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Travelling salesman model in fuzzy environment

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Abstract

In classical travelling salesman model, the objective is to visit n cities, starting from his home city and returning to home city, with minimum cost. In this paper, travelling cost is represented by trapezoidal fuzzy number. TrFN is defuzzified by using linear ranking function proposed by Maleki [22]. Classical travelling salesman model is extended to solve FNTSP.

Keywords

Trapezoidal fuzzy number, Linear Ranking function.

AMS Subject Classification

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Contents

1. Introduction

The traveling salesman problem (TSP) was first proposed by Irish Mathematician W.R. Hamilton in the 19th century. Given n cities and distance, the salesman starts from city 1, then any permutation of $2,3,\ldots,n$ represent the number of possible ways for his tour. So there are $(n-1)!$ possible ways for his tour. The problem is to select an optimal route that could achieved his objective.

The (TSP) can be classified as:

(i) Symmetrical: If the distance between every pair of cities is independent of the direction of his journey.

(ii) Asymmetrical: If for one or more pair if cities the distance changes with the direction.

In real world problem time and cost are not fixed. it may be due to:-

- The actual cost of travelling depends on the current fuel consumptions, which is partly affected by the speed but the speed is externally determined by the current traffic.
- The fuel price is also fluctuating depending the international scenario.
- A relatively large proportion of transportation cost is the cost of labour, which is calculated on driving time.
- As transportation sector is a capital intensive unit, hence, utilization of vehicle in time is a crucial problem.

If the cost or time or distance is represented by fuzzy numbers, then it becomes a fuzzy travelling salesman problem (FTSP).

In recent years, several technique are adopted for solving FTSP, [7, 10–17].

Here classical traveling salesman model is extended to solve FNTSP. It is organized as, in part 2, fundamental concepts on TrFN & ranking function are given. In part 3 formulation of FNTSP as fuzzy number assignment problem. In part 4, the procedure of the proposed algorithm is discussed. In part 5, two numerical examples given & in part 6, the paper is concluded.

2. Fundamental of fuzzy set theory

The term fuzzy was proposed by Zadeh [20] in 1962. In [21], he published the paper entitled fuzzy sets in the year 1965. For more details on this theory, we suggest the reader to refer [1–6].

Definition 2.1. *Let X is a collections of objects denoted generically by X, then a fuzzy set in X is a set of ordered pairs* $\underline{A} = \{x, \mu_{\underline{A}}(x) | x \in X, \mu_{\underline{A}}(x) \in [0,1]\}$*, where* $\underline{A}$ *is called the membership function.*

Definition 2.2. *The support of a fuzzy set* $\underline{A}$ *is the crisps set defined by* $\underline{A} = \{x \in X | \mu_{\underline{A}} > 0\}$.

Definition 2.3. *The core of a fuzzy set* $\underline{A}$ *is the crisp set of points* $x \in X$ *with* $\mu_{\underline{A}} = 1$.

Definition 2.4. *The boundary of a fuzzy set* $\underline{A}$ *are defined set of points* $x \in X$ *such that* $0 < \mu_{\underline{A}} < 1$.

It is evident that the boundary is defined as the region of the universal set containing elements that have non-zero membership but not complete membership. The Fig.1 illustrates the region.

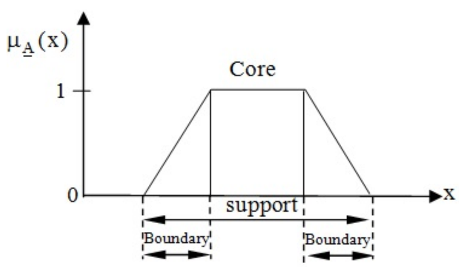

Figure 1

Definition 2.5. *A fuzzy set* $\underline{A}$ *is normal if and only there exists* $x_i \in X$ *such that* $\mu_{\underline{A}}(x_i) = 1$.

Definition 2.6. *A fuzzy set* $\underline{A}$ *is sub normal if* $\mu_{\underline{A}} < 1$.

Definition 2.7. *The* α*-cut of a fuzzy set* $\underline{A}$ *denoted by* $[\underline{A}]_\alpha$ *and is defined by* $[\underline{A}]_\alpha = \{x \in X | \mu_{\underline{A}}(x) \geq \alpha\}$. *If* $\mu_{\underline{A}}(x) > \alpha$*, then* $[\underline{A}]_\alpha$ *is called strong* α*-cut. It is clear that* α*-cut (strong* α*-cut) is a crisp set.*

Definition 2.8. *A fuzzy set* $\underline{A}$ *on X is convex if for any* x_1, x_2 *and* $\lambda \in [0,1], \mu_{\underline{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\underline{A}}(x_1), \mu_{\underline{A}}(x_2)\}$.

It is to be noted that a fuzzy set is convex if and only if its α–cut is convex.

Definition 2.9. *A fuzzy number is a fuzzy subset in universal set X which is both convex and normal.*

Definition 2.10. *A fuzzy number* $\underline{A} = \{a, b, c, s\}$ *is said to be a trapezoidal fuzzy number if its membership function is given by*

$$\mu_{\underline{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)}, & q \leq x < b \\ 1, \quad b \leq x \leq c \\ \dfrac{(x-d)}{(c-d)}, & c < x \leq d \\ 0, \quad \textit{otherwise}. \end{cases}$$

Definition 2.11. *Let* $\underline{A} = \{a^L, a^U, \alpha, \beta\}$ *be the TrFN, where* $(a^L - \alpha, a^U + \beta)$ *is the support of* $\underline{A}$ *and* $[a^L, a^U]$ *is the core of* $\underline{A}$.

Arithmetic on Trapezoidal Fuzzy Numbers

Let $F(\mathbb{R})$ be set of all trapezoidal fuzzy numbers over the real line $\mathbb{R}$. The arithmetic operations on trapezoidal fuzzy numbers are defined as follows:

Let $\underline{a} = (a^L, a^U, \alpha, \beta)$ and $\underline{b} = (b^L, b^U, \gamma, \delta) \left(\frac{\pi}{2} - \theta\right)$ be two trapezoidal fuzzy numbers and $x \in \mathbb{R}$. We define

$$\begin{aligned} & x > 0, x \in \mathbb{R}; x\underline{a} = (xa^L, xa^U, x\alpha, x\beta), \\ & x < 0, x \in \mathbb{R}; x\underline{a} = (xa^U, xa^L, -x\beta, -x\alpha), \\ & \underline{a} + \underline{b} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \delta), \\ & \underline{a} - \underline{b} = (a^L - b^L, a^U - b^U, \alpha + \gamma, \beta + \delta). \end{aligned}$$

Ranking Function

Ranking is one of the effective method for ordering fuzzy numbers. Various types of ranking function have been introduced and some have been used for solving linear programming problems with fuzzy parameters. An effective approach for ordering the element of $F(\mathbb{R})$ is to define a ranking function.

Let $\mathbb{R} : F(\mathbb{R}) \to \mathbb{R}$. We define order on $F(\mathbb{R})$ as follow:

(i) $\underline{a} \geq \underline{b}$ iff $\mathbb{R}(\underline{a}) \geq \mathbb{R}(\underline{b})$,

(ii) $\underline{a} > \underline{b}$ iff $\mathbb{R}(\underline{a}) > \mathbb{R}(\underline{b})$,

(iii) $\underline{a} = \underline{b}$ iff $\mathbb{R}(\underline{a}) = \mathbb{R}(\underline{b})$,

(iv) $\underline{a} \leq \underline{b}$ iff $\mathbb{R}(\underline{a}) \leq \mathbb{R}(\underline{b})$.

Here $\mathbb{R}$ is the ranking functions, such that $\mathbb{R}(k\underline{a} + \underline{b}) = k\mathbb{R}(\underline{a}) + \mathbb{R}(\underline{b})$.

Here, we introduce a linear ranking function that is similar to the ranking function adopted by Maleki [22]. For a trapezoidal fuzzy number $\underline{a} = (a^L, a^U, \alpha, \beta)$, we use ranking function as follows:

$$\mathbb{R}(\underline{a}) = \int_0^1 (\inf \underline{a}_\alpha + \sup \underline{a}_\alpha) d\alpha$$

which reduced to $\mathbb{R}(\underline{a}) = (a^L + a^U) + \frac{1}{2}(\beta - \alpha)$.

For any trapezoidal fuzzy numbers $\underline{a} = (a^L, a^U, \alpha, \beta)$ and $\underline{b} = (b^L, b^U, \gamma, \delta)$, we have $a \geq b$ if and only if $a^L + a^U + \frac{1}{2}(\beta - \alpha) \geq b^L + b^U + \frac{1}{2}(\delta - \gamma)$.

3. Formulation of fuzzy number travelling salesman problem as fuzzy number assignment problems

A travelling salesman has to visit n cities & return to the starting city. He has to start from any one city & visit each city only once.

Let distance or cost or time of travel from ith city to jth city be $\underline{C}_i j$. Let the variable x_{ij} be defined as

$$x_{ij} = \begin{cases} 1 & \text{if from city} \quad i \quad \text{to city} \quad j \\ 0 & \text{otherwise.} \end{cases}$$

Then the objective function is

$$\min \underline{Z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \underline{C}_{ij} x_{ij}$$

subject to the constraint $\sum_{j=1}^{n} x_{ij} = 1$ for $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} x_{ij} = 1$ for $j = 1, 2, \ldots, n$ with $x_{ij} = 0$ or 1 for all i, j with additional restriction that x_{ij} must be so chosen that no city is visited twice before the tour is completed. In particular, he cannot go directly from city i to i. So, $\underline{C}_{ij} = \infty$. However, all $\mathbb{R}(\underline{C}_{ij}) >)$ and $\underline{C}_{ij} + \underline{C}_{jk}$ for all i, j, k. Remove x_{ij} from the problem & only single $x_{ij} = 1$ for each value of i and j.

The fuzzy cost matrix is shown in a tabular form as:

		To City					
		1	**2**	**3**	**...**	**...**	**n**
	1	∞	$\underline{C}_{12}$	$\underline{C}_{13}$	...	...	$\underline{C}_{1n}$
	2	$\underline{C}_{21}$	∞	$\underline{C}_{23}$	...	...	$\underline{C}_{2n}$
From City	**3**	$\underline{C}_{31}$		∞	...	...	$\underline{C}_{3n}$
	M	M	M		...	...	
	n	$\underline{C}_{n1}$	$\underline{C}_{n2}$	$\underline{C}_{n3}$	...	...	∞

4. Procedure

Step 1: Express the given fuzzy travelling salesman problem in a tabular form.
Step 2: Find the smallest fuzzy cost in each row of cost matrix. Subtract this smallest fuzzy cost element from each element of that row. This is called the first reduced fuzzy cost matrix.
Step 3: In the reduced fuzzy cost matrix, find the smallest element in each column. Subtract this smallest fuzzy cost element from each element in that column. This is called second reduced fuzzy cost matrix.
Step 4: Using linear ranking function express the second reduced fuzzy cost matrix in crisp form.
Step 5: Determine an optimum assignment as follows:

(i) Examine the rows successively until a row with exactly one zeros is found. Box around the zero element as an assigned cell & cross out all other zero in its column. Proceed in this manner until all the rows have been examined. If there are more than one zero in any row, then do not consider that row and pass on to the next row.

(ii) Repeat the procedure for the columns of the reduced cost matrix. If there is no single zero in any row or column of the reduced matrix, then arbitrarily choose a row or column having the minimum number of zeros. Arbitrarily, choose zero in the row or column and cross the remaining zeros in that row or column.

Step 6: An optimal assignment is found, if the number of assigned cells equal the number of rows (and columns). If a zero cell is arbitrarily chosen, there may be an alternate optimum. If no optimum solution is found (some rows or columns without an assignment), then go to next step.
Step 7: Draw the minimum number of horizontal and/or vertical lines through all the zeros as follows:

(i) Mark (✓) to those rows where no assignment has been made.

(ii) Mark (✓) to those columns which have zeros in the marked rows.

(iii) Mark (✓) rows (not already marked) which have assignments in marked columns.

(iv) The process may be repeated until no more rows or columns can be checked.

(v) Draw straight lines through all unmarked rows and marked columns.

Step 8: If the minimum number of lines passing through all the zeros is equal to the number of rows or columns, the optimum solution is attained by an arbitrary allocation in the positions of the zeros not crossed in Step 7. Otherwise go to the next step.
Step 9: Revise the fuzzy cost matrix as follows:

(i) Find the element that are not covered by a line. Select the smallest of there elements and subtract this element from all the uncrossed elements and add the same at the point of intersection of the two lines.

(ii) Other elements crossed by the lines remain unchanged.

Step 10: Go to Step 5 and repeat the procedure till an optimum fuzzy solution is attained.
Step 11: If the optimum fuzzy solution obtained in Step 10 satisfies the route condition of travelling salesman problem. Then it gives the optimum solution of fuzzy travelling salesman problem. If not necessary adjustment is to be made assigning the next best solution so as to satisfy the route condition of travelling salesman problem.

5. Numerical Examples

In this section, we provide some examples to illustrate the obtained theoretical results.

Example 5.1. *Solve fuzzy number travelling salesman problem*

		To City			
		1	2	3	4
	1	∞	(1, 1.5, 1, 1)	(6, 9, 2, 3)	(2, 3,1, 2)
From City	2	(6,7,1,2)	∞	(9,11,3,4)	(5,5,1,1)
	3	(9,9,1,1)	(8,9,2, 4)	∞	(4, 4,3,2)
	4	(10,11,3,4)	(3,4,2,3)	(7,8,1,3)	∞

Step 1: First reduced fuzzy cost matrix.

From City \ To City	1	2	3	4
1	∞	(-0.5, 0.5,2,2)	(4.5,8,3,4,)	(0.5,2,2,3)
2	(1, 2,2,3)	∞	(4, 6, 4, 5)	(0, 0, 2, 2)
3	(5,5,3,3)	(4,5,4,6)	∞	(0,0, 4,4)
4	(6, 8,6,6)	(-1,1,5,5)	(3,5,4,5)	∞

Step 2: Second reduced fuzzy cost matrix.

From City \ To City	1	2	3	4
1	∞	(-1.5, 1.5,7,7)	(-0.5,5,8,8)	(0.5,2,4,5)
2	(-1,1,5,5)	∞	(-1,3,9,9)	(0,0,4,4)
3	(3,4,6,5)	(3,6,9,11)	∞	(0,0,6,6)
4	(4,7,9,8)	(-2,2,10,10)	(-2,2,9,9)	∞

Step 3: Using the linear ranking function, expressing the reduced fuzzy cost matrix in crisp form.

From City \ To City	1	2	3	4
1	∞	0	4.5	3
2	0	∞	2	0
3	6.5	10	∞	0
4	10.5	0	0	∞

Step 4: : Examine each row successively, since first row contains one zero, make an assignment in first row and cross (x) all of the zeros present in its column. Again third row contains one zero, so make an assignment in the third row and cross (x) all of the zeros present in its column. Similarly examine each column successively and make an assignment in first column and third column and cross (x) all other zeros present in its row. Thus, we get

From City \ To City	1	2	3	4
1	∞	$\boxed{0}$	4.5	3.7
2	$\boxed{0}$	∞	2	~~0~~
3	6	11	∞	$\boxed{0}$
4	10	~~0~~	$\boxed{0}$	∞

Since each row and each column contains exactly one assignment, the current assignment is optimal. Hence optimal assignment is $1 \to 2, 2 \to 1, 3 \to 4, 4 \to 3$.

City ↓ \ City →	1	2	3	4
1	∞	$\boxed{-1.5, 1.5, 7, 7}$	(-0.5,5,8,8)	(0.5, 2, 4, 5)
2	$\boxed{-1, 1, 5, 5}$	∞	(-1, 3, 9, 9)	~~(0,0,4,4)~~
3	(3, 4, 6, 5)	(3, 6,9, 11)	∞	$\boxed{0, 0, 6, 6}$
4	(4,7,9,8)	~~(−2,2,10,10)~~	$\boxed{-2, 2, 9, 9}$	∞

The optimum fuzzy assignment is $1 \to 2, 2 \to 1, 3 \to 4, 4 \to 3$ with minimum travelling cost (18, 20.5, 5, 8). The fuzzy assignment schedule does not provide us the solution of the fuzzy travelling salesman problem as it gives $1 \to 2, 2 \to 1$, without passing through 3 and 4. Then we try to find the next best solution which satisfies the route restrictions. The next non zero minimum in the fuzzy cost matrix is (-1, 3, 9, 9). So we bring (-1, 3, 9, 9) into the solution but if does not satisfy the restriction of FTSP. Next we try with the next minimum fuzzy cost (0.5,2,4,5) which all does not satisfy the route restrictions. Ultimately the next smallest fuzzy cost (-0.5, 5, 8, 8) satisfies the route restrictions. Thus the fuzzy optimum assignment which satisfies the travelling salesman restriction is give by $1 \to 3, 2 \to 1, 3 \to 4, 4 \to 2$ i.e. $1 \to 3 \to 4 \to 2 \to 1$.

Hence, the optimal fuzzy travelling salesman assignment is

City ↓ \ City →	1	2	3	4
1	∞	~~$\boxed{-1.5, 1.5, 7, 7}$~~	$\boxed{-0.5, 5, 8, 8}$	(0.5, 2, 4, 5)
2	$\boxed{-1, 1, 5, 5}$	∞	(-1, 3, 9, 9)	~~(0,0,4,4)~~
3	(3, 4, 6, 5)	(3, 6,9, 11)	∞	$\boxed{0, 0, 6, 6}$
4	(4,7,9,8)	$\boxed{-2, 2, 10, 10}$	~~(−2,2,9,9)~~	∞

The optimum solution of the original fuzzy travelling salesman problem is

City ↓ \ City →	1	2	3	4
1	∞	(1,1.5,1,1)	$\boxed{6, 9, 2, 3}$	(2, 3, 1, 2)
2	$\boxed{6, 7, 1, 2}$	∞	(9,11, 3, 4)	(5, 5, 1, 1)
3	(9, 9, 1, 1)	(8, 9,2, 4)	∞	$\boxed{4, 4, 2, 2}$
4	(10,11,3,4)	$\boxed{3, 4, 2, 3}$	(7, 8, 1, 3)	∞

The optimal route for the fuzzy travelling salesman problem is $1 \to 3, 2 \to 1, 3 \to 4, 4 \to 2$ i.e. $1 \to 3 \to 4 \to 2 \to 1$ and the fuzzy minimum cost is (19, 24, 7,10).

Example 5.2. *Solve fuzzy travelling salesman problem*

From City \ To City	1	2	3	4
1	∞	(9, 10, 1, 3)	(6, 8, 3, 5)	(8, 9,1, 3)
2	(9,10,2,4)	∞	(10,11,3,1)	(4,5,1,3)
3	(7,8,1,3)	(10,11,3, 4)	∞	(7, 8,2,3)
4	(9,10,3,5)	(10,11,3,4)	(6,8,1,5)	∞

Step 1: First reduced fuzzy cost matrix.

From City \ To City	1	2	3	4
1	∞	(1, 4, 6, 6)	(-2,2,8, 8)	(10,3,6,6)
2	(4,6,5,5)	∞	(5,7,6,2)	(-1,1,4,4)
3	(-1,1,4,5)	(2,4,6, 6)	∞	(-1, 1,5,5)
4	(1,4,8,6)	(1,5,8,5)	(-2,2,6,6)	∞

Step 2: Second reduced fuzzy cost matrix.

From City \ To City	1	2	3	4
1	∞	(-4,3,11, 14)	(-4,4,14,14)	(9,4,10,10)
2	(3, 7,10,9)	∞	(3,9,12,8)	(-2,2,8,8)
3	(-2,2,9,9)	(-3,3,11,14)	∞	(-2,2,9,9)
4	(0,5,13,10)	(-4,4,13,13)	(-4,4,12,12)	∞

Step 3: Using the linear ranking function, expressing the reduced fuzzy cost matrix in crisp form.

		To City			
		1	2	3	4
From City	1	∞	0.5	0	13
	2	9.5	∞	10	0
	3	0	1.5	∞	0
	4	3.5	0	0	∞

Step 4: Examine each row successively, since first row contains one zero, make an assignment in first row and cross (x) all of the zeros present in its column. Again second row contains one zero, so make an assignment in the second row and cross (x) all of the zeros present in its column. Similarly examine each column successively and make an assignment in first column and second column and cross (x) all other zeros present in its row. Thus, we get,

		1	2	3	4
From City	1	∞	2	$\boxed{0}$	13
	2	9	∞	8	$\boxed{0}$
	3	$\boxed{0}$	3	∞	$\cancel{0}$
	4	8	$\boxed{0}$	$\cancel{0}$	∞

Since each row and each column contains exactly one assignment, the current assignment is optimal. Hence optimal assignment is $1 \to 3, 2 \to 4, 3 \to 1, 4 \to 2$.

City → / City↓	1	2	3	4
1	∞	(-4,3,11,14)	$\boxed{-4,4,14,14}$	(9,4, 10, 10)
2	(3, 7, 10,9)	∞	(3, 9, 12, 8)	$\boxed{-2,2,8,8}$
3	$\boxed{-2,2,9,9}$	(-3, 3,11, 14)	∞	~~(−2,2,9,9)~~
4	(0,5,13,10)	$\boxed{-4,4,13,13}$	~~(−4,4,12,12)~~	∞

The optimum fuzzy assignment is $1 \to 3, 2 \to 4, 3 \to 1, 4 \to 2$ with minimum travelling cost (26, 32, 8, 13). The fuzzy assignment schedule does not provide us the solution of the fuzzy travelling salesman problem as it gives $1 \to 3, 3 \to 1$, without passing through 2 and 4. Then we try to find the next best solution which satisfies the route restrictions. The next non zero minimum in the fuzzy cost matrix is (-4, 3, 11, 14). So we bring (-4, 3, 11, 14) into the solution with satisfies the route restriction of FTSP. Thus the fuzzy optimum assignment which satisfies the travelling salesman restriction is give by $1 \to 2, 2 \to 4, 3 \to 1, 4 \to 3$ i.e. $1 \to 2 \to 4 \to 3 \to 1$.

Hence, the optimal fuzzy travelling salesman assignment is

City → / City↓	1	2	3	4
1	∞	$\boxed{-4,3,11,14}$	~~$\boxed{-4,4,14,14}$~~	(9,4, 10, 10)
2	(3, 7, 10,9)	∞	(3, 9, 12, 8)	$\boxed{-2,2,8,8}$
3	$\boxed{-2,2,9,9}$	(-3, 3,11, 14)	∞	~~(−2,2,9,9)~~
4	(0,5,13,10)	~~$\boxed{-4,4,13,13}$~~	$\boxed{-4,4,12,12}$	∞

The optimum solution of the original fuzzy travelling salesman problem is

City → / City↓	1	2	3	4
1	∞	$\boxed{9,10,1,3}$	(6,8,3,5)	(8,9, 1,3)
2	(9,10, 2,4)	∞	(10,11,3,1)	$\boxed{4,5,1,3}$
3	$\boxed{7,8,1,3}$	(10,11,3, 4)	∞	(7,8,2,3)
4	(9,10,3,5)	(9,11,3,4)	$\boxed{6,8,1,5}$	∞

The optimal route for the fuzzy travelling salesman problem is $1 \to 2, 2 \to 4, 3 \to 1, 4 \to 3$ i.e. $1 \to 2 \to 4 \to 3 \to 1$ and the fuzzy minimum cost is (26, 31, 4,14).

6. Conclusion

The proposed method for solving fuzzy number travelling salesman problem is based on the classical method used for solving travelling salesman problem. The result so obtained in example 4.1 and 4.2 are matched with the existing technique. This proposed algorithm is effective and easy to understand, can be applied for solving real life problems.

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