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$\delta\hat{g}$ Closed sets in grill topological spaces

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Abstract
In this paper we introduce a new type of closed sets called $\varsigma\delta\hat{g}$ closed sets in a topological space X, defined in terms of grills. The relations between $\varsigma\delta\hat{g}$ closed sets and already available closed sets are discussed.

Keywords
δ closed sets, ω closed sets, $\varsigma\omega$ closed sets, r closed sets, ςg closed sets.

AMS Subject Classification
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Contents

1. Introduction

The concept of grill topological spaces was first introduced by Choquet [1] in 1947. B. Roy and M. N. Mukherjee [4] defined on a typical topology induced by a grill. Shyamapada Modak & Sukalyan Mistry [5] invented grill on generalized topological spaces. Rodyna A. Hosny [3] introduce the concept of δ using grill and obtain grill delta space. M. Lellis Thivagar et al [2] introduced $\delta\hat{g}$ closed sets. Veera Kumar [6] introduced $\hat{g}$ closed sets in topological spaces.

2. Preliminaries

Let (X,τ) be a topological space with no separation properties assumed. A sub-collection ς of $P(X)$ is called a grill on X if ς satisfies the conditions

(i) $\phi \notin \varsigma$

(ii) $A \in \varsigma$ and $A \subseteq B \subseteq X \Rightarrow B \in \varsigma$

(iii) $A, B \subseteq X \& A \cup B \in \varsigma$ implies that $A \in \varsigma$ or $B \in \varsigma$.

For any point x of a topological space (X,τ), $\tau(x)$ denotes the collection of all open neighborhoods of x.

Definition 2.1. *Let $[X, \tau^{\delta}, \varsigma]$ be a grill delta space. A mapping $\varphi_{\delta} : P(X) \to P(X)$ denoted by $\varphi_{\delta}(A)$ is called the operator associated with grill ς and the topology τ^{δ} and is defined by $\varphi_{\delta} = \{x \in X / A \cap U \in \varsigma$ for all $U \in \delta O(X,\tau)\}$. A mapping $\psi_{\delta} : P(X) \to P(X)$ is defined as $A \cup \varphi_{\delta}(A)$ for all $A \in P(X)$.*

Definition 2.2. *To a grill delta space $(X, \tau^{\delta}, \varsigma)$ there exists a unique topology $\tau_{\varsigma}^{\delta}$ on X given by*

$$\tau_{\varsigma}^{\delta} = \{U \subseteq X : \psi_{\delta}(X \backslash U) = (X \backslash U)\}$$

where for any $A \subseteq X$, $\psi_{\delta}(A) = A \cup \varphi_{\delta}(A) = \tau_{\varsigma}^{\delta} - cl_{\delta}(A)$.

3. $\varsigma\delta\hat{g}$-closed sets

Definition 3.1. *Let (X,τ) be a topological space and ς be a grill on X. Then a subset A of X is said to be a $\varsigma\delta\hat{g}$ closed if $\varphi_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\varsigma\omega$ open in (X,τ,ς).*

Proposition 3.2. *Every δ closed set is $\varsigma\delta\hat{g}$ closed.*

Proof. Let A be a δ closed set and U be any $\varsigma\omega$ open set containing A. Since A is δ closed then $\varphi_{\delta}(A) \subseteq cl_{\delta}(A) = A$ for every subset A of X. Therefore $\varphi_{\delta}(A) = A \subseteq U$ and hence A is $\varsigma\delta\hat{g}$ closed. □

Remark 3.3. *The converse of the above theorem is false as shown in the following example.*

Example 3.4. *Let $X = \{p,q,r\}$, $\tau = \{\phi, X, \{p\}, \{q,r\}\}$,*
$\varsigma = \{X, \{p\}, \{r\}, \{p,q\}, \{q,r\}, \{p,r\}\}$
δ closed $= \{\phi, X, \{p\}, \{q,r\}\}$,
$\varsigma\delta\hat{g}$ closed$=\{\phi, X, \{p\}, \{q\}, \{p,q\}, \{q,r\}\}$.
Here $\{p,q\}$ is $\varsigma\delta\hat{g}$ closed but not δ closed in (X,τ).

Proposition 3.5. *Every regular closed set is $\varsigma\delta\hat{g}$ closed.*

Proof. Assume A is regular closed set and U be any $\varsigma\omega$ open set containing A. Here A is regular closed set. This implies A is δ closed and $cl_\delta(A) = A$ for any subset A in (X,τ) and $\varphi_\delta(A) \subseteq cl_\delta(A) \subseteq U$.

Hence A is $\varsigma\delta\hat{g}$ closed. □

Remark 3.6. *The converse of the above theorem is false as shown in the following example.*

Example 3.7. *Let $X = \{p,q,r\}$, $\tau = \{\phi,X,\{q\},\{p,r\}\}$, $\varsigma = \{X,\{p\},\{q\},\{p,q\}\}$ $\varsigma\delta\hat{g}$ closed = $\{\phi,X,\{q\},\{r\},\{q,r\},\{p,r\}\}$ rclosed= $\{\phi,X,\{p,r\}\}$.*

Here $\{q\}$ is $\varsigma\delta\hat{g}$ closed but not regular closed.

Proposition 3.8. *Every $\varsigma\delta\hat{g}$ closed set is ςg closed set.*

Proof. Consider A to be a $\varsigma\delta\hat{g}$ closed set and U be any open set containing A in (X,τ). Since every open set is $\varsigma\omega$ open and A is $\varsigma\delta\hat{g}$ closed. Then $\varphi_\delta(A) \subseteq U$ for every subset A of X. $\varphi(A) \subseteq \varphi_\delta(A) \subseteq U \Rightarrow \varphi(A) \subseteq U$ and hence A is ςg closed. □

Remark 3.9. *The converse of the above theorem is false as shown in the following example.*

Example 3.10. *Let (X,τ) be a topological space with $X = \{p,q,r\}$, $\tau = \{\phi,X,\{p\}\}$, $\varsigma = \{X,\{p\},\{p,q\},\{q,r\}\}$ ςg closed $= \{\phi,X,\{q\},\{r\},\{p,q\},\{q,r\},\{p,r\}\}$ $\varsigma\delta\hat{g}$ closed $= \{\phi,X,\{q\},\{r\},\{q,r\},\{p,r\}\}$. Here $\{p,q\}$ is ςg closed but not $\varsigma\delta\hat{g}$ closed .*

Proposition 3.11. *Every $\varsigma\delta\hat{g}$ closed set is $\varsigma g\delta s$ closed set.*

Proof. Let A be a $\varsigma\delta\hat{g}$ closed set and U be any δ- open set containing A in (X,τ). Since δ- open set is $\varsigma\omega$ open and A is $\varsigma\delta\hat{g}$ closed. Then $\varphi_\delta(A) \subseteq U$ for every subset A of X.

Therefore A is $\varsigma g\delta s$ closed. □

Remark 3.12. *The converse of the above theorem is not true as shown in the following example.*

Example 3.13. *Let $X = \{p,q,r\}$, $\tau = \{\phi,X,\{q\}\}$, $\varsigma = \{X,\{q\},\{p,r\},\{q,r\}\}$*

$\varsigma\delta\hat{g}$ closed= $\{\phi,X,\{p\},\{r\},\{p,q\}\}$

$\varsigma g\delta s$ closed= $\{\phi,X,\{p\},\{q\},\{r\},\{p,q\},\{q,r\},\{p,r\}\}$.

Here $\{q\}$ is $\varsigma g\delta s$ closed but not $\varsigma\delta\hat{g}$ closed .

Remark 3.14. *The following example shows that $\varsigma\delta\hat{g}$ closedness is independent from closedness , ω- closedness , $\varsigma\omega$ closedness, δg closedness and δg^* closedness.*

Example 3.15. *Let $X = \{p,q,r\}$, $\tau = \{\phi,X,\{q\},\{q,r\}\}$, $\varsigma = \{X,\{p\},\{q\},\{p,q\}\}$*

$\varsigma\delta\hat{g}$ closed=$\{\phi,X,\{r\},\{q,r\},\{p,r\}\}$

δg closed= δg^* closed $= \{\phi,X,\{p\},\{p,q\},\{p,r\}\}$

$\varsigma\omega$ closed= $\{\phi,X,\{p\},\{r\},\{q,r\},\{p,r\}\}$.

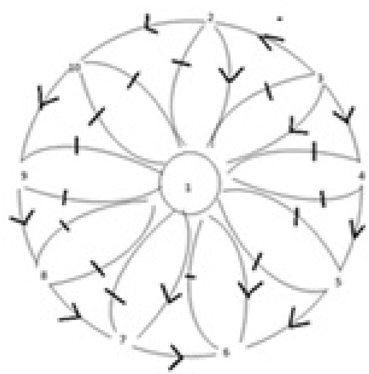
Here $\{p\}$ is δg closed, δg^* closed and $\varsigma\omega$ closed but not $\varsigma\delta\hat{g}$ closed.

Example 3.16. *Let $X = \{p,q,r\}$, $\tau = \{\phi,X,\{r\},\{q,r\}\}$, $\varsigma = \{X,\{p\},\{r\},\{p,q\},\{q,r\},\{p,r\}\}$.*

Here $\{q\}$ is $\varsigma\delta\hat{g}$ closed but not δg closed, δg^* closed and $\varsigma\omega$ closed in (X,τ).

Example 3.17. *Let $X = \{p,q,r\}$, $\tau = \{\phi,X,\{q\},\{p,q\}\}$, $\varsigma = \{X,\{p\},\{r\},\{p,q\},\{q,r\},\{p,r\}\}$. Here $\{r\}$ is closed and ω closed but not $\varsigma\delta\hat{g}$ closed whereas $\{q\}$ is $\varsigma\delta\hat{g}$ closed but not ω closed and closed in (X,τ).*

Remark 3.18. *The relationship of $\varsigma\delta\hat{g}$ closed sets with known existing sets is given below. $A \to B$ represents A implies B but not conversely.*



1. $\varsigma\delta\hat{g}$ closed, 2. δ closed, 3. r closed, 4. δg^* closed,
5. δg closed, 6. $\varsigma g\delta s$ closed, 7. ςg closed,
8. $\varsigma\omega$ closed, 9. ω closed, 10. closed.

Theorem 3.19. *Let $(X,\tau^\delta,\varsigma)$ be a grill delta space. If a subset A of X is $\varsigma\delta\hat{g}$ closed then $\tau^\delta_\varsigma - cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $\varsigma\omega$ open.*

Proof. Let A be a $\varsigma\delta\hat{g}$ closed set and U be $\varsigma\omega$ open in X such that $A \subseteq U$ then $\varphi_\delta(A) \subseteq U \Rightarrow A \cup \varphi_\delta(A) \subseteq U \Rightarrow \tau^\delta_\varsigma - cl_\delta(A) \subseteq U$.

Thus $\tau^\delta_\varsigma - cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $\varsigma\omega$ open. □

Theorem 3.20. *Let $(X,\tau^\delta,\varsigma)$ be a grill delta space. If a subset A of X is $\varsigma\delta\hat{g}$ closed then for all $x \in \tau^\delta_\varsigma - cl_\delta(A) \subseteq U cl_\delta(\{x\}) \cap A \neq \phi$.*

Proof. Let $x \in \tau^\delta_\varsigma - cl_\delta(A)$. If $cl_\delta(\{x\}) \cap A = \phi$ $\Rightarrow A \subseteq X \backslash cl_\delta(\{x\})$ then by Theorem 3.19 $\tau^\delta_\varsigma - cl_\delta(A) \backslash A \subseteq X \backslash cl_\delta(\{x\})$ which is a contradiction to our assumption that $x \in \tau^\delta_\varsigma - cl_\delta(A)$. Therefore, $cl_\delta(\{x\}) \cap A \neq \phi$. □

Lemma 3.21. *Let (X,τ^δ) be a space and ς be a grill on X. If $A (\subseteq X)$ is τ^δ_ς- dense in itself, then $\varphi_\delta(A) = cl_\delta(\varphi_\delta(A)) = \tau^\delta_\varsigma - cl_\delta(A) = cl_\delta(A)$.*

Proof. Assume A to be τ^δ_ς- dense in itself. $\therefore A \subseteq \varphi_\delta(A)$.

Thus $cl_\delta(A) \subseteq cl_\delta(\varphi_\delta(A)) = \varphi_\delta(A) \subseteq cl_\delta(A)$.

This implies that $cl_\delta(A) = \varphi_\delta(A) = cl_\delta(\varphi_\delta(A))$.

Now by definition

$$\tau^\delta_\varsigma - cl_\delta(A) = A \cup \varphi_\delta(A) = A \cup cl_\delta(A) = cl_\delta(A).$$

Therefore

$$\varphi_\delta(A) = cl_\delta(\varphi_\delta(A)) = \tau_\varsigma^\delta - cl_\delta(A) = cl_\delta(A).$$

□

Theorem 3.22. *Let ς be a grill on a space (X, τ^δ). If $A(\subseteq X)$ is τ_ς^δ-dense in itself and $\varsigma\delta\hat{g}$ closed, then A is $\delta\omega$ closed.*

Proof. From Lemma 3.21 □

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