ in $X$ is semi-closed if $Int(Cl(S)) \subseteq S$. Any union of semi-open sets is semi-open, while the intersection of two semi-open sets need not be semi-open. Every open set is semi-open but the converse is not always true. The semi-closure of a subset $S$ of $X$, denoted by $sCl(S)$, is the intersection of all semi-closed subsets of $X$ containing $S$. In other words, the semi-closure of a subset $S$ of $X$ is the smallest semi-closed subset of $X$ containing $S$. The union of all semi-open sets in $X$ that are contained in $S \subseteq X$ is called the semi-interior of $S$ and is denoted by $sInt(S)$. It is known that a set $S$ in $X$ is semi-closed (resp. semi-open) if and only if $sCl(S) = S$ (resp. $sInt(S) = S$). In [2], it is proved that $x \in sCl(S)$ if and only if $S \cap U \neq \emptyset$ for any semi-open set $U$ in $X$ containing $x$. A point $x \in X$ is called a semi-interior point of $S$ if there exists a semi-open set $U$ in $X$ such that $x \in U \subseteq S$. The set of all semi-interior points of $S$ is equal to $sInt(S)$. Further development on semi-open sets and semi-closed sets can be seen in [2, 7]. The family of all semi-open (resp. semi-closed) sets in $X$ is denoted by $SO(X)$ (resp. $SC(X)$).

**Definition 2.1.** *A subset $A \subseteq X$ is a semi-neighborhood of a point $x \in X$ if there exists a semi-open set $U$ in $X$ such that $x \in U \subseteq A$. If a semi-neighborhood $A$ of a point $x \in X$ is semi-open, then we say $A$ is semi-open neighborhood of $x$. The collection of all semi-open neighborhoods of a point $x \in X$ is denoted by $\mathscr{N}_x$.*

**Definition 2.2.** [6] *Let $X$ and $Y$ be topological spaces. A mapping $f : X \to Y$ is called semi-continuous if $f^{-1}(U) \in SO(X)$, for each open set $U$ in $Y$. Equivalently, $f$ is semi-continuous if for each $x \in X$ and each open neighborhood $V$ of $f(x)$ in $Y$, there exists a semi-open neighborhood $U$ of $x$ in $X$ such that $f(U) \subseteq V$.*

Clearly, every continuous function is semi-continuous but the converse need not be true. For example, let $X = \mathbb{R}$, the set of reals, with its usual topology. Then the function $f : X \to X$ defined by $f(x) = 0$, if $x \leq 0$ and $f(x) = 1$, if $x > 0$, is semi-continuous which is obviously not continuous.

## 3. Semi-topological rings

We start this section with some notations. By $R$, we mean a ring $(R,+,.)$ without unity unless stated explicitly. We denote the multiplication of two elements $x$ and $y$ in $R$ by $xy$. We define semi-topological rings, elaborate this notion by some examples of semi-topological rings. We mention in brief, the relation between topological rings and semi-topological rings.

**Definition 3.1.** *A semi-topological ring is a ring $R$ with a topology $\tau$ on $R$ such that the following three conditions are satisfied:*

*(1) For each $x, y \in R$ and each open neighborhood $W$ of $x+y$ in $R$, there exist semi-open neighborhoods $U$ and $V$ of $x$ and $y$ respectively, in $R$ such that $U+V \subseteq W$,*

*(2) For each $x \in R$ and each open neighborhood $V$ of $-x$ in $R$, there exists a semi-open neighborhood $U$ of $x$ in $R$ such that $-U \subseteq V$, and*

*(3) For each $x, y \in R$ and each open neighborhood $W$ of $xy$ in $R$, there exist semi-open neighborhoods $U$ and $V$ of $x$ and $y$ respectively, in $R$ such that $U.V \subseteq W$*

For any subsets $A$ and $B$ of $R$, we define $A+B = \{a+b : a \in A, b \in B\}$, $A.B = \{ab : a \in A, b \in B\}$ and $-A = \{-a : a \in A\}$.

**Some examples of semi-topological rings.**

**Example 3.2.** *Consider the ring $R$ of reals with its standard topology $\mathscr{U}$. Then $(R, \mathscr{U})$ is a semi-topological ring.*

**Example 3.3.** *Let $R$ be any ring and $\mathscr{D}$ be the discrete topology on $R$. Then $(R, \mathscr{D})$ is a semi-topological ring.*

It is obvious from the definition that every topological ring is a semi-topological ring but the other way fails to hold. Below is an example of a semi-topological ring which is not a topological ring.

**Example 3.4.** *Consider the ring $R$ of reals and let $\tau$ be the topology on $R$ generated by the family of sets $\mathscr{B} = \{(a,b) : a,b \in \mathbb{R}\} \cup \{[c,d) : c,d \in \mathbb{R}, 0 < c < d\}$. We show that $(R,\tau)$ is a semi-topological ring which is clearly not a topological ring.*

*(1) Let $x$ and $y$ be any elements of $R$. For open neighborhood $W = [x+y, x+y+\varepsilon)$ (resp. $(x+y-\varepsilon, x+y+\varepsilon)$) of $x+y$ in $R$, we can opt for semi-open neighborhoods $U = [x, x+\delta)$ (resp. $(x-\delta, x+\delta)$) and $V = [y, y+\delta)$ (resp. $(y-\delta, y+\delta)$) of $x$ and $y$ respectively, in $R$ such that $U+V \subseteq W$ for each $\delta < \frac{\varepsilon}{2}$.*

*(2) Let $x \in R$. We have following cases:*

*(i) If $x = 0$. In this case, for open neighborhood $V = (-\varepsilon, \varepsilon)$ of $-x$ in $R$, we can choose the same open neighborhood $U = V$ of $x$ such that $-U \subseteq V$.*

*(ii) If $x > 0$, then for open neighborhood $V = (-x-\varepsilon, -x+\varepsilon)$ of $-x$, choose semi-open set $U = (x-\varepsilon, x+\varepsilon)$ in $R$ containing $x$ such that $-U \subseteq V$.*

*(iii) If $x < 0$, then for open set $V = [-x, -x+\varepsilon)$ containing $-x$, choose semi-open set $U = (x-\varepsilon, x]$ in $R$ containing $x$ that satisfies $-U \subseteq V$.*

*Thus, second condition of the definition of semi-topological rings is verified.*

*(3) Let $x, y \in R$ be arbitrary. Consider open neighborhood $W = [xy, xy+\varepsilon)$ (resp. $(xy-\varepsilon, xy+\varepsilon)$) of $xy$ in $R$. We have following cases:*

*Case (i). If $x>0$ and $y>0$. We can choose semi-open sets $U=[x,x+\delta)$ (resp. $(x-\delta,x+\delta)$) in $R$ containing $x$ and $V=[y,y+\delta)$ (resp. $(y-\delta,y+\delta)$) in $R$ containing $y$ such that $U.V\subseteq W$ for each $\delta<\frac{\varepsilon}{x+y+1}$.*

*Case (ii). Suppose $x<0$ and $y<0$. We can choose semi-open sets $U=(x-\delta,x]$ (resp. $(x-\delta,x+\delta)$) and $V=(y-\delta,y]$ (resp. $(y-\delta,y+\delta)$) in $R$ such that $U.V\subseteq W$ for sufficiently appropriate $\delta\leq\frac{-\varepsilon}{x+y-1}$.*

*Case (iii). If $x=0$ and $y>0$ ( resp. $x>0$ and $y=0$). Then $\lambda x=0$. Consider any open neighborhood $W=(-\varepsilon,\varepsilon)$ of $0$ in $R$. We can go for semi-open sets $U=(-\delta,\delta)$ (resp. $U=(x-\delta,x+\delta)$) in $R$ containing $x$ and $V=(y-\delta,y+\delta)$ (resp. $V=(-\delta,\delta)$) in $R$ containing $y$ such that $U.V\subseteq W$ for each $\delta<\frac{\varepsilon}{y+1}$ (resp. $\delta<\frac{\varepsilon}{x+1}$).*

*Case (iv). If $x=0$ and $y<0$ (resp. $x<0$ and $y=0$). Consider any open neighborhood $W=(-\varepsilon,\varepsilon)$ of $0$ in $R$. Then, for the selection of semi-open sets $U=(-\delta,\delta)$ (resp. $U=(x-\delta,x+\delta)$) and $V=(y-\delta,y+\delta)$ (resp. $V=(-\delta,\delta)$) in $R$, we have $U.V\subseteq W=(-\varepsilon,\varepsilon)$ for each $\delta<\frac{\varepsilon}{1-y}$ (resp. $(\delta<\frac{\varepsilon}{1-x})$).*

*Case (v). If $\lambda=0$ and $x=0$. Consider any open neighborhood $W=(-\varepsilon,\varepsilon)$ of $0$ in $R$, we can find semi-open sets $U=(-\delta,\delta)$ and $V=(-\delta,\delta)$ in $R$, such that $U.V\subseteq W$ for each $\delta<\sqrt{\varepsilon}$.*

*Case (vi). If $x<0,\ y>0$ (resp. $x>0,\ y<0$). In this case, there is only one type of open neighborhood $W=(xy-\varepsilon,xy+\varepsilon)$ of $xy$ in $R$. Choose semi-open sets $U=(x-\delta,x+\delta)$ and $V=(y-\delta,y+\delta)$ in $R$ containing $x$ and $y$ respectively, we have $U.V\subseteq W$ for each $\delta<\frac{\varepsilon}{y-x+1}$ (resp. $\delta<\frac{\varepsilon}{x-y+1}$).*

*Therefore, $(R,\tau)$ is a semi-topological ring.*

## 4. Characterizations

In this section, we prove that translation of an open (resp. closed) set in a semi-topological ring is semi-open (resp. semi-closed). Later, we also show that multiplication of an open (resp. closed) set by an invertible element of a semi-topological ring is semi-open (resp. semi-closed). We further investigate some permanence properties of semi-topological rings.

**Theorem 4.1.** *Let $A$ be an open set in a semi-topological ring $R$. Then*

*(1) $-A\in SO(R)$.*
*(2) $x+A\in SO(R)$ for each $x\in R$.*

*Proof.* (1) Let $x$ be any element from $-A$. Then there exists $U\in\mathscr{N}_x(R)$ such that $-U\subseteq A$. This gives $x\in U\subseteq -A \Rightarrow x\in sInt(-A)$ and hence $-A=sInt(-A)$. Therefore, $-A\in SO(R)$.

(2) Let $y$ be an element from $x+A$. Our task is to prove $x+A=sInt(x+A)$. Since $A$ is open, let $U$ and $V$ be semi-open sets in $R$ such that $-x\in U, y\in V$ and $U+V\subseteq A$. In particular, $-x+V\subseteq A \Rightarrow V\subseteq x+A$. This proves that $y\in sInt(x+A)$. Consequently, $x+A=sInt(x+A)$. That is, $x+A\in SO(R)$. □

**Corollary 4.2.** *Let $A$ be an open set in a semi-topological ring $R$. Then*

*(1) $-A\subseteq Cl(Int(-A))$.*
*(2) $x+A\subseteq Cl(Int(x+A))$ for each $x\in R$.*

**Corollary 4.3.** *For any closed set $F$ in a semi-topological ring $R$, the following hold:*

*(1) $-F\in SC(R)$.*
*(2) $x+F\in SC(R)$ for each $x\in R$.*

**Theorem 4.4.** *Let $A$ be any subset of a semi-topological ring $R$. The following are valid:*

*(1) $x+sCl(A)\subseteq Cl(x+A)$ for each $x\in R$.*
*(2) $sCl(x+A)\subseteq x+Cl(A)$ for each $x\in R$.*
*(3) $x+Int(A)\subseteq sInt(x+A)$ for each $x\in R$.*
*(4) $Int(x+A)\subseteq x+sInt(A)$ for each $x\in R$.*

*Proof.* (1) Suppose $z\in x+sCl(A)$ be arbitrary. Then $z=x+y$ for some element $y$ from $sCl(A)$. Our aim is to show $z\in Cl(x+A)$. For, let $W$ be an open neighborhood of $z$ in $R$. Then we find semi-open neighborhoods $U$ of $x$ and $V$ of $y$ in $R$ satisfying $U+V\subseteq W$. This follows from the definition of semi-topological rings.

Since $y\in sCl(A)$, there is a common element $g$ of $A$ and $V$. This leads $x+g\in(x+A)\cap W \Rightarrow (x+A)\cap W\neq\emptyset$. That is, $z\in Cl(x+A)$. Hence the assertion follows.

(2) Let $y$ be an element from $sCl(x+A)$. We have to show that $y\in x+Cl(A)$. That is, $-x+y\in Cl(A)$. Let $W$ be any open set in $R$ containing $-x+y$. Then there exist semi-open sets $U$ in $R$ containing $-x$ and $V$ in $R$ containing $y$ such that $U+V\subseteq W$. By assumption, $(x+A)\cap V\neq\emptyset$. Let $g$ be a common element of $x+A$ and $V$. Then $-x+g\in A\cap(U+V)\subseteq A\cap W$. Therefore, $-x+y\in Cl(A)$; that is, $y\in x+Cl(A)$. Hence $sCl(x+A)\subseteq x+Cl(A)$.

(3) Let $y$ be any element from $x+Int(A)$. Then $-x+y\in Int(A)$. By using definition of a semi-topological ring, we obtain $U,V\in SO(R)$ such that $-x\in U, y\in V$ and $U+V\subseteq A$. In particular, $-x+V\subseteq A$ implies that $V\subseteq x+A$, thereby it follows that $y\in sInt(x+A)$. Thus, $x+Int(A)\subseteq sInt(x+A)$.

(4) Let $y\in Int(x+A)$. Then $y=x+a$ for some $a\in A$. Further we obtain semi-open sets $U$ and $V$ in $R$ containing $x$ and $a$ respectively, such that $U+V\subseteq x+A$. Whence we find that $y=x+a\in x+sInt(A)$. Therefore, job is done. □

By similar arguments as above, we obtain the following result:

**Theorem 4.5.** *Let $A$ be any subset of a semi-topological ring $R$. Then*

*(1) $-sCl(A)\subseteq Cl(-A)$.*
*(2) $sCl(-A)\subseteq -Cl(A)$.*
*(3) $-Int(A)\subseteq sInt(-A)$.*
*(4) $Int(-A)\subseteq -sInt(A)$.*

We say that $(R, \tau)$ (or simply, $R$) is a semi-topological ring with unity if $(R, \tau)$ is a semi-topological ring and $R$ is a ring with unity. In this case, we denote the set of all invertible elements in $R$ by $R^*$.

**Theorem 4.6.** *Let $(R, \tau)$ be a semi-topological ring with unity. If $A \in \tau$, then $rA, Ar \in SO(R)$ for each $r \in R^*$.*

*Proof.* We begin to show $rA$ is semi-open. For any element $x$ from $rA$, we find $r^{-1}x \in A$. We will use the definition of a semi-topological ring without further mention. Let $U$ and $V$ be semi-open sets in $R$ satisfying $r^{-1} \in U, x \in V$ and $U.V \subseteq A$. Then $V \subseteq rA \Rightarrow x \in sInt(rA)$. Thus, $rA = sInt(rA)$. Hence, $rA \in SO(R)$.

Analogously, we can show that $Ar \in SO(R)$. □

**Theorem 4.7.** *Let $F$ be any closed set in a semi-topological ring with unity $R$. Then $rF, Fr \in SC(R)$ for each $r \in R^*$.*

*Proof.* We only show that $rF$ is semi-closed. The proof for semi-closedness of $Fr$ follows analogously.

Let $x$ be any element of $sCl(rF)$ and let $W$ be an open neighborhood of $r^{-1}x$. There exist semi-open neighborhoods $U$ and $V$ of $r^{-1}$ and $x$ respectively, in $R$ such that $U.V \subseteq W$. Since $x \in sCl(rF)$, there is $g \in (rF) \cap V$. Consequently, $r^{-1}g \in F \cap (U.V) \subseteq F \cap W \Rightarrow F \cap W \neq \emptyset \Rightarrow r^{-1}x \in Cl(F)$. Since $F$ is closed, $x \in rF$. Hence $rF = sCl(rF)$; that is, $rF \in SC(R)$. □

**Theorem 4.8.** *Let $R$ be a semi-topological ring with unity. For any $A \subseteq R$, the following hold:*

*(1) $rsCl(A) \subseteq Cl(rA)$ for each $r \in R^*$.*
*(2) $sCl(rA) \subseteq rCl(A)$ for each $r \in R^*$.*
*(3) $rInt(A) \subseteq sInt(rA)$ for each $r \in R^*$.*
*(4) $Int(rA) \subseteq rsInt(A)$ for each $r \in R^*$.*

*Proof.* We only prove (1) and (2). The proof for part (3) and (4) can be obtained analogously.

(1) Let $y$ be any element from $rsCl(A)$. Then $y = rx$ for some $x \in sCl(A)$. We have to show that for any open neighborhood $W$ of $y$, $(rA) \cap W \neq \emptyset$. Let $W$ be an open neighborhood of $y$. We obtain semi-open neighborhoods $U$ and $V$ of $r$ and $x$ respectively, in $R$ such that $U.V \subseteq W$. Now, $x \in sCl(A) \Rightarrow A \cap V \neq \emptyset \Rightarrow$ there is $a \in A \cap V$. This gives $ra \in (rA) \cap (U.V) \subseteq (rA) \cap W \Rightarrow y \in Cl(rA)$. Hence $rsCl(A) \subseteq Cl(rA)$.

(2) Let $x \in sCl(rA)$. We have to show that $x \in rCl(A)$. For any open set $W$ in $R$ containing $y = r^{-1}x$, we obtain semi-open sets $U$ and $V$ in $R$ satisfying $r^{-1} \in U$, $x \in V$ and $U.V \subseteq W$. By assumption, we always have $(rA) \cap V \neq \emptyset$. So, there is $a \in (rA) \cap V$ wherefrom we have $r^{-1}a \in A \cap (U.V) \subseteq A \cap W$; that is, $A \cap W \neq \emptyset$. Thus, $x \in rCl(A)$. □

**Theorem 4.9.** *Let $A$ and $B$ be any subsets of a semi-topological ring $R$. Then $sCl(A) + sCl(B) \subseteq Cl(A+B)$.*

*Proof.* Suppose $x \in sCl(A)$ and $y \in sCl(B)$. We have to show that $x + y \in Cl(A+B)$. Let $W$ be an open neighborhood of $x+y$. Then some semi-open neighborhoods $U$ of $x$ and $V$ of $y$ will satisfy $U + V \subseteq W$. Assumptions yield, there are $a \in A \cap U$ and $b \in B \cap V$. This helps to produce the fact $a + b \in (A+B) \cap W \Rightarrow (A+B) \cap W \neq \emptyset$. Thus, $x + y \in Cl(A+B)$ and thereby the proof is complete. □

**Theorem 4.10.** *Let $R$ be a semi-topological ring. Then the following mappings:*

*(1) $\phi_x : R \to R$ defined by $\phi_x(y) = x + y$, for all $y \in R$ ($x \in R$ is fixed),*
*(2) $\psi : R \to R$ defined by $\psi(x) = -x$ for all $x \in R$*

*are semi-continuous.*

*Proof.* (1) In order to show that $\phi_x$ is semi-continuous, we will show that the inverse image of any open set in $R$ under $\phi_x$ is semi-open set in $R$. Let $U$ be any open set in $R$. Then $\phi_x^{-1}(U) = -x + U$. By Theorem 4.1, $-x + U$ is semi-open. Hence $\phi_x$ is semi-continuous.

(2) Let $x$ be any element of $R$ and let $V$ be any open neighborhood of $\psi(x)$. Since $R$ is semi-topological ring, there exists a semi-open neighborhood $U$ of $x$ such that $-U \subseteq V$. This amounts to the relation $\psi(U) \subseteq V$. This proves that $\psi$ is semi-continuous at $x$ and hence $\psi$ is semi-continuous. □

**Theorem 4.11.** *Let $R$ be a semi-topological ring with unity. Then the mapping $h : R \to R$ defined by $h(x) = rx$, for all $x \in R$ ($r \in R^*$ is fixed), is semi-continuous.*

*Proof.* This is a direct consequence of Theorem 4.4. □

**Theorem 4.12.** *Let $R$ be a semi-topological rings, $S$ be a topological ring and let $f : R \to S$ be a ring homomorphism. If $f$ is continuous at zero, then $f$ is semi-continuous.*

*Proof.* Let $x$ be any element of $R$ and $W$ be an open neighborhood of $f(x)$ in $S$. According to hypothesis, $W - f(x)$ is open neighborhood of $0 = f(0)$ in $S$. Since $f$ is continuous at zero, there exists an open neighborhood $U$ of 0 in $R$ such that $f(U) \subseteq W - f(x)$. This gives $f(x+U) \subseteq W$. By Theorem 4.1, $x + U$ is semi-open and hence $f$ is semi-continuous at $x$. Thus, $f$ is semi-continuous. □

## 5. Conclusion

In this paper, we developed semi-topological ring which is basically one of the generalization of topological ring. The concept is further elaborated with examples and counter examples. Moreover, some permanence results and properties of semi-topological ring are characterized and explained throughout the paper.

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