Malaya Journal of Matematik, Vol. 8, No. 1, 42-47, 2020
https://doi.org/10.26637/MJM0801/0008

Soft semiseparated sets via operations

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Abstract
In this paper, we introduce the notion γ-soft semiseparated sets and study some of their basic properties.

Keywords
Soft topological spaces, γ-soft semiseparated set, γ-soft semiconnected space.

AMS Subject Classification
54D05, 54C08, 54B05.

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Article History: Received **13** August **2019**; Accepted **22** November **2019**

Contents

1. Introduction

The concept of soft sets was first introduced by Molodtsov [11]. After the introduction of the definition of a soft sets by Molodtsov, a large number of topologists have turned their attention to the generalization of different concepts of a classical sets in this sets. Recently, the concept of soft topological spaces was introduced and studied by Shabir and Naz [17]. A Good number of results are studied in this paper. The study of topological properties via operations was introduced and studied by Biswas and Prasannan in [2]. In this paper, we introduce the notion γ-soft semiseparated sets and study some of their basic properties. Also the concept of γ-soft semiconnected spaces are also introduced and studied in this paper.

2. Preliminaries

Let U be an initial universe set and E_U be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. We will call E_U the universe set of parameters with respect to U.

Definition 2.1. *[11] A pair (F,A) is called a soft set over U if $A \subset E_U$ and $F : A \to P(U)$, where $P(U)$ is the set of all subsets of U.*

Definition 2.2. *[6] Let U be an initial universe set and E_U be a universe set of parameters. Let (F,A) and (G,B) be soft sets over a common universe set U and $A,B \subset E$. Then (F,A) is a subset of (G,B), denoted by $(F,A)\widetilde{\subset}(G,B)$, if $A \subset B$ and for all $e \in A, F(e) \subset G(e)$. Also (F,A) equals (G,B), denoted by $(F,A) = (G,B)$, if $(F,A)\widetilde{\subset}(G,B)$ and $(G,B)\widetilde{\subset}(F,A)$.*

Definition 2.3. *[12] A soft set (F,A) over U is called a null soft set, denoted by $\widetilde{\emptyset}$, if $e \in A$, $F(e) = \emptyset$.*

Definition 2.4. *[12] A soft set (F,A) over U is called an absolute soft set, denoted by $\widetilde{A}$, if $e \in A$, $F(e) = U$.*

Definition 2.5. *[12] The union of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \backslash B, \\ G(e) & \text{if } e \in B \backslash A, \\ F(e) \cup G(e) & \text{if } e \in B \cap A. \end{cases}$$

We write $(F,A) \cup (G,B) = (H,C)$.

Definition 2.6. *[6] The intersection of two soft sets of (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A \cap B$, and for all $e \in C$, $H(e) = F(e) \cap G(e)$. We write $(F,A) \cap (G,B) = (H,C)$.*

Now we recall some definitions and results defined and discussed in [16, 17]. Henceforth, let X be an initial universe set and E be the fixed nonempty set of parameter with respect to X unless otherwise specified.

Definition 2.7. *For a soft set (F,A) over U, the relative complement of (F,A) is denoted by $(F,A)'$ and is defined by $(F,A)' = (F',A)$, where $F' : A \to P(U)$ is a mapping given by $F'(e) = U \setminus F(e)$ for all $e \in A$.*

Definition 2.8. *Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies the following axioms.*

1. *$\widetilde{\emptyset}$, $\widetilde{X}$ belong to τ.*
2. *The union of any number of soft sets in τ belongs to τ.*
3. *The intersection of any two soft sets in τ belongs to τ.*

The triplet (X,τ,E) is called a soft topological space.

Definition 2.9. *Let (X,τ,E) be a soft topological space, then the members of τ are said to be soft open sets in X.*

Definition 2.10. *Let (X,τ,E) be a soft topological space. A soft set (F,E) over X is said to be a soft closed set in X, if its relative complement $(F,E)'$ belongs to τ.*

Proposition 2.11. *Let (X,τ,E) be a soft topological space. Then one has the following*

1. *$\widetilde{\emptyset}$, $\widetilde{X}$ are soft closed sets over X.*
2. *The intersection of any number of soft closed sets is a soft closed set over X.*
3. *The union of any two soft closed sets is a soft closed set over X.*

Definition 2.12. *Let (X,τ,E) be a soft topological space and (A,E) a soft set over X.*

1. *The soft interior of (A,E) is the soft set $\mathrm{Int}(A,E) = \cup\{(O,E) : (O,E)$ is soft open and $(O,E)\widetilde{\subset}(A,E)\}$.*
2. *The soft closure of (A,E) is the soft set $\mathrm{Cl}(A,E) = \cap\{(F,E) : (F,E)$ is soft closed and $(A,E)\widetilde{\subset}(F,E)\}$.*

Definition 2.13. *Let (F,E) be a soft set over X and $x \in X$. We say that $x \in (F,E)$ read as x belongs to the soft set (F,E), whenever $x \in F(\alpha)$ for all $\alpha \in E$. Note that for $x \in X$, $x \notin (F,E)$ if $x \notin F(\alpha)$ for $\alpha \in E$.*

Definition 2.14. *Let $x \in X$, then (x,E) denotes the soft set over X for which $x(\alpha) = \{x\}$ for all $\alpha \in E$.*

Definition 2.15. *[2] An operation on a soft topology τ over X is called a γ-operation if a mapping from τ to the set $P(X)^E$ and defined by $\gamma : \tau \to P(X)^E$ such that for each $(V,E) \in \tau$, $(V,E)\widetilde{\subset}\gamma(V,E)$.*

Definition 2.16. *[2] A soft set (P,E) is said to be γ-soft open set if for each $x\widetilde{\in}(P,E)$, there exists a soft open set (V,E) such that $x\widetilde{\in}(V,E)\widetilde{\subset}\gamma(V,E)\widetilde{\subset}(P,E)$. The complement of a γ-soft open set is called a γ-soft closed set. The family of all γ-soft open sets of (X,τ,E,γ) is denoted by τ_γ.*

Definition 2.17. *[2] Let (X,τ,E,γ) be an operation-soft topological space and (A,E) a soft set over X. Then*

1. *the τ_γ-soft interior of (A,E) is the soft set τ_γ-$\mathrm{Int}(A,E) = \cup\{(O,E) : (O,E)$ is γ-soft open and $(O,E)\widetilde{\subset}(A,E)\}$.*
2. *the τ_γ-soft closure of (A,E) is the soft set τ_γ-$\mathrm{Cl}(A,E) = \cap\{(F,E) : (F,E)$ is γ-soft closed and $(A,E)\widetilde{\subset}(F,E)\}$.*

Lemma 2.18. *Let (X,τ,E,γ) be an operation-soft topological space. Then*

1. *for every γ-soft open set (G,E) and every soft subset (A,E) over X we have τ_γ-$\mathrm{Cl}((A,E)) \cap (G,E) \subset \tau_\gamma$-$\mathrm{Cl}((A,E) \cap (G,E))$,*
2. *for every γ-soft closed set (F,E) and every soft subset (A,E) over X we have τ_γ-$\mathrm{Int}((A,E) \cap (F,E) \subset \tau_\gamma$-$\mathrm{Int}((A,E)) \cap (F,E)$.*

Definition 2.19. *A subset (A,E) of an operation-soft topological space (X,τ,E,γ) is said to be γ-soft semiopen [7] if $(A,E) \subset \tau_\gamma$-$\mathrm{Cl}(\tau_\gamma$-$\mathrm{Int}((A,E)))$.*

The complement of a γ-soft semiopen set is called a γ-soft semiclosed set. The family of all γ-soft semiopen sets of (X,τ,E,γ) is denoted by γ-$SSO(X)$.

Definition 2.20. *[7] Let (X,τ,E,γ) be an operation-soft topological space and (A,E) a soft set over X. Then*

1. *τ_γ-$s\,\mathrm{Int}(A,E) = \cup\{(O,E) : (O,E)$ is γ-soft semiopen and $(O,E)\widetilde{\subset}(A,E)\}$.*
2. *τ_γ-$s\,\mathrm{Cl}(A,E) = \cap\{(F,E) : (F,E)$ is γ-soft semiclosed and $(A,E)\widetilde{\subset}(F,E)\}$.*

3. On operation-soft semiseparated sets

Definition 3.1. *Two nonempty soft subsets (A,E) and (B,E) of an operation-soft topological space (X,τ,E,γ) are said to be γ-soft semiseparated if $(A,E)\widetilde{\cap}\tau_\gamma$-$s\,\mathrm{Cl}((B,E)) = \tau_\gamma$-$s\,\mathrm{Cl}((A,E))\widetilde{\cap}(B,E) = \widetilde{\emptyset}$. If $\widetilde{X} = (A,E)\widetilde{\cup}(B,E)$ such that (A,E) and (B,E) are γ-soft semiseparated sets, then we say that (A,E) and (B,E) form a γ-soft semiseparation of $\widetilde{X}$.*

Remark 3.2. *Each two γ-soft semiseparated sets are always disjoint, since $(A,E)\widetilde{\cap}(B,E)\widetilde{\subset}(A,E)\widetilde{\cap}\tau_\gamma$-$s\,\mathrm{Cl}((B,E)) = \widetilde{\emptyset}$.*

Theorem 3.3. *For the soft subsets (A,E) and (B,E) of an operation-soft topological space (X,τ,E,γ), the following statements are equivalent:*

1. *(A,E) and (B,E) are γ-soft semiseparated.*
2. *There exist γ-soft semiclosed sets (F_1,E), (F_2,E) satisfying $(A,E)\widetilde{\subset}(F_1,E)\widetilde{\subset}\widetilde{X}\setminus(B,E)$ and $(B,E)\widetilde{\subset}(F_2,E)$ $\widetilde{\subset}\widetilde{X}\setminus(A,E)$.*

3. *There exist γ-soft semiopen sets (G_1,E) and (G_2,E) satisfying $(A,E)\widetilde{\subset}(G_1,E)$ $\widetilde{\subset}(\widetilde{X}\setminus(B,E))$ and $(B,E)\widetilde{\subset}(G_2,E)\widetilde{\subset}\widetilde{X}\setminus(A,E)$.*

Proof. The proof is clear. □

Proposition 3.4. *Let (A,E) and (B,E) be soft subsets of an operation-soft topological space (X,τ,E,γ). If (A,E) and (B,E) are γ-soft semiseparated, $\widetilde{\emptyset}\neq(C,E)\widetilde{\subset}(A,E)$ and $\widetilde{\emptyset}\neq(D,E)\widetilde{\subset}(B,E)$, then (C,E) and (D,E) are γ-soft semiseparated.*

Proof. Since (A,E) and (B,E) are γ-soft semiseparated sets, $(A,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((B,E))=\widetilde{\emptyset}$ and $\tau_\gamma\text{-}s\text{Cl}((A,E))\widetilde{\cap}(B,E)=\widetilde{\emptyset}$. Then $(C,E)\widetilde{\subset}(A,E)$, $\tau_\gamma\text{-}s\text{Cl}((C,E))\widetilde{\cap}(D,E)=\widetilde{\emptyset}$. Similarly, we have $(C,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((D,E))=\widetilde{\emptyset}$. Therefore, (C,E) and (D,E) are γ-soft semiseparated sets. □

Theorem 3.5. *If (A,E) and (B,E) are γ-soft semiseparated sets and (S,E) is a γ-soft semiclosed subset of an operation-soft topological space such that $(S,E)=(A,E)\widetilde{\cup}(B,E)$, then (A,E) and (B,E) are γ-soft semiclosed sets.*

Proof. Let $(S,E)=(A,E)\widetilde{\cup}(B,E)$, where $\tau_\gamma\text{-}s\text{Cl}((A,E))\widetilde{\cap}(B,E)=\widetilde{\emptyset}=(A,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((B,E))$. It is clear that $(S,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((A,E))=((A,E)\widetilde{\cup}(B,E))\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((A,E))=(A,E)$. As the intersection of γ-soft semiclosed sets is γ-soft semiclosed, (A,E) is γ-soft semiclosed. Similarly (B,E) is γ-soft semiclosed. □

Theorem 3.6. *Let (A,E) and (B,E) be nonempty soft subsets in an operation-soft topological space (X,τ,E,γ). The following statements hold:*

1. *If (A,E) and (B,E) are γ-soft semiseparated such that $(A_1,E)\widetilde{\subset}(A,E)$, $(B_1,E)\widetilde{\subset}(B,E)$, then (A_1,E), (B_1,E) are so.*

2. *If $(A,E)\widetilde{\cap}(B,E)=\widetilde{\emptyset}$ such that (A,E) and (B,E) are γ-soft semiclosed (γ-soft semiopen), then (A,E) and (B,E) are γ-soft semiseparated.*

3. *If (A,E), (B,E) are γ-soft semiclosed (γ-soft semiopen) and $(H,E)=(A,E)\cap(\widetilde{X}\setminus(B,E))$ and $(G,E)=(B,E)\cap(\widetilde{X}\setminus(A,E))$, then (H,E) and (G,E) are γ-soft semiseparated.*

Proof. (1). Since $(A_1,E)\widetilde{\subset}(A,E)$, we have $\tau_\gamma\text{-}s\text{Cl}((A_1,E))\widetilde{\subset}\tau_\gamma\text{-}s\text{Cl}((A,E))$. Then $(B,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((A,E))=\widetilde{\emptyset}$ implies that $(B_1,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((A,E))=\widetilde{\emptyset}$ and $(B_1,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((A_1,E))=\widetilde{\emptyset}$. Similarly $(A_1,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((B_1,E))=\widetilde{\emptyset}$. Hence (A_1,E) and (B_1,E) are γ-soft semiseparated.
(2). Since $(A,E)=\tau_\gamma\text{-}s\text{Cl}((A,E))$, $(B,E)=\tau_\gamma\text{-}s\text{Cl}((B,E))$ and $(A,E)\widetilde{\cap}(B,E)=\widetilde{\emptyset}$, $\tau_\gamma\text{-}s\text{Cl}((A,E))\widetilde{\cap}(B,E)=\widetilde{\emptyset}$ and $\tau_\gamma\text{-}s\text{Cl}((B,E))\widetilde{\cap}(A,E)=\widetilde{\emptyset}$. Hence (A,E) and (B,E) are γ-soft semiseparated sets. If (A,E) and (B,E) are γ-soft semiopen, then their complements are γ-soft semiclosed.
(3). If (A,E) and (B,E) are γ-soft semiopen, then $\widetilde{X}\setminus(A,E)$ and $\widetilde{X}\setminus(B,E)$ are γ-soft semiclosed. Since $(H,E)\widetilde{\subset}\widetilde{X}\setminus(B,E)$, $\tau_\gamma\text{-}s\text{Cl}((H,E))\widetilde{\subset}\tau_\gamma\text{-}s\text{Cl}(\widetilde{X}\setminus(B,E))=\widetilde{X}\setminus(B,E)$ and hence $\tau_\gamma\text{-}s\text{Cl}((H,E))\widetilde{\cap}(B,E)=\widetilde{\emptyset}$. Thus $(G,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((H,E))=\widetilde{\emptyset}$. Similarly, $(H,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((G,E))=\widetilde{\emptyset}$. Hence (H,E) and (G,E) are γ-soft semiseparated sets. □

Theorem 3.7. *The soft sets (A,E) and (B,E) of an operation-soft topological space are γ-soft semiseparated if, and only if there exist $(U,E),(V,E)\in\gamma SSO(X)$ such that $(A,E)\widetilde{\subset}(U,E)$, $(B,E)\widetilde{\subset}(V,E)$, $(A,E)\widetilde{\cap}(V,E)=\widetilde{\emptyset}$ and $(B,E)\widetilde{\cap}(U,E)=\widetilde{\emptyset}$.*

Proof. Suppose that $(V,E)=\widetilde{X}\setminus\tau_\gamma\text{-}s\text{Cl}((A,E))$ and $(U,E)=\widetilde{X}\setminus\tau_\gamma\text{-}s\text{Cl}((B,E))$. Then $(U,E),(V,E)\in\gamma SSO(X)$ such that $(A,E)\widetilde{\subset}(U,E)$, $(B,E)\widetilde{\subset}(V,E)$, $(A,E)\widetilde{\cap}(V,E)=\widetilde{\emptyset}$ and (B,E) $\widetilde{\cap}(V,E)=\widetilde{\emptyset}$. On the other hand, let $(U,E),(V,E)\in\gamma SSO(X)$ such that $(A,E)\widetilde{\subset}(U,E)$, $(B,E)\widetilde{\subset}(V,E)$, $(A,E)\widetilde{\cap}(V,E)=\widetilde{\emptyset}$ and $(B,E)\widetilde{\cap}(U,E)=\widetilde{\emptyset}$. Since $\widetilde{X}\setminus(V,E)$ and $\widetilde{X}\setminus(U,E)$ are γ-soft semiclosed sets, $\tau_\gamma\text{-}s\text{Cl}((A,E))\widetilde{\subset}\widetilde{X}\setminus(V,E)\widetilde{\subset}\widetilde{X}\setminus(B,E)$ and $\tau_\gamma\text{-}s\text{Cl}((B,E))\widetilde{\subset}\widetilde{X}\setminus(U,E)\widetilde{\subset}\widetilde{X}\setminus(A,E)$. It follows that $\tau_\gamma\text{-}s\text{Cl}((A,E))\widetilde{\cap}(B,E)=\widetilde{\emptyset}$ and $\tau_\gamma\text{-}s\text{Cl}((B,E))\widetilde{\cap}(A,E)=\widetilde{\emptyset}$. □

Theorem 3.8. *Let (A,E) and (B,E) be nonempty disjoint soft subsets of an operation-soft topological space (X,τ,E,γ) and $(G,E)=(A,E)\widetilde{\cup}(B,E)$. Then (A,E) and (B,E) are γ-soft semiseparated if and only if each of (A,E) and (B,E) are γ-soft semiclosed (γ-soft semiopen) in (G,E).*

Proof. Let (A,E) and (B,E) are γ-soft semiseparated sets. By Definition 3.1, (A,E) contains no γ-soft semilimit points of (B,E). Then (B,E) contains all γ-soft semilimit points of (B,E) which are in $(A,E)\widetilde{\cup}(B,E)$ and (B,E) is γ-soft semiclosed in $(A,E)\widetilde{\cup}(B,E)$. Therefore (B,E) is γ-soft semiclosed in (G,E). Similarly (A,E) is γ-soft semiclosed in (G,E). □

Theorem 3.9. *Let (X,τ,E,γ) be an operation-soft topological space. If (A,E) and (B,E) are γ-soft semiseparation of $\widetilde{X}$ itself, then (A,E) and (B,E) are γ-soft semiclosed sets of (X,τ,E,γ).*

Proof. Since (A,E) and (B,E) are γ-soft semiseparated, we have $(A,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((B,E))=\tau_\gamma\text{-}s\text{Cl}((A,E))\widetilde{\cap}(B,E)=\widetilde{\emptyset}$. Then $(A,E)\widetilde{\cap}\tau_\gamma\text{-}s\text{Cl}((B,E))=\widetilde{\emptyset}$ if, and only if (B,E) is γ-soft semiclosed in $(A,E)\widetilde{\cup}(B,E)=\widetilde{X}$. Similarly, we can show that (A,E) is γ-soft semiclosed in $\widetilde{X}$. □

4. Operation-soft semiconnected spaces

Definition 4.1. *A subset (A,E) of an operation-soft topological space (X,τ,E,γ) is said to be γ-soft semiconnected if it cannot be expressed as the union of two γ-soft semiseparated sets. Otherwise, the set (A,E) is called γ-soft semidisconnected.*

Lemma 4.2. *Let $(A,E)\widetilde{\subset}(B,E)\widetilde{\cup}(C,E)$ such that (A,E) be a nonempty γ-soft semiconnected set in an operation-soft topological space (X,τ,E,γ) and $(B,E),(C,E)$ be γ-soft semiseparated sets. Then only one of the following conditions holds:*

1. $(A,E)\widetilde{\subset}(B,E)$ *and* $(A,E)\widetilde{\cap}(C,E)=\widetilde{\emptyset}$.
2. $(A,E)\widetilde{\subset}(C,E)$ *and* $(A,E)\widetilde{\cap}(B,E)=\widetilde{\emptyset}$.

Proof. Since $(A,E)\widetilde{\cap}(C,E)=\widetilde{\emptyset}$, we have $(A,E)\widetilde{\subset}(B,E)$. If $(A,E)\widetilde{\cap}(B,E)=\widetilde{\emptyset}$, then $(A,E)\widetilde{\subset}(C,E)$. Since $(A,E)\widetilde{\subset}(B,E)$ $\widetilde{\cap}(C,E)$, then both $(A,E)\widetilde{\cap}(B,E)=\widetilde{\emptyset}$ and $(A,E)\widetilde{\cap}(C,E)=\widetilde{\emptyset}$ cannot hold. Similarly, suppose that $(A,E)\widetilde{\cap}(B,E)\neq\widetilde{\emptyset}$ and $(A,E)\widetilde{\cap}(C,E)\neq\widetilde{\emptyset}$, then by Theorem 3.6 (1), $(A,E)\widetilde{\cap}(B,E)$ and $(A,E)\widetilde{\cap}(C,E)$ are γ-soft semiseparated sets such that $(A,E)=((A,E)\widetilde{\cap}(B,E))\widetilde{\cup}((A,E)\widetilde{\cap}(C,E))$ which contradicts with the γ-soft semiconnectedness of (A,E). Hence one of the conditions (1) and (2) must be hold. □

Theorem 4.3. *If a γ-soft semiconnected soft set (S,E) of an operation-soft topological space (X,τ,E,γ) is contained in $(A,E)\widetilde{\cup}(B,E)$, where (A,E) and (B,E) are γ-soft semiseparated sets, then either $(S,E)\widetilde{\subset}(A,E)$ or $(S,E)\widetilde{\subset}(B,E)$.*

Proof. Let $(S,E)=((S,E)\widetilde{\cap}(A,E))\widetilde{\cup}((S,E)\widetilde{\cup}(B,E))$ where $(S,E)\widetilde{\cap}(A,E)$ and $(S,E)\widetilde{\cap}(B,E)$ are γ-soft semiseparated sets. So either $(S,E)\widetilde{\cap}(A,E)=\widetilde{\emptyset}$ or $(S,E)\widetilde{\cap}(B,E)=\widetilde{\emptyset}$ and hence either $(S,E)\widetilde{\subset}(B,E)$ or $(S,E)\widetilde{\subset}(A,E)$. □

Theorem 4.4. *A soft subset (M,E) of an operation-soft topological space (X,τ,E,γ) is a γ-soft semiconnected if there exists a γ-soft semiconnected set (C,E) such that $(C,E)\widetilde{\subset}(M,E)$ $\widetilde{\subset}\tau_\gamma\text{-}s\mathrm{Cl}((C,E))$.*

Proof. Let $(M,E)=(A,E)\widetilde{\cup}(B,E)$, where (A,E) and (B,E) are γ-soft semiseparated sets. Then either $(C,E)\widetilde{\subset}(A,E)$ and $(C,E)\widetilde{\subset}(B,E)$ and hence either $(M,E)\subset\tau_\gamma\text{-}s\mathrm{Cl}((C,E))\subset\tau_\gamma\text{-}s\mathrm{Cl}((A,E))\widetilde{\subset}(\widetilde{X}\setminus(B,E))$ or $(M,E)\widetilde{\subset}(\widetilde{X}\setminus(A,E))$. Therefore either $(B,E)=\widetilde{\emptyset}$ or $(A,E)=\widetilde{\emptyset}$. □

Corollary 4.5. *If (C,E) is a γ-soft semiconnected soft set of an operation-soft topological space (X,τ,E,γ), then $\tau_\gamma\text{-}s\mathrm{Cl}((C,E))$ is so.*

Proof. Follows from Theorem 4.4. □

Theorem 4.6. *If $\{(M_\alpha,E):\alpha\in\Delta\}$ is a family of γ-soft semiconnected sets of an operation-soft topological space (X,τ,E,γ) satisfying the property that any two of which are not γ-soft semiseparated, then $(M,E)=\underset{\alpha\in\Delta}{\cup}(M_\alpha,E)$ is γ-soft semiconnected.*

Proof. Let $(M,E)=(A,E)\widetilde{\cup}(B,E)$, where (A,E) and (B,E) are γ-soft semiseparated sets. Then for each $\alpha\in\Delta$ either $(M_\alpha,E)\widetilde{\subset}(A,E)$ or $(M_\alpha,E)\widetilde{\subset}(B,E)$. Since any two members of the family $\{(M_\alpha,E):\alpha\in\Delta\}$ are not γ-soft semiseparated, either $(M_\alpha,E)\widetilde{\subset}(A,E)$ for each $\alpha\in\Delta$ or $(M_\alpha,E)\widetilde{\subset}(B,E)$ for each $\alpha\in\Delta$. So either $(B,E)=\widetilde{\emptyset}$ or $(A,E)=\widetilde{\emptyset}$. □

Corollary 4.7. *If $(M,E)=\underset{\alpha\in\Delta}{\cup}(M_\alpha,E)$, where each (M_α,E) is γ-soft semiconnected set in an operation-soft topological space (X,τ,E,γ) and also $(M_\alpha,E)\widetilde{\cap}(M_{\alpha'},E)\neq\widetilde{\emptyset}$ for $\alpha,\alpha'\in\Delta$, then (M,E) is γ-soft semiconnected.*

Proof. Follows from Theorem 4.6. □

Corollary 4.8. *If $(M,E)=\underset{\alpha\in\Delta}{\cup}(M_\alpha,E)$, where each (M_α,E) is γ-soft semiconnected in an operation-soft topological space (X,τ,E,γ) and $\underset{\alpha\in\Delta}{\cap}(M_\alpha,E)\neq\widetilde{\emptyset}$ for each $\alpha\in\Delta$, then (M,E) is γ-soft semiconnected.*

Proof. Suppose that $\underset{\alpha\in\Delta}{\cup}(M_\alpha,E)$ is not γ-soft semiconnected. Then $\underset{\alpha\in\Delta}{\cup}(M_\alpha,E)=(H,E)\widetilde{\cup}(G,E)$, where (H,E) and (G,E) are γ-soft semiseparated sets in $\widetilde{X}$. Since $\underset{\alpha\in\Delta}{\cap}(M_\alpha,E)\neq\widetilde{\emptyset}$, we have a soft point $e_M\widetilde{\in}\underset{\alpha\in\Delta}{\cap}(M_\alpha,E)$. Since $e_M\in\underset{\alpha\in\Delta}{\cup}(M_\alpha,E)$, either $e_M\in(G,E)$ or $e_M\in(H,E)$. Suppose that $e_M\in(H,E)$. Since $e_M\in(M_\alpha,E)$ for each $\alpha\in\Delta$, then (M_α,E) and (H,E) intersect for each $\alpha\in\Delta$. By Theorem 4.3, $(M_\alpha,E)\widetilde{\subset}(H,E)$ or $(M_\alpha,E)\widetilde{\subset}(G,E)$. Since (H,E) and (G,E) are disjoint, $(M_\alpha,E)\widetilde{\subset}(H,E)$ for all $\alpha\in\Delta$ and hence $\underset{\alpha\in\Delta}{\cup}(M_\alpha,E)\widetilde{\subset}(H,E)$. Then $(G,E)=\widetilde{\emptyset}$, which is a contradiction. Suppose that $e_M\in(G,E)$. By similar way, we have $(H,E)=\widetilde{\emptyset}$, which is a contradiction. Thus $\underset{\alpha\in\Delta}{\cup}(M_\alpha,E)$ is γ-soft semiconnected. □

Theorem 4.9. *The following statements are equivalent for an operation-soft topological space (X,τ,E,γ):*

1. *$\widetilde{X}$ is γ-soft semiconnected.*
2. *$\widetilde{X}$ can not be expressed as the union of two nonempty disjoint γ-soft semiopen sets.*
3. *$\widetilde{X}$ contains no nonempty soft subset which is both γ-soft semiopen and γ-soft semiclosed.*

Proof. (1) ⇒ (2): Suppose that $\widetilde{X}$ is γ-soft semiconnected and if $\widetilde{X}$ can be expressed as the union of two nonempty disjoint sets (A,E) and (B,E) such that (A,E) and (B,E) are γ-soft semiopen sets. Consequently $(A,E)\ \widetilde{\subset}\ \widetilde{X}\setminus(B,E)$. Then $\tau_\gamma\text{-}s\mathrm{Cl}((A,E))\ \widetilde{\subset}\ \tau_\gamma\text{-}s\mathrm{Cl}(\widetilde{X}\setminus(B,E))=\widetilde{X}\setminus(B,E)$. Therefore, $\tau_\gamma\text{-}s\mathrm{Cl}((A,E))\ \widetilde{\cap}\ (B,E)=\widetilde{\emptyset}$. Similarly we can prove $(A,E)\widetilde{\cap}\tau_\gamma\text{-}s\mathrm{Cl}((B,E))=\widetilde{\emptyset}$. This is a contradiction to the fact that X is γ-soft semiconnected. Then $\widetilde{X}$ cannot be expressed as the union of two nonempty disjoint γ-soft semiopen sets.
(2) ⇒ (3): Suppose that $\widetilde{X}$ cannot be expressed as the union of two nonempty disjoint sets (A,E) and (B,E) such that (A,E) and (B,E) are γ-soft semiopen sets. If $\widetilde{X}$ contains a nonempty proper subset (A,E) which is both γ-soft semiopen and γ-soft semiclosed. Then $\widetilde{X}=(A,E)\widetilde{\cup}(\widetilde{X}\setminus(A,E))$. Hence (A,E) and $\widetilde{X}\setminus(A,E)$ are disjoint γ-soft semiopen sets whose union is $\widetilde{X}$. This is the contradiction to our assumption. Hence $\widetilde{X}$ contains no nonempty proper subset which is both γ-soft semiopen and γ-soft semiclosed.

(3) $\Rightarrow$ (1): Suppose that $\widetilde{X}$ contains no nonempty soft subset which is γ-soft semiopen and γ-soft semiclosed and $\widetilde{X}$ is not γ-soft semiconnected. Then $\widetilde{X}$ can be expressed as the union of two nonempty disjoint soft sets (A,E) and (B,E) such that $((A,E)\widetilde{\cap}\tau_\gamma\text{-}s\mathrm{Cl}((B,E)))\widetilde{\cup}(\tau_\gamma\text{-}s\mathrm{Cl}((A,E))\widetilde{\cap}(B,E))=\widetilde{\emptyset}$. Since $(A,E)\widetilde{\cap}(B,E)=\widetilde{\emptyset}$, $(A,E)=\widetilde{X}\setminus(B,E)$ and $(B,E)=\widetilde{X}\setminus(A,E)$. Since $\tau_\gamma\text{-}s\mathrm{Cl}((A,E))\widetilde{\cap}(B,E)=\widetilde{\emptyset}$, $\tau_\gamma\text{-}s\mathrm{Cl}((A,E))\widetilde{\subset}\widetilde{X}\setminus(B,E)$. Hence $\tau_\gamma\text{-}s\mathrm{Cl}((A,E))\widetilde{\subset}(A,E)$. Therefore, (A,E) is γ-soft semiclosed. Similarly, (B,E) is γ-soft semiclosed. Since $(A,E)=\widetilde{X}\setminus(B,E)$, (A,E) is γ-soft semiopen. Therefore, there exists a nonempty set (A,E) which is both γ-soft semiopen and γ-soft semiclosed. This is a contradiction to our assumption. Therefore, $\widetilde{X}$ is γ-soft semiconnected. □

Theorem 4.10. *An operation-soft topological space is γ-soft semiconnected if, and only if $\widetilde{X}$ is not the union of any two γ-soft semiseparated sets.*

Proof. Let (A,E) and (B,E) be any two γ-soft semiseparated sets such that $\widetilde{X}=(A,E)\ \widetilde{\cup}\ (B,E)$. Therefore $\tau_\gamma\text{-}s\mathrm{Cl}((A,E))\ \widetilde{\cap}\ (B,E)=(A,E)\ \widetilde{\cap}\ \tau_\gamma\text{-}s\mathrm{Cl}((B,E))=\widetilde{\emptyset}$. Since $(A,E)\ \widetilde{\subset}\ \tau_\gamma\text{-}s\mathrm{Cl}((A,E))$ and $(B,E)\ \widetilde{\subset}\ \tau_\gamma\text{-}s\mathrm{Cl}((B,E))$, $(A,E)\widetilde{\cap}(B,E)=\widetilde{\emptyset}$. Now $\tau_\gamma\text{-}s\mathrm{Cl}((A,E))\widetilde{\subset}\widetilde{X}\setminus(B,E)=(A,E)$. So $\tau_\gamma\text{-}s\mathrm{Cl}((A,E))=(A,E)$. Then (A,E) is γ-soft semiclosed. By the same way we can show that (B,E) is γ-soft semiclosed which contradicts with Theorem 4.9 (2). Conversely, let (A,E) and (B,E) be any two disjoint nonempty and γ-soft semiclosed sets over X such that $\widetilde{X}=(A,E)\widetilde{\cup}(B,E)$. Then $\tau_\gamma\text{-}s\mathrm{Cl}((A,E))\widetilde{\cap}(B,E)=(A,E)\widetilde{\cap}\tau_\gamma\text{-}s\mathrm{Cl}((B,E))=(A,E)\widetilde{\cap}(B,E)=\widetilde{\emptyset}$, which contradicts with the hypothesis. □

Theorem 4.11. *An operation-soft topological space is γ-soft semiconnected if, and only if for every pair of soft points x_α, y_β in X, there is a γ-soft semiconnected subset of X which contains both x_α and y_β.*

Proof. The necessity is immediate since the γ-soft semiconnected space itself contains these two points. For the sufficiency, suppose that for any two soft points x_α and y_β, there is a γ-soft semiconnected subset $(C,E)_{x_\alpha,y_\beta}$ of $\widetilde{X}$ such that $x_\alpha, y_\beta\in(C,E)_{x_\alpha,y_\beta}$. Let a_μ be a fixed soft point and $\{C_{a_\mu,x_\alpha}: x_\alpha\widetilde{\in}\widetilde{X}\}$ be a class of all γ-soft semiconnected subsets of $\widetilde{X}$ which contain the points a_μ and x_α. Then $\widetilde{X}=\underset{x_\alpha\widetilde{\in}\widetilde{X}}{\cup}C_{a_\mu,x_\alpha}$ and $\underset{x_\alpha\in X}{\cap}C_{a_\mu,x_\alpha}\neq\widetilde{\emptyset}$. Therefore by Corollary 4.8, $\widetilde{X}$ is γ-soft semiconnected. □

Theorem 4.12. *Let (X,τ,E,γ) be an operation-soft topological space and $\{(F_i,E): i\in\Delta\}$ a family of γ-soft semiconnected sets. If a pair $((F_i,E),(F_j,E))$ is not a γ-soft semiseparation for any $i,j\in\Delta$, then $\widetilde{\cup}\{(F_i,E): i\in\Delta\}$ is γ-soft semiconnected.*

Proof. Suppose $\widetilde{\cup}\{(F_i,E): i\in\Delta\}$ is not γ-soft semiconnected. Then there exist γ-soft semiseparated sets (A,E), (B,E) such that $\widetilde{\cup}\{(F_i,E): i\in\Delta\}=(A,E)\widetilde{\cup}(B,E)$. Since (F_i,E) is γ-soft semiconnected for each $i\in\Delta$ and $(F_i,E)\widetilde{\subset}(A,E)\widetilde{\cup}(B,E)$, by Theorem 4.3, $(F_i,E)\widetilde{\subset}(A,E)$ or $(F_i,E)\widetilde{\subset}(B,E)$. Now, put $\Delta_a=\{i\in\Delta:(F_i,E)\widetilde{\subset}(A,E)\}$, $\Delta_b=\{i\in\Delta:(F_i,E)\widetilde{\subset}(B,E)\}$. Then $\Delta_a\neq\widetilde{\emptyset}$, $\Delta_b\neq\widetilde{\emptyset}$ and $\Delta_a\widetilde{\cup}\Delta_b=\Delta$. Let $i_a\in\Delta_a$ and $i_b\in\Delta_b$, then $(F_{i_a},E)\widetilde{\subset}(A,E)$ and $(F_{i_b},E)\widetilde{\subset}(B,E)$. By Proposition 3.4, we obtain (F_{i_a},E) and (F_{i_b},E) are γ-soft semiseparated sets. This is contrary to our hypothesis. □

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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