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Server-Vacation-Active service-Break down and repair-Equilibrium state queuing model

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Abstract

This paper consists of an queuing system Server-Vacation-Active service-Breakdown Model I and Repair-Equilibrium State Model II in which the server begins a vacation of random duration in each time that the system becomes empty. This complicated recover model from vacation to equilibrium model has not been studied widely, which suggests that the that the wide number of clients of customers present in the queueing system at a random time factor is distributed as the sum of two independent random variables: (i) the number of Poisson arrivals over a allotted time period distributed as the time of a vacation's forward recurrence, and (ii) the range and the number of customers present in the corresponding regular queuing. This note provides an intuitive reason for this outcome, while offering a simpler and more elegant solution method at the same time.

Keywords

Modified Bernoulli Vacation, Retrial Queue, Single server Queue, Server Utilization.

AMS Subject Classification

60K25.

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Contents

1. Introduction

Many researchers have done huge innovative research in queueing systems with general bulk service and vacation because they thought that these kinds of queue dealt with efficient use of the idle time of the server for the primary as well as secondary work. Such queueing systems, brought enormous applications in diverse field like production sector, inventory systems, automated communications, computer networks and so on. These have a broad variety of uses in many real-life scenarios. Doshi [3] performed a detailed study of vacation queueing schemes. The bulk service provider queueing system with many working vacations and server breakdown became analyzed by means of Fadhil R [7]. He finds an M / G/1 queueing model, in which the server provides all arriving customers with the first critical service. These will exit the system with probability $(1-\theta)$ as soon as the first service is executed, and the second optional service is given. A bulk arrival queue with optional re-service provider and queueing model with working vacation and crowd sourcing was taken into consideration by means of S R. Chakravarthy [4] . He finds a queueing model of M / G/1, in which the first server performs Q. Ye et al, [15] addressed, on request for re-service, a bulk queue with several vacations and a control policy. Baruah, et al [2] have studied a batch service queueing system with several vacations; set-up times and re-service admission server range. In the real world, no machine is found to be flawless, because all systems fail more or less often. Kalita P. et al [9] did an analysis of Single Server Queue with Modified Vacation Policy. E. Shoukry et al [12], discussed about matrix geometric method for M/M/1 queueing model.

Therefore, random failures and system wise maintenance of machining system components have a huge effect on the production and efficiency of the machining system. The transient solution to a $M^{[x]}/G/1$ queueing method with feedback was extracted by Choudhury, G. Deka, M. [5] further they investigated an M / G/1 queue with two phases of operation subject to random breakdown and delayed repair. In a $M^{[x]}/G(a,b)/1$ queueing method with several vacations and closed-down time, of the server breakdown without interruption. J. Li et al [11] studied about the discrete-time Geo/G/1 queue with vacations in random environment. This random environment in Geo concepts focused many applications in the real life scenario. Vacation based priority systems were analysed by Takagi et al [14]. Analysis of a finite-capacity system with breakdowns and retention of impatient customers, were conferred by D.Y. Yang et al [14].

Dimou S et al [6], studied an M / G/1 two-phase multi-optional retrial queue with breakdown and repair. Recently, Artalejo J R [1] has examined a discrete time queueing system with server breakdowns and shifts in repair times. W.M. Kempa et al [10] discussed about transient solution for the queue-size distribution. T. Jiang et al [8] studied, the GI/M/1 queue in a multi-phase service environment with disasters and working breakdowns. In a certain convinced situations, the operating machine can malfunction, but because of the queue's standby machines, it remains operational and continues to perform the assigned job. The smooth running of the system is ensured by the availability of standby support and repair systems to the queueing system. Applications of queueing models with standby support aid may be mentioned inside the area of computer and communication structured systems, delivery and service systems, production / manufacturing systems, and so on.

The outline of the paper is structured as follows. Section II provides few necessary basic notations and assumptions. In section III, balance equations for the queue length distribution of two proposed models, namely Vacation - Active service - Breakdown Model I and Repair - Equilibrium State Model II are discussed. Appropriate queue length generating functions are arrived in Section IV. Results and discussions are given as a conclusion in section V.

2. Basic notations and Assumptions

When the queue is empty, and obviously the server goes on vacation, then at the ending stages of the vacation, it continues to take exponentially long vacations until it finds a client in the queue and ready to proceed with very long (time as well as multi level of service to the customer) customer service. Failures in service mechanisms occur solitarily during active operation, and the time of failure after repair or vacation is leads to exponential. If failure after repair leads to exponential then repair times are calculated exponentially. Arrival procedures, active services and breakdowns during the holidays could be evaluated through Poisson with different approaches. Simultaneously, the time of arrival, breakdown, service, repair does not depend on each other.

For the infinite source (λ_0 and) :

(a). Probability between arrivals - λ_n

(b). Probability between completions of the service (exponentially distributed)-μ_n

The notations used in this article are as follows:

λ_0 arrival rate during vacation

λ arrival rate during active service

λ_1 arrival rate during breakdown

λ_2 arrival rate during repair

λ_3 arrival rate when the system is in equilibrium state

ω vacation rate

μ service rate

k breakdown rate

q repair rate

$$\psi = \frac{\lambda}{\mu}$$

$$\psi_0 = \frac{\lambda_0}{\lambda_0 + \omega}$$

Let us imagine that $\lambda_0, \lambda, \mu, \omega, k, q > 0$ and $\lambda_1, \lambda_2, \lambda_3 \geq 0$.

3. Balance equations for the queue length distribution of the proposed models

In many machine / systems, communication, output and other stochastic structures, related to queueing theory leads that the server operates on primary and secondary (vacation) activities. It is also possible to model these systems as queueing systems for vacations. Here we offer an impression of few general results of decomposition and the methods used for two vacation models (Server - Vacation - Active service - Breakdown Model I and Server - Repair - Equilibrium State Model II) to obtain these results. In terms of the outcomes for these simple models, we also explain how other similar models can be solved.

3.1 Server-Vacation-Active service-Breakdown Model I

The proposed model Vacation - Active service - Breakdown for discrete time queuing system's status when λ_0, λ, λ_1 are active and possible are as follows:

- $(0, j)$ is where the client j is in the queue and server is on vacation, $j > 0$ and the probability is given by $p(0, j)$.
- $(1, j)$ is where the client j is in the system while the service is active, $j > 0$ and the probability is given by $p(1, j)$.
- $(2, j)$ is where the client j is logged in during the recovery process, $j \geq 0$ and the probability is given by $p(2, j)$.

The partial sum of generating functions of this model is as follows :

$$H_{\lambda_0}(x) = \sum_{j=1}^{\infty} p(0,j)x^j,$$

$$H_{\lambda}(x) = \sum_{j=1}^{\infty} p(1,j)x^j,$$

$$H_{\lambda_1}(x) = \sum_{j=0}^{\infty} p(2,j)x^j.$$

The queue length the generating function is given by

$$H_{\lambda_0+\lambda+\lambda_1}(x) = H_{\lambda_0}(x) + H_{\lambda}(x) + H_{\lambda_1}(x).$$

The relationship between the steady - steady probabilities of this proposed model Vacation - Active service - Breakdown are given below:

$$\lambda_0 p(0,0) = \mu p(1,1), \tag{3.1}$$

$$(\lambda_0+\omega)p(0,j) = \lambda_0 p(0,j-i), j>0 \tag{3.2}$$

$$(\lambda+\rho+k)p(1,1) = \omega p(0,1)+\rho p(1,2)+qp(2,1), \tag{3.3}$$

$$(\lambda+\rho+k)p(1,j) \tag{3.4}$$
$$= \lambda p(1,j-1)+\omega p(0,j)+\rho p(1,j+1)+qp(2,j), j>0,$$

$$(\lambda_1+q)p(2,1) = kp(1,1) \tag{3.5}$$

$$(\lambda_1+q)p(2,j) = kp(1,j)+\lambda_1 p(2,j-1), j\geq 0. \tag{3.6}$$

From equation (3.1), we get

$$p(1,1) = \frac{\lambda_0}{\mu}p(0,0), \tag{3.7}$$

From equations (3.5) and (3.7), we have

$$p(2,1) = \frac{k}{\lambda_1+q}p(1,1) = \frac{k\lambda_0}{\mu(\lambda_1+q)}p(0,0). \tag{3.8}$$

From equation (3.2), we get

$$p(0,j) = \frac{\lambda_0 p(0,j-1)}{\lambda_0+\omega} \tag{3.9}$$

$p(0,j) = \psi_0^j p(0,j-1)$, where $\psi_0^j = \dfrac{\lambda_0}{\lambda_0+\omega}$.

3.2 Server - Repair - Equilibrium State Model II

The proposed model Repair-Equilibrium State for discrete time queuing system's status when λ_2 and λ_3 are active and possible are as follows:

- $(3,j)$ is where the client j is in the system while the service is active, $j\geq 0$ and the probability is given by $p(3,j)$.
- $(4,j)$ is where the client j is logged in during the recovery process, $j\geq 0$ and the probability is given by $p(4,j)$.

The partial sum of generating functions of this model is as follows :
$H_{\lambda_2}(x) = \sum_{j=0}^{\infty} p(3,j)x^j,$
$H_{\lambda_3}(x) = \sum_{j=0}^{\infty} p(4,j)x^j.$
The queue length the generating function is given by

$$H_{\lambda_2+\lambda_3}(x) = H_{\lambda_2}(x) + H_{\lambda_3}(x).$$

We can construct the equations 3.1 to 3.9 for the repair - equilibrium model II also.

4. Queue length generating functions

From equation (3.9), we now get

$$H_{\lambda_0}(x) = \sum_{j=0}^{\infty} p(0,j)x^j = \frac{\lambda_0 p(0,j-1)}{\lambda_0+\omega}x^j \tag{4.1}$$

This can be rewritten like

$$H_{\lambda_0}(x) = \sum_{j=0}^{\infty} p(0,j)x^j = \frac{p(0,j-1)}{1-\psi_0 x}.$$

Let us multiply Equation (3.4) by x^j where $j=2,3,\ldots$ and simplifying along with (3.3) we obtain

$$(\lambda+\rho+k)[H_{\lambda}(x)-xp(1,1)] = \lambda x H_{\lambda}(x) \tag{4.2}$$
$$+\omega[H_{\lambda_0}(x)-xp(0,1)-p(0,0)]+$$
$$\frac{\rho}{x}[H_{\lambda}(x)-xp(1,1)-x^2p(1,2)]+q[H_{\lambda_2}(x)-zp(2,1)].$$

In a similar manner let us multiply (3.6) x^j where $j=2,3,\ldots,$ we get,

$$(\lambda_1+q-\lambda_1 x)H_{\lambda_1}(x) = kH_{\lambda}(x)+(\lambda_1+q)xp(2,1)-kxp(1,1). \tag{4.3}$$

Simplifying (3.5), we now have

$$H_{\lambda}(x) = \frac{k}{\lambda_1+q-\lambda_1 x}H_{\lambda_1}(x). \tag{4.4}$$

Substitute (4.4) in (4.2) and apply various algebraic operations, we found that

$$\frac{(x-1)R(x)}{x(\lambda_1+q-\lambda_1 x)}H_\lambda(x) \quad (4.5)$$
$$= x[(\lambda+\rho+k)p(1,1)-\omega p(0,1)-\rho p(1,2)-qp(2,1)]$$
$$-[\omega p(0,0)\rho p(1,1)]+\frac{\omega p(0,0)}{1-\psi_0 x},$$

where $R(x)=\lambda\lambda_1 x^2-(\lambda_1\rho+\lambda_1 k+\lambda\lambda_1+\lambda q)x+\rho(q+\lambda_1)$. (4.6)

For the existence Queue length distribution, the RHS of (4.5) must be absent if $x=1$.
From (3.7) to (3.9) we obtain $p(1,1)$, $p(2,1)$ and $p(0,1)$ this helps us to find $\rho p(1,2)$ when

$$\rho p(1,2)=\frac{\lambda\lambda_1+k\lambda_1+\lambda q}{\lambda_1+q}p(1,1)+\lambda_0\rho_0 p(0,0) \quad (4.7)$$

While substitute (4.7) in (4.5) and further simplifications, we now obtain the modified RHS of (4.5) as
$(x-1)\dfrac{\lambda_0}{1-\psi_0 x}p(0,0)$ and then

$$H_\lambda(x)=\frac{\lambda_0 x(\lambda_1+q-\lambda_1 x)}{R(x)(1-\psi_0 x)}p(0,0). \quad (4.8)$$

Let us establish the nature of the roots of $R(x)$ for positive λ_1. Hence, $R(x)$ is a Quadratic expression where its discriminant Δ satisfy
$\Delta \geq \lambda_1^2k^2+\lambda_1^2\rho^2+\lambda_1^2\lambda^2+\lambda^2q^2-2\lambda_1^2\lambda\mu-2\lambda_1\lambda\rho q+2\lambda_1\lambda^2 q=\lambda_1^2k^2+(\lambda\lambda_1^+\lambda q-\lambda_1\mu)^2>0.$
Here the equation $R(x)$ has two distinct real roots. In the steady state queue length distribution, the two roots of $R(x)$, must be greater than 1. The coefficient of x^2 in $R(x)$ is positive, then the two roots of $R(x)$ are greater than 1 only for $R(1)>0$ and $R(1)<0$, because $R(1)=\mu q-\lambda_1 k-\lambda q=R(x)$.
Hence

$$\mu q>\lambda_1 k+\lambda q \text{ or } \frac{\lambda_1 k}{q\mu}+\frac{\lambda}{\mu}<1. \quad (4.9)$$

Equation (4.9) shows that $\mu>\lambda$, and (3.9) satisfies $R(1)=\lambda_1(\lambda-\rho)-\lambda_1 k-\lambda q<0$.
If (4.9) satisfies, then the two roots x_1 and x_2 of $R(x)$ must be greater than 1. The recurrence relation respect to the generating function of queue length distribution, as of (4.1), (4.4) and (4.8) is simplified as,

$$H(x)=\frac{R(x)+\lambda_0(\lambda_0+k+q-\lambda_1 x)x}{R(x)(1-\psi_0 x)}p(0,0). \quad (4.10)$$

By (4.10), the normalizing condition of $H(1)=1$, and

$$p(0,0)=\frac{(\mu q-\lambda_1 k-\lambda q)(1-\psi_0)}{\mu q-\lambda_1 k-\lambda q+\lambda_0(k+q)}. \quad (4.11)$$

From the condition (4.9), note down that $0<p(0,0)<1$. At this instant, let us take $\lambda_1>0$ and assume $\gamma=\dfrac{1}{x_1}$, $\delta=\dfrac{1}{x_2}$, where x_1 and x_2 be the two roots of $R(x)$. By (4.11),

$$p(0,0)=\frac{\mu(q+\lambda_1)(1-\gamma)(1-\delta)(1-\psi_0)}{\mu q-\lambda_1 k-\lambda q+\lambda_0(k+q)}. \quad (4.12)$$

By (4.10) and (4.12)

$$H(x)=T(x)\frac{(1-\gamma)(1-\delta)(1-\psi_0)}{(1-\gamma x)(1-\delta x)(1-\psi_0 x)}, \quad (4.13)$$

$$\text{where } T(x)=\frac{R(x)+\lambda_0(\lambda_1+k+q-\lambda_1 x)x}{\mu q-\lambda_1 k-\lambda q+\lambda_0(k+q)}. \quad (4.14)$$

Notice that $T(1)=1$. When there is no client in the queue during repair process, then $\lambda_1=0$. From (4.6)

$$R(x)=\mu q(1-\psi x). \quad (4.15)$$

Here $R(x)$ have only one root and the root is $\dfrac{1}{\psi}$. Now consider (4.10)

$$H(x)=\frac{\mu q(1-\psi x+\lambda_0(k+q)x}{\mu q(1-\psi x)(1-\psi_0 x)}p(0,0). \quad (4.16)$$

Proceeding like this, we get

$$p(0,0)=\frac{\mu q(1-\psi)(1-\psi_0)}{\mu q(1-\psi)+\lambda_0(k+q)}, \quad (4.17)$$

and

$$H(x)=T(x)\frac{(1-\psi)(1-\psi_0)}{(1-\psi x)(1-\psi_0 x)} \quad (4.18)$$

$$\text{where } T(x)=\frac{\mu q(1-\psi x)+\lambda_0(k+q)x}{\mu q(1-\psi)+\lambda_0(k+q)}. \quad (4.19)$$

Finally the total queue length distribution appropriation can be resolved from (4.13) when $\lambda_1>0$ and from (4.18) when $\lambda_1=0$, at the same time, the condition (4.9) holds the essential and adequate conditions for the queue length distribution to be present where ever applicable. Table 1 shows Expected average waiting time in the queue during Server - Vacation - Active service - Breakdown Model I.

λ	p(0,j)	H(x)	p(1,j)	T(x)	$H_\lambda(x)$
5	0.171985	1.29966	0.534024	1.02231	7.55684
5.2	0.179469	1.37521	0.525902	1.02024	7.66263
5.4	0.18697	1.45201	0.517964	1.01818	7.76602
5.6	0.194485	1.53003	0.510204	1.01613	7.86709
5.8	0.202013	1.60922	0.502618	1.01408	7.96591
6	0.209554	1.68954	0.495199	1.01205	8.06253
6.2	0.217107	1.77095	0.487943	1.01002	8.15704
6.4	0.224671	1.85341	0.480846	1.008	8.24948
6.6	0.232244	1.9369	0.473903	1.00599	8.33992
6.8	0.239827	2.02136	0.467108	1.00398	8.42841
7	0.247419	2.10678	0.460459	1.00199	8.51502
7.2	0.255019	2.19311	0.453951	1	8.59979
7.4	0.262627	2.28033	0.44758	0.99802	8.68277
7.6	0.270242	2.36841	0.441342	0.996047	8.76402
7.8	0.277864	2.45731	0.435233	0.994083	8.84358
8	0.285492	2.54702	0.429251	0.992126	8.9215

Table 1: Expected average waiting time in the queue during Server - Vacation - Active service - Breakdown Model I

In the Equation (4.9), if $\lambda_1 = 0$, the constraints towards occurrence and existence of the consistent steady - state queue length distribution is equivalent to a M/M/1 queue, and is autonomous of the break-down and administration rates. Notice that when $\lambda_1 = 0$, breakdowns essentially have the impact of suspending the activity of the G/M/1 vacation queue in a exponential time frame.

In both (4.13) and (4.18), the factor $\frac{(1-\psi_0)}{1-\psi_0 x)}$ shows up in the generating function. Various analogous terms leads to identify the generating function of the breakdown model without vacations and in which the principal client of a bustling period shows up with rate λ_0. The number W, of clients in the framework is the aggregate of two free irregular factors W_1 and W_2, where W_1 is the quantity of clients in the framework because of worker vacation, and W_2 is the number in the breakdown model without vacations.

For $\lambda_1 > 0$, the mean queue length S, can be found by registering $H(x)$ from (4.13) and (4.14)

$$S = \frac{\gamma}{1-\gamma} + \frac{\delta}{1-\delta} + \frac{\psi_0}{1-\psi_0} + \frac{\lambda_1(\lambda-\mu-k)+\lambda_0(k+q-\lambda_1)-\lambda q}{\mu q - \lambda_1 k - \lambda q + \lambda_0(k+q)}.$$

When $\lambda_1 = 0$, the mean queue length is found from (4.18) and (4.19)

$$S = \frac{\psi}{1-\psi} + \frac{\psi_0}{1-\psi_0} + \frac{\lambda_0(k+q)-\mu q\psi}{\mu q(1-\psi)+\lambda_0(k+q)}.$$

Conclusion

This paper demonstrates recursive technique of an queuing system Server - Vacation - Active service - Breakdown Model I and Repair - Equilibrium State Model II, in which the server starts a vacation in a random period of time. Concurrently multi-level queueing systems in the steady state constraints are utilized model II to arrive the optimal queue length. In varying the system parameters, this method was preferred to produce several measurements of system performance. The failure of servers has affected system reliability measures when adjusting system parameters. When server going on vacation, we identified a queueing model for the server was breaking down, server repairs, and a backup server assisting the server during vacations and repairs. Finally the qualitative analysis of the steady-state model across a variety of device output tests for different scenarios was done.

References

[1] Artalejo JR, Economou A, Lopez-Herrero MJ (2005) Analysis of multiserver queue with setup times, Queueing Syst 52(1-2):53-76

[2] Baruah, M.; Madan, K.C.; Eldabi, T. A Two Stage Batch Arrival Queue with Reneging during Vacation and Breakdown Periods. Am. J. Oper. Res. 2013, 3, 570-580.

[3] B.T. Doshi, Queueing systems with vacations - a survey, Queueing Syst., 1 (1986), pp. 29-66

[4] S.R. Chakravarthy, S. Ozkar, MAP/PH/1 Queueing model with working vacation and crowdsourcing, Math. Appl., 44 (2016), pp. 263-294

[5] Choudhury, G. Deka, M. A batch arrival unreliable server delaying repair queue with two phases of service and Bernoulli vacation under multiple vacation policy. Qual. Technol. Quant. Manag. 2018, 15, 157-186.

[6] Dimou S, Economou A, Fakinos D, The single vacation queueing model with geometric abandonments, Journal of Statistical Plannin and Inference, 141 (2011), 2863-2877.

[7] Fadhil, R.; Madan, K.C.; Lukas, A.C. An M(X)/G/1 Queue with Bernoulli Schedule General Vacation Times, Random Breakdowns, General Delay Times and General Repair Times. Appl. Math. Sci. 2011, 5, 35-51.

[8] T. Jiang and L. Liu, The GI/M/1 queue in a multi-phase service environment with disasters and working breakdowns, International Journal of Computer Mathematics, vol. 94, no. 4, pp. 707-726, 2015.

[9] Kalita, P., Choudhury, G. & Selvamuthu, D. Analysis of Single Server Queue with Modified Vacation Policy. Methodol Comput Appl Proba. 22, 511-553 (2020). https://doi.org/10.1007/s11009-019-09713-9

[10] W.M. Kempa, M. Kobielnik, Transient solution for the queue-size distribution in a finite-buffer model with general independent input stream and single working vacation policy, Appl. Math. Model., 59 (2018), pp. 614-628

[11] J. Li and L. Liu, On the discrete-time Geo/G/1 queue with vacations in random environment, Discrete Dynamics in Nature and Society. An International Multidisciplinary Research and Review Journal, Art. ID 4029415, 9 pages, 2016.

[12] E. Shoukry, M.A. Salwa, A.S. Boshra, Matrix geometric method for M/M/1 queueing model with and without

breakdown ATM machines, Am. J. Eng. Res. (AJER), 7 (2018), pp. 246-252.

[13] Takagi, H. Vacation and Priority Systems. Queuing Analysis: A Foundation of Performance Evaluation; North-Holland: Amsterdam, The Netherlands, 1991; Volume I.

[14] D.Y. Yang, Y.Y. Wu, Analysis of a finite-capacity system with working breakdowns and retention of impatient customers, J. Manufactur. Syst., 44 (2017), pp. 207-216.

[15] Q. Ye, L. Liu, Analysis of MAP/M/1 queue with working breakdowns Commun. Stat. Theory Methods, 47 (2018), pp. 3073-3084.

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