Malaya Journal of Matematik, Vol. 7, No. 4, 818-822, 2019

https://doi.org/10.26637/MJM0704/0030

Strongly multiplicative labeling of certain tree derived networks

M. Joice Punitha[1] and A. Josephine Lissie [2*]

Abstract

A graph $G=(V(G),E(G))$ with p vertices is said to be strongly multiplicative if the vertices of G can be labeled with p distinct integers $1,2,...,p$ such that the labels induced on the edges by the product of labels of the end vertices are all distinct [3].

In this paper we prove that the $X-$ tree, Hypertree and shuffle Hypertree are strongly multiplicative for all $n\geq 2$.

Keywords

$X-$ tree, Hypertree, Shuffle Hypertree and Strongly multiplicative labeling.

AMS Subject Classification

26A33, 30E25, 34A12, 34A34, 34A37, 37C25, 45J05.

[1] *Department of Mathematics, Bharathi Women's College, Chennai-600108, Tamil Nadu, India.*

[2] *Department of Mathematics, Stella Maris College, Chennai-600086, Tamil Nadu, India.*

***Corresponding author**: [2] lissie84@gmail.com.

Article History: Received **02** September **2019**; Accepted **19** November **2019**

Contents

1. Introduction

Graph labeling concerns the assigning of values, usually represented by integers, to the edges and/or vertices of a graph [1]. It plays an important role in Neural Networks, Communication Networks, Circuit Analysis, Coding theory, particularly for missile guidance codes, design of good radar type codes and convolution codes with optimal autocorrelation properties and also used in the study of X-ray Crystallography, etc. Graph labeling serves as a frontier between number theory and structure of graphs [2].

In this paper we have considered tree derived networks like $X-$ tree, Hypertree and shuffle Hypertree. Hypertree is an interconnection Topology for incrementally expansible multi computer system, which combines the easy expansibility of the tree structure with the compactness of Hypercube, that it combines the best feature of binary tree and the hypercubes [7].Shuffle Hypertree $SHT(n)$ is a modification of Hypertree $HT(n)$. The basic skeleton of a X-tree, hypertree and shuffle hypertree are a complete binary tree T_n.

In this paper we prove that X-tree, Hypertree and Shuffle Hypertree networks are strongly multiplicative.

2. Preliminaries

Definition 2.1. If every internal vertex of the rooted tree has exactly two children then the tree is called a *complete binary tree.*

Remark 2.1. For any non negative integer n, the complete binary tree of height n denoted by T_n, is the binary tree where each internal vertex has exactly two children and all the leaves are at the same level. T_n has n levels namely $1,2,3,...,n$ and level i, $1\leq i\leq n$ contains 2^{i-1} vertices.

Remark 2.2. At each level, T_n has two sets of edges i.e. left edges and right edges. $T_n, n\geq 3$ contains 2^{n-2} number of internal vertices in the $(n-1)^{th}$ level.

Remark 2.3. At each level, T_n has exactly 2^n-1 vertices and 2^n-2 edges.

Definition 2.2. [4]The Slim tree $ST(n)$ is a complete binary tree T_n with the set of edges $L=[(i,i+1)/\ 2^{n-1}\leq i\leq 2^n-2]$ to the complete binary tree T_n.

Definition 2.3. The X-tree is a slim tree $ST(n)$ along with edges obtained by joining the left and the right children of every parent at all intermediate levels $2,3,...,n$ and is denoted by $XT(n)$.[6].

Remark 2.4. The number of vertices and edges in the X-tree $XT(n)$ are 2^n-1 and $3(2^{n-1}-1)$ respectively.

Remark 2.5. For convenience the vertices of the X tree $XT(n)$ are labeled as shown in figure 1.

Definition 2.4. A Hypertree is a graph is a complete binary tree along with the edges obtained by joining the two intermediate nodes in the same level l of the tree if the label difference is $2^{l-2}, 2 \le l \le n$.We denote the $n-$ level Hypertree as $HT(n)$ [8].

Remark 2.6. The number of vertices and edges in Hypertree graph are 2^n-1 and $3(2^{n-1}-1)$ respectively.

Definition 2.5. The Shuffle Hyper Tree is a Hyper tree where the intermediate edges of $HT(n)$ are removed and replaced by the hyper edges $\{(v_{2^{i-1}}, v_{2^{i}-1})/\ 2 \le i \le n\}$ and $\{(v_{2^{i-1}+2k-1}, v_{2^{i-1}+2k})\ /3 \le i \le n\ , 1 \le k \le 2^{i-2}-1\}$ and is denoted by $SHT(n)$. [5]

Remark 2.7. The number of vertices and edges in shuffle Hypertree graph are 2^n-1 and $3(2^n-1)$ respectively.

3. Main Results

Theorem 3.1. *The X-tree $XT(n)$ is strongly multiplicative for all n*

Proof. To prove that the *X-tree* $XT(n)$ is strongly multiplicative,
Define the vertex set $V = \{v_i\ /1 \le i \le 2^n-1\}$ and
the edge set $E = E_1 \cup E_2 \cup E_3$ where
$E_1 = \left\{e_i = (v_i, v_{2i})\ /\ 1 \le i \le 2^{n-1}-1\right\}$,
$E_2 = \left\{\gamma_i = (v_i, v_{2i+1})\ /\ 1 \le i \le 2^{n-1}-1\right\}$, and
$E_3 = \left\{\delta_i = (v_{2^{l-1}+2i}, v_{2^{l-1}+2i+1})/0 \le i \le (2^{l-2}-1)\ , 2 \le l \le n\right\}$,
where l denotes each level of the tree.
Define the vertex labeling by a bijective map $f: V \longrightarrow N$ such that $f(v_i) = i$ for all $1 \le i \le 2^n-1$.
We shall prove that all the edge labelings in E_1 are distinct.
Define an edge induced function $g: E_1 \to N$ such that for all $e_i \in E_1$, $g(e_i) = f(v_i)f(v_{2i})\ , 1 \le i \le 2^{n-1}-1$,
For $i \ne p$,let $e_i, e_p \in E_1$ be distinct edges,
we claim that $g(e_i) \ne g(e_p)$
Assume that

$$\begin{aligned} g(e_i) &= g(e_p) \\ g(v_i, v_{2i}) &= g(v_p, v_{2p}) \\ f(v_i)f(v_{2i}) &= f(v_p)f(v_{2p}) \\ i(2i) &= p(2p) \\ i^2 &= p^2 \Rightarrow i = p. \end{aligned}$$

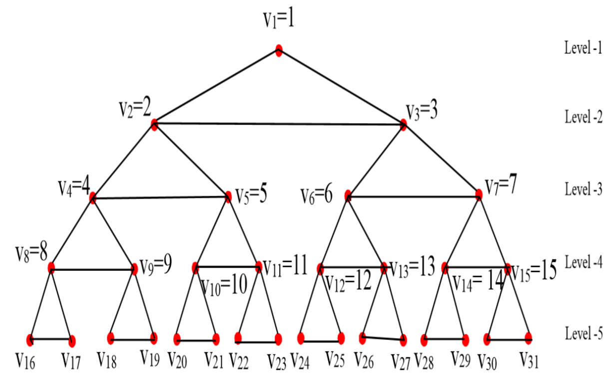

Fig. 1. X tree XT(5)

This is a contradiction for i.
Hence $g(e_i) \ne g(e_p)$, for all $1 \le i \le 2^{n-1}-1$.
Hence all edge labelings in E_1 are distinct.
We shall show that the labeling of edges within E_2 are distinct.
Define an edge induced function $g: E_2 \to N$ such that for all $e_i \in E_2$, $g(e_i) = f(v_i)f(v_{2i+1}), 1 \le i \le 2^{n-1}-1$.
For $i \ne p$,let $e_i, e_p \in E_2$ be distinct edges.
We claim that $g(e_i) \ne g(e_p)$.
Assume that $g(e_i) = g(e_p)$

$$\begin{aligned} g(v_i, v_{2i+1}) &= g(v_p, v_{2p+1}) \\ f(v_i)f(v_{2i+1}) &= f(v_p)f(v_{2p+1}) \\ i(2i+1) &= p(2p+1) \\ 2(i^2-p^2) &= -(i-p) \Rightarrow i = -(p+1/2) \end{aligned}$$

This is a contradiction for i.
Hence $g(e_i) \ne g(e_p)$, for all $1 \le i \le 2^{n-1}-1$.
Hence all the labelings of edges in E_2 are distinct.
We shall show that the labeling of edges within E_3 are distinct.
Define an edge induced function $g: E_3 \to N$ such that for all $e_i \in E_3$, $g(e_i) = f(v_{2^{l-1}+2i})f(v_{2^{l-1}+2i+1}), 0 \le i \le (2^{l-2}-1)$, $2 \le l \le n$.
Case 1. For $i \ne p$,let $e_i, e_p \in E_3$ be distinct edges in the same level l, we claim that $g(e_i) \ne g(e_p)$.
Assume that $g(e_i) = g(e_p)$

$$\begin{aligned} g(v_{2^{l-1}+2i}, v_{2^{l-1}+2i+1}) &= g(v_{2^{l-1}+2p}, v_{2^{l-1}+2p+1}) \\ f(v_{2^{l-1}+2i})f(v_{2^{l-1}+2i+1}) &= f(v_{2^{l-1}+2p})f(v_{2^{l-1}+2p+1}) \\ (2^{l-1}+2i)(2^{l-1}+2i+1) &= (2^{l-1}+2p)(2^{l-1}+2p+1) \\ \Rightarrow x(x+1) &= y(y+1), \text{ where } x = 2^{l-1}+2p \\ \Rightarrow x &= -1-y \text{ or } 2^{l-1}+2i = -1-(2^{l-1}+2p) \end{aligned}$$

$i = -1/2(1+2^l+2p)$,this is a contradiction for i.
Hence $g(e_i) \ne g(e_p)$, for all $0 \le i \le (2^{l-2}-1), 2 \le l \le n$.
Case 2. For $i \ne p$,let $e_i, e_p \in E_3$ be distinct edges at different level l_r and l_s, we claim that $g(e_i) \ne g(e_p)$.
Assume that $g(e_i) = g(e_p)$

$$\begin{aligned} g(v_{2^{l_r-1}+2i}, v_{2^{l_r-1}+2i+1}) &= g(v_{2^{l_s-1}+2p}, v_{2^{l_s-1}+2p+1}) \\ f(v_{2^{l_r-1}+2i})f(v_{2^{l_r-1}+2i+1}) &= f(v_{2^{l_s-1}+2p})f(v_{2^{l_s-1}+2p+1}) \end{aligned}$$

$$(2^{l_r-1}+2i)(2^{l_r-1}+2i+1) = (2^{l_s-1}+2p)(2^{l_s-1}+2p+1)$$

$\Rightarrow x(x+1) = y(y+1)$, where $x = 2^{l_r-1}+2i$ and $y = 2^{l_s-1}+2p$
$\Rightarrow x = -1-y$ or $2^{l_r-1}+2i = -1-(2^{l_s-1}+2p)$
$i = -1/2(1+2^{l_r-1}+2^{l_s-1}+2p)$
This is a contradiction for i.
Hence $g(e_i) \neq g(e_p)$, for all $0 \leq i \leq (2^{l-2}-1), 2 \leq l \leq n$.
Hence all the labelings of edges in E_3 is distinct.
We shall show that the labeling of edges E_1 and E_2 are distinct.
If $e_i \in E_1$ and $e_p \in E_2$ be two distinct edges then to prove $g(e_i) \neq g(e_p)$.

For $i \neq p, 1 \leq i, p \leq 2^{n-1}-1$, assume that $g(e_i) = g(e_p)$.

$$g(v_i, v_{2i}) = g(v_p, v_{2p+1})$$
$$f(v_i)f(v_{2i}) = f(v_p)f(v_{2p+1})$$
$$i(2i) = p(2p+1) \Rightarrow i = \sqrt{\frac{p(2p+1)}{2}}$$

This is a contradiction for i.
Hence $g(e_i) \neq g(e_p)$, for all $1 \leq i \leq 2^{n-1}-1$.
Hence all the edge labelings set in E_1 and E_2 are distinct.
We shall show that the labeling of edges E_1 and E_3 are distinct.
If $e_i \in E_1$ and $e_p \in E_3$ be two distinct edges at levels l, then to prove $g(e_i) \neq g(e_p)$.
For $i \neq p, 1 \leq i \leq 2^{n-1}-1, 0 \leq p \leq (2^{l-2}-1), 2 \leq l \leq n$
Assume that $g(e_i) = g(e_p)$.

$$g(v_i, v_{2i}) = g(v_{2^{l-1}+2p}, v_{2^{l-1}+2p+1})$$
$$f(v_i)f(v_{2i}) = f(v_{2^{l-1}+2p})f(v_{2^{l-1}+2p+1})$$
$$i(2i) = (2^{l-1}+2p)(2^{l-1}+2p+1)$$

$$\Rightarrow i = \sqrt{\frac{(2^{l-1}+2p)(2^{l-1}+2p+1)}{2}}$$
This is a contradiction for i.
Hence $g(e_i) \neq g(e_p)$, for all $1 \leq i \leq 2^{n-1}-1$.
Therefore all the induced edge labeling in E_1 and E_3 are distinct.
Hence all the induced edge labelings in E are distinct.
Hence the X-tree $XT(n)$ is strongly multiplicative for all positive $n \geq 2$. □

Theorem 3.2. *The Hypertree as $HT(n)$ is strongly multiplicative for all $n \geq 2$.*

Proof. Let $HT(n)$ be the Hypertree, with the vertex set $V = \{v_i/\ 1 \leq i \leq 2^n-1\}$ and the edge set $E = E_1 \cup E_2 \cup E_3$ where
$E_1 = \{e_i = (v_i, v_{2i})\ /\ 1 \leq i \leq 2^{n-1}-1\}$,
$E_2 = \{\gamma_i = (v_i, v_{2i+1})\ /\ 1 \leq i \leq 2^{n-1}-1\}$ and
$E_3 = \{\delta_i = (v_i, v_{2^{l-2}+i})\ /\ 2^{l-1} \leq i \leq 2^{l-1}+(2^{l-2}-1), 2 \leq l \leq n\}$.
To prove that $HT(n)$ is strongly multiplicative.
Define a bijective mapping $f : V \longrightarrow N$ such that $f(v_i) = i$ for all $1 \leq i \leq 2^n-1$.
Proving that all the edge labelings in E_1 and E_2 are distinct is similar to that of X - tree.

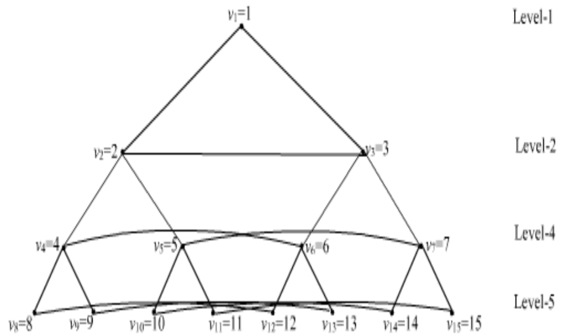

Fig. 2. Hypertree $HT(4)$

We shall prove the edge labelings in E_3 are distinct.
Define an edge induced function $g : E_3 \rightarrow N$ such that for any $e_i \in E_3$, $g(e_i) = f(v_i)f(v_{v_{2^{l-2}+i}})$, where $2^{l-1} \leq i \leq 2^{l-1}+2^{l-2}-1,\ 2 \leq l \leq n$.
Case 1. If e_i and e_p are distinct edges at different levels l_r and l_s in E_3, then to prove $g(e_i) \neq g(e_p)$.
For $i \neq p, 2^{l-1} \leq i,\ p \leq 2^{l-1}+2^{l-2}-1,\ 2 \leq l \leq n$
Assume that $g(e_i) = g(e_p)$

$$g(v_i, v_{2^{l_r-2}+i}) = g(v_p, v_{_{2^{l_s-2}+p}})$$
$$f(v_i)f(v_{2^{l_r-2}+i}) = f(v_p)f(v_{v_{2^{l_s-2}+p}})$$
$$i(2^{l_r-2}+i) = p(2^{l_s-2}+p)$$
$$(i+2^{l_r-3}) = \sqrt{(p+2^{l_s-3})^2-(2^{l_s-3})^2+(2^{l_r-3})^2}$$

This is a contradiction for i.
Hence $g(e_i) \neq g(e_p)$, for all $2^{l-1} \leq i \leq 2^{l-1}+2^{l-2}-1, 2 \leq l \leq n$.
Case 2. If e_i and e_p are distinct edges at the same level in E_3 then to prove $g(e_i) \neq g(e_p)$.
For $i \neq p,\ 2^{l-1} \leq i,\ p \leq 2^{l-1}+2^{l-2}-1,\ 2 \leq l \leq n$
Assume that $g(e_i) = g(e_p)$

$$g(v_i, v_{2^{l-2}+i}) = g(v_p, v_{2^{l-2}+p})$$
$$f(v_i)f(v_{2^{l-2}+i}) = f(v_p)f(v_{2^{l-2}+p})$$
$$i(2^{l-2}+i) = p(2^{l-2}+p)$$
$$i = -p-2^{l-2}, \text{ which is a contradiction for } i$$

Hence $g(e_i) \neq g(e_p)$, for all $2^{l-1} \leq i \leq 2^{l-1}+2^{l-2}-1$, $2 \leq l \leq n$.
Hence all the edge labelings in E_3 are distinct.
We shall show that the labeling of edges E_1 and E_3 are distinct.
If e_i and e_p are distinct edges in E_1 and E_3, then to prove $g(e_i) \neq g(e_p)$.
For $i \neq p,\ 1 \leq i \leq 2^{n-1}-1,\ 2^{l-1} \leq p \leq 2^{l-1}+2^{l-2}-1$, $2 \leq l \leq n$, assume that $g(e_i) = g(e_p)$

$$g(v_i, v_{2i}) = g(v_p, v_{2^{l-2}+p})$$
$$f(v_i)f(v_{2i}) = f(v_p)f(v_{2^{l-2}+p})$$
$$i(\ 2i) = p(2^{l-2}+p) \Rightarrow i = \sqrt{\frac{p(2^{l-2}+p)}{2}}$$

This is a contradiction for i.
Hence $g(e_i) \neq g(e_p)$, for all $1 \leq i \leq 2^{n-1}-1$.
Therefore all the induced edge labelings in E_1 and E_3 are distinct. The induced edge labeling in E_1 and E_2 can be similarly proved to be distinct as in the case of $X-$Tree.
Hence all the induced edge labelings in E are distinct.
Hence the hypertree $HT(n)$ is strongly multiplicative for all positive $n \geq 2$. □

Theorem 3.3. *The Shuffle Hypertree $SHT(n)$ is strongly multiplicative for all $n \geq 2$.*

Proof. Let $SHT(n)$ be the Shuffle Hypertree, with the vertex set $V = \{v_i / \ 1 \leq i \leq 2^n - 1\}$ and the edge set $E = E_1 \cup E_2 \cup E_3$ where
$E_1 = \left\{e_i = (v_i, v_{2i}) \ / \ 1 \leq i \leq 2^{n-1}-1\right\}$,
$E_2 = \left\{\gamma_i = (v_i, v_{2i+1}) \ / \ 1 \leq i \leq 2^{n-1}-1\right\}$ and
$E_3 = \{\delta_i = [(v_{2^{i-1}+2k-1}, v_{2^{i-1}+2k})/3 \leq i \leq n, 1 \leq k \leq 2^{i-2}-1] \cup [(v_{2^{i-1}}, v_{2^i-1})/2 \leq i \leq n]\}$.
To prove that $SHT(n)$ is strongly multiplicative,
define a bijective mapping $f : V \longrightarrow N$ such that $f(v_i) = i$ for all $1 \leq i \leq 2^n - 1$.

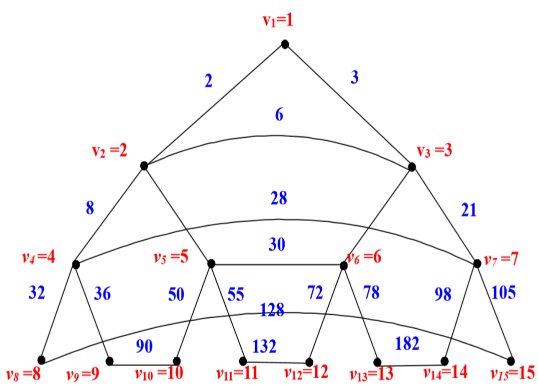

Fig. 3. Shuffle Hypertree $SHT(4)$

To prove in shuffle hypertree that all the edge labelings in E_1 and E_2 are distinct is similar to that X - tree.

We shall prove the edge labelings in E_3 are distinct.
Case1: Define an edge induced function $g : E_3 \rightarrow N$ such that for all $e_i \in E_3$,
$g(e_i) = f(v_{2^{i-1}+2k-1})f(v_{2^{i-1}+2k}), 3 \leq i \leq n, 1 \leq k \leq 2^{i-2}-1$
If e_i and e_p are distinct edges in E_3 then to prove $g(e_i) \neq g(e_p)$.
For $i \neq p, 3 \leq i, p \leq n, 1 \leq k, r \leq 2^{i-2}-1$
Assume that $g(e_i) = g(e_p)$.

$$g(v_{2^{i-1}+2k-1}, v_{2^{i-1}+2k}) = g(v_{2^{p-1}+2r-1}, v_{2^{p-1}+2r})$$
$$f(v_{2^{i-1}+2k-1})f(v_{2^{i-1}+2k}) = f(v_{2^{p-1}+2r-1})f(v_{2^{p-1}+2r})$$
$$(2^{i-1}+2k-1)(2^{i-1}+2k) = (2^{p-1}+2r-1)(2^{p-1}+2r)$$
$$(x-1)x = (y-1)y, \text{ where } x = 2^{i-1}+2k, y = 2^{p-1}+2r$$
$$x = -1-y \Rightarrow k = -1/2[1+2^{i-1}+2^{p-1}+2r]$$

This is a contradiction for k. Hence $g(e_i) \neq g(e_p)$.
Case2: Define an edge induced function $g : E_3 \rightarrow N$ such that for all $e_i \in E_3$, $g(e_i) = f(v_{2^{i-1}})f(v_{2^i-1}), 2 \leq i \leq n$
If e_i and e_p are distinct edges in E_3,
then to prove $g(e_i) \neq g(e_p)$.
For $i \neq p, \ 2 \leq i, p \leq n$, assume that $g(e_i) = g(e_p)$.

$$g(v_{2^{i-1}}, v_{2^i-1}) = g(v_{2^{p-1}}, v_{2^p-1})$$
$$f(v_{2^{i-1}})f(v_{2^i-1}) = f(v_{2^{p-1}})f(v_{2^p-1})$$
$$2^{i-1}(2^i-1) = 2^{p-1}(2^p-1)$$
$$2^{i-1}x = 2^{p-1}y, \text{ where } x = 2^i-1, y = 2^p-1$$
$$\Rightarrow 2^i = 1+2^{p-i}(2^p-1)$$

This is a contradiction for i.
Hence $g(e_i) \neq g(e_p)$, for all $2 \leq i, p \leq n$.
Case 3: If $e_i = (v_{2^{i-1}+2k-1}, v_{2^{i-1}+2k})$ and $e_p = g(v_{2^{p-1}}, v_{2^p-1})$ be distinct edges in $E_3, 3 \leq i \leq n, 1 \leq k \leq 2^{i-2}-1, 2 \leq p \leq n$, then to Prove $g(e_i) \neq g(e_p)$.
For $i \neq p$, assume that $g(e_i) = g(e_p)$

$$g(v_{2^{i-1}+2k-1}, v_{2^{i-1}+2k}) = g(v_{2^{p-1}}, v_{2^p-1})$$
$$f(v_{2^{i-1}+2k-1})f(v_{2^{i-1}+2k}) = f(v_{2^{p-1}})f(v_{2^p-1})$$
$$(2^{i-1}+2k-1)(2^{i-1}+2k) = 2^{p-1}(2^p-1)$$
$$(x-1)x = 2^{p-1}(2^p-1), \text{ where } x = 2^{i-1}+2k$$
$$(x-1/2)^2 = 2^{p-1}(2^p-1)+1/4$$
$$2^{i-1} = 1/2-2k+\sqrt{2^{p-1}(2^p-1)+1/4}$$

This is a contradiction for i.
Hence $g(e_i) \neq g(e_p)$, for all $3 \leq i \leq n$.
Hence all th edge labeling set in E_3 is distinct.
The induced edge labeling in E_1 and E_2 can be similarly proved to be distinct as in the case of $X-$Tree.
We shall show that the labeling of edges E_1 and E_3 are distinct.
If e_i and e_p are distinct edges in $E_1$and E_3, to prove $g(e_i) \neq g(e_p)$.
Case 1: For $i \neq p, 1 \leq i \leq 2^{n-1}, 3 \leq p \leq n, 1 \leq r \leq 2^{i-2}-1$
Assume that $g(e_i) = g(e_p)$

$$g(v_i, v_{2i}) = g(v_{2^{p-1}+2r-1}, v_{2^{p-1}+2r})$$
$$f(v_i)f(v_{2i}) = f(v_{2^{p-1}+2r-1})f(v_{2^{p-1}+2r})$$
$$i(2i) = (2^{p-1}+2r-1)(2^{p-1}+2r)$$
$$\Rightarrow i = \sqrt{\frac{(2^{p-1}+2r-1)(2^{p-1}+2r)}{2}}$$

This is a contradiction for i. Hence $g(e_i) \neq g(e_p)$.
Case 2: For $i \neq p, 1 \leq i, p \leq 2^{n-1}$,
Assume that $g(e_i) = g(e_p)$

$$g(v_i, v_{2i}) = g(v_{2^{p-1}}, v_{2^p-1})$$
$$f(v_i)f(v_{2i}) = f(v_{2^{p-1}})f(v_{2^p-1})$$

$$i(2i) = 2^{p-1}(2^p - 1)$$
$$\Rightarrow i = \sqrt{\frac{2^{p-1}(2^p - 1)}{2}},$$

This is a contradiction for i. Hence $g(e_i) \neq g(e_p)$.
Therefore all the induced edge labeling in E_1 and E_3 are distinct.
Thus all the edge labeling of $E_{1,}$ E_2 and E_3 are distinct. Also the induced edge labeling of E_1 & E_2 and E_2 & E_3 are distinct. Hence the Shuffle Hypertree Network has strongly multiplicative labeling for all $n \geq 2$. □

4. Conclusion

In this paper we have proved that $X-$Tree $XT(n)$, Hypertree $HT(n)$ and Shuffle Hypertree $SHT(n)$ are strongly multiplicative for all $n \geq 2$. Finding strongly multiplicative labeling for other tree derived networks is quite challenging.

References

[1] B. D. Acharya, K. A. Germina and V. Ajitha, *Multiplicatively Indexable Graphs in Labeling Discrete Structures and Applications*, Narosa Publishing House, New Delhi, pp. 29–40, 2008.

[2] L. W. Beineke and S. M. Hegde, Strong Multiplicative Graphs, *Discuss. Math Graph Theory*, 21(2001), 63–75.

[3] G. S. Bloom and S. W. Golomb, Applications of Numbered Undirected Graphs, *Proceeding of IEEE*, 165(4)(1977), 562–570.

[4] Chun-Nan Hung, Lih-Hsing Hsu, Ting-Yi Sung, Christmas Tree, A versatile 1-fault tolerant design for token rings, *Information Processing Letters*, 72(1999), 55–63.

[5] J. A. Gallian, A Dynamic Survey of Graph Labeling, *Electronic Journal of Combinatorics*, 2015.

[6] K. K. Kanani and T. M. Chhaya, Strongly Multiplicative Labeling of some path related graphs, *International Journal of Mathematics and Computer Applications Research*, 5(5)(2015), 1–4.

[7] M. A. Seoud and A. Zid, Strong multiplicativity of unions and corona of paths and complete graphs, *Proc. Math. Phys. Soc. Egypt*, 74(1999), 59–71.

[8] D.Paul, Indra Rajasingh, B.Sundara Rajan, Acyclic edge colouring of Hypertreeand Shuffle Hypertree, *International Journal of Pure and Applied Mathematics*, 2(101)(2015), 623–629.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆