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Mahgoub transform of Boehmians

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Abstract

In literature many integral transforms are applied for solving the theory of differential equations, from the motivation of classical Sumudu transform, Elzki transform, Mahgoub transform was also developed. We extend this transform for Schwartz space of distribution of compact support and to Boehmian space. More results are also established.

Keywords

Mahgoub Transform, Convolution, Natural Transform, Boehmians.

AMS Subject Classification

54C40,14E20,46E25, 20C20.

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Contents

1. Introduction

Mohand M.A. Mahgoub [12] introduced a new transform called Mahgoub transform. It is derived from classical Fourier integral. The Mahgoub transform has a strong relation with other integral transform. This relation is given by [9]. Mahgoub transform is used to facilitate the process of solving ordinary and partial differential equation in the time domain. From the motivation of Natural transform [10] the Mahgoub transform of a function $f(x)$ over the set where the set $\mathscr{A}$ is

$$\mathscr{A} = \{f(x)/\exists K, J_1, J_2 > 0\,|f(x)| < K\,e^{|x|/J_i}\}$$

for the given function in the set $\mathscr{A}$. The constant K must be finite. J_1 , J_2 may be finite or infinite.

$$M[f(x)] = \hat{M}(v) = v\int_0^\infty f(x)\,e^{-vx}\,\mathrm{d}x \quad x \geq 0, \quad J_1 \leq v \leq J_2 \tag{1.1}$$

Taha [9] gave the dualities between Mahgoub transform and other integral transform Sudhanshu Aggarwal [1, 3] gave the Solution of Linear volterra Integro Differential Equation of Second kind as well as the application for solving population growth and decay problems using Mahgoub transform, Aggarwal [2] also discussed the Linear ordinary differential equation using Mahgoub transform. S.K.Q. Al-omari [4] discussed the application of Natural transform and extended the Natural transform to generalized functions and Boehmians. Also, S.K.Q. Al-omari [5–7] gave the Hartly transform for L^p Boehmians, Boehmians for optical fresnal warelet transform & generalized Hartly Hilbert and Fourier Hilbert transform. A. H. Zemanian & R.S. Pathak[13, 14] gave applications of generalized integral transforms in detail. In this paper we give the properties of classical Mahgoub transform, distributional Mahgoub transform and it's extension to Boehmian space.

2. Preliminaries

1. The Mahgoub transform of a constant function i.e. $f(t) = \alpha$ is

$$M(\alpha) = v\int_0^\infty \alpha \cdot e^{-vt}\,\mathrm{d}t = \alpha \tag{2.1}$$

2. The Mahgoub transform of f(t)=1 is

$$M(1) = v\int_0^\infty 1\cdot e^{-vt}\,\mathrm{d}t = 1 \tag{2.2}$$

3. The Mahgoub transform of f(t)=t is

$$M(t) = \frac{1}{v} \tag{2.3}$$

4. The Mahgoub transform of f(t)=e^{at} is

$$M(e^{at}) = \frac{v}{v-a} \tag{2.4}$$

5. The Mahgoub transform of f(t)=$\sin at$, $f(t) = \cos at$ is

$$\begin{aligned} M(\sin at) &= \frac{av}{v^2+a^2} \\ M(\cos at) &= \frac{v^2}{v^2+a^2} \end{aligned} \tag{2.5}$$

6. The Mahgoub transform of a delta function

$$M(\delta) = \hat{M}_\delta(v) = v \tag{2.6}$$

7. Change of Scale by nonzero integer α

$$M(f(\alpha t)) = \hat{M}(\frac{v}{\alpha}) \tag{2.7}$$

8. The Mahgoub transform of derivative $F'(t), F''(t)$

$$\begin{aligned} M(F'(t)) &= v(\hat{M}(v) - F(0)) \\ M(F''(t)) &= v^2\hat{M}(v) - vF'(0) - v^2F(0) \end{aligned} \tag{2.8}$$

9. The inverse Mahgoub transform is related with Bromwich contour integral

$$f(x) = M^{-1}(f(v)) = \lim_{v\to\infty}\frac{1}{2\pi i}\int_{\gamma-iv}^{\gamma+iv}\hat{M}(f(v))(x)\,e^{vx}\,\mathrm{d}x \tag{2.9}$$

3. Relation between Mahgoub transform and other integral transforms

i) Connection between Kamal transform and Mahgoub transform :- If $\hat{M}(v)$ and $\hat{K}(v)$ are Mahgoub and Kamal transform of f(x) then

$$\hat{K}(v) = v\hat{M}\left(\frac{1}{v}\right)$$

ii) Mahgoub Laplace Duality :- If $\hat{M}(v)$ and $F(v)$ are the Mahgoub and Laplace transform of f(x) then

$$\hat{M}(v) = vF(v)$$

iii) Similarity Mahgoub-Sumudu Duality is given by

$$\hat{M}(v) = S\left(\frac{1}{v}\right)$$

iv) Mahgoub Elzaki Duality is

$$\hat{M}(v) = v^2E(v)$$

v) Mahgoub Aboodh Duality is

$$\hat{M}(v) = v^2A(v)$$

Linearity

If $f(x)$ and $g(x)$ have Mahgoub transform as $M(f)$ and $M(g)$ then

$$M(\alpha f(x) + \beta g(x)) = \alpha M(f) + \beta M(g) \quad where \quad \alpha\beta \in \mathbb{R}_+.$$

Proof.

$$\begin{aligned} &M(\alpha f(x)+\beta g(x)) \\ &= v\int_0^\infty(\alpha f(x)+\beta g(x))\cdot e^{-vx}\,\mathrm{d}x \qquad (3.1)\\ &= v\int_0^\infty \alpha f(x)\cdot e^{-vx}\,\mathrm{d}x + v\int_0^\infty \beta g(x)\cdot e^{-vx}\,\mathrm{d}x \\ &= \alpha[\int_0^\infty v f(x)\cdot e^{-vx}\,\mathrm{d}x] + \beta[\int_0^\infty v g(x)\cdot e^{-vx}\,\mathrm{d}x] \\ &M(\alpha f(x)+\beta g(x)) = \alpha M(f) + \beta M(g) \qquad (3.2) \end{aligned}$$

□

Convolution

The Convolution of two functions f and g is given by

$$(f*g)(x) = \int_0^\infty f(u)\,g(x-u)\,\mathrm{d}u \tag{3.3}$$

The Convolution theorem of two function f, g of Mahgoub transform is given by,

$$M(f*g)(v) = \frac{1}{v}M(f)\cdot M(g)$$

$$\begin{aligned} &M(f*g)(v) \\ &= \int_0^\infty (f*g)(x)\,[ve^{-vx}]\,\mathrm{d}x \qquad (3.4)\\ &= \int_0^\infty\int_0^\infty f(u)\,g(x-u)\,\mathrm{d}u\,(ve^{-xv})\,\mathrm{d}x \\ &= \int_0^\infty f(u)\,\mathrm{d}u\int_0^\infty g(x-u)\,ve^{-xv}\,\mathrm{d}x \qquad (3.5) \end{aligned}$$

Substitute $x-u=y$, we have

$$M(f*g)(v) = \int_0^\infty f(u)\,\mathrm{d}u \int_0^\infty g(y)\,v\,e^{-(y+u)v}\,\mathrm{d}y \tag{3.6}$$

$$= \int_0^\infty f(u)\cdot e^{-uv}\mathrm{d}u \int_0^\infty g(y)\,v\,e^{-yv}\,\mathrm{d}y$$

$$= \frac{1}{v}\left\{ v\int_0^\infty f(u)\,e^{-uv}\mathrm{d}u\cdot v\int_0^\infty g(y)\,e^{-yv}\,\mathrm{d}y \right\}$$

$$= \frac{1}{v}M(f)\cdot M(g)$$

$$M(f*g)(v) = \frac{1}{v}M(f)\cdot M(g) \tag{3.7}$$

4. Mahgoub Transform of Distribution

Let $H(\mathbb{R})$ be the space of infinitely smooth function on $\mathbb{R}$ and $H'(\mathbb{R})$ be the dual of $H(\mathbb{R})$i.e. $H'(\mathbb{R})$ is dual space of distribution of compact support.

Let $D(\mathbb{R})$ denote the subspace of $H(\mathbb{R})$ of testing function space of compact support and $D'(\mathbb{R})$ is its dual consists of Schwartz space of distributions.Now $D\subset H$, $H'\subset D'$.

The kernel of Mahgoub transform ve^{-vx} is clearly a member of $H(\mathbb{R})$. Hence we define the generalized Mahgoub transform M on $H'(\mathbb{R})$ by

$$M(f(x))(v) = < f(x), v\,e^{-vx} > \tag{4.1}$$

for every distribution $f\in H'(\mathbb{R})$

Theorem 4.1. *M is well defined mapping in the space $H(\mathbb{R})$*

Proof. Proof is immediate since, $v\,e^{-vx}\in H(\mathbb{R})$ □

Theorem 4.2. *M is infinitely smooth and*

$$\frac{d^k}{dv^k}M(f)(v) = < f(x)\,,\,\frac{d^k}{dv^k}v\,e^{-vx} > \quad f\in H'(\mathbb{R})$$

Proof. This theorem can be proved with the help of [[13],Theorem 2.9.1]. □

Theorem 4.3. *The generalized Mahgoub transform M is Linear.*

Proof. Let $f,g\in H'(\mathbb{R})$ and α,β non negative real number then

$$M(\alpha f+\beta g)(v)$$
$$= < \alpha f+\beta g\,,\,ve^{-vx} >$$
$$= < \alpha f, ve^{-vx} > + < \beta g, ve^{-vx} >$$
$$= \alpha < f, ve^{-vx} > +\beta < g, ve^{-vx} >$$
$$= < \alpha M(f)(v)+\beta M(g)(v) >$$

□

Theorem 4.4. *If $f\in H'(\mathbb{R})$ and $g(x)=\begin{cases} f(x-\tau) & x\geq\tau \\ 0 & x<\tau\end{cases}$ then*

$$M(g)(u) = e^{-v\tau}M(f)(v) \tag{4.2}$$

Proof. Let $f\in H'(\mathbb{R})$anf g is defined as above then by translation property of distributional Zemanian [14]

$$M(g)(v) = < g(x), ve^{-vx} >$$
$$= < f(x-\tau), ve^{-vx} >$$
$$= < f(x), ve^{-v(x+\tau)} >$$
$$= < g(x), ve^{-vx}\cdot e^{-v\tau} >$$
$$= e^{-v\tau} < f(x), ve^{-vx} >$$
$$M(g)(v) = e^{-v\tau}M(f)(v)$$

□

Let us define the Convolution of two functions.
Let $f,g\in H'(\mathbb{R})$ we define the generalized Convolution between f and g by

$$< (f*g), \psi(x) > = < f(x), < g(t), \psi(t+x) >>$$

for every $\psi\in H(\mathbb{R})$

Theorem 4.5. *Let $f,g\in H'(\mathbb{R})$ then*

$$M(f*g)(v) = \frac{M(f)M(g)}{v}.$$

Proof. By definition of M

$$M(f*g)(v) = < (f*g), ve^{-vx} >$$
$$= < f(x), < g(t), ve^{-v(x+t)} >>$$
$$= < f(x), < g(t), ve^{-vx}\cdot e^{-vt} >>$$
$$= < f(x), ve^{-vx} >< g(t), e^{-vt} >$$
$$M(f*g)(v) = \frac{1}{v} < f(x), ve^{-vx} >< g(t), ve^{-vt} >$$
$$M(f*g)(v) = \frac{1}{v}M(f)\cdot M(g)$$

□

Theorem 4.6. *Let $f\in H'(\mathbb{R})$ then the following holds*

$$M(xf(x))(v) = \frac{1}{v}[M(f)(v) - \frac{d}{dv}M(f)(v)]$$

Proof. By using the properties of Mahgoub transform and

Theorem (2.2) we get

$$\frac{d}{dv}M(f)$$
$$= \frac{d}{dv} < f(x), ve^{-vx} >$$
$$=< f(x), \frac{d}{dv} ve^{-vx} >$$
$$=< f(x), ve^{-vx}(-x) + e^{-vx} >$$
$$=< f(x), -xve^{-vx} + e^{-vx} >$$
$$=< f(x), e^{-vx} > - < f(x), xve^{-vx} >$$
$$=< f(x), e^{-vx} > - < xf(x), ve^{-vx} >$$
$$= \frac{1}{v} < f(x), ve^{-vx} > - < xf(x), ve^{-vx} >$$
$$= \frac{d}{v} M(f)(v) - M(xf)(v)$$
$$M(xf)(v) = \frac{1}{v} M(f)(v) - \frac{d}{dv} M(f)$$

□

Theorem 4.7 (Shifting Theorem)**.** *Let $f \in H'(\mathbb{R})$ then*

$$\begin{aligned} M(e^{ax} f(x))(v) &=< e^{ax} f(x), ve^{-vx} > \\ &=< f(x), ve^{-(v-a)x} > \\ &=< f(x), \frac{v}{(1-a/v)} e^{-(1-\frac{a}{v})vx} > \\ &=< f(x), \frac{1}{(1-a/v)} \cdot ve^{-(v-a)x} > \\ &= \frac{v}{(v-a)} < f(x), ve^{-(v-a))x} > \\ &= \frac{v}{(v-a)} Mf(v-a) \end{aligned}$$

5. Mahgoub transform of Boehmians

Distributions or generalized function are the objects that generalize functions.To differentiate functions whose derivative do not exist in classical sense is possible in distributional sense. In 1983 Boehmians are the objects obtained by abstract algebraic construction to generalized distribution [11]. The original construction was motivated by regular operators [8]. Boehmians are defined as equivalence classes of convolution quotients of functions that are subclass of Mikusinski's operator. The most youngest generalization of functions is the theory of Boehmians.

For Linear Space G and subspace F of G assume to all pair (f,ϕ), (g,ψ) of elements, f,g $\in$G $\phi,\psi \in$F the product f$*\phi$, g$*\psi$ such that the following conditions are satisfied.

1. $\phi * \psi \in F$ and $\phi * \psi = \psi * \phi$
2. $(f * \phi) * \psi = f * (\phi * \psi)$
3. $(f+g) * \phi = (f * \phi) + (g * \phi)$
4. $\lambda(f * \phi) = (\lambda f) * \phi = f * (\lambda \phi) \quad \lambda \in \mathbb{R}$
   Let $\triangle$ be the family of sequences from F such that for $f, g \in G$
5. If $(\varepsilon_n) \in \triangle$ and $(f * \varepsilon_n) = (g * \varepsilon_n)$ n=1,2,$\cdots$ then f=g
6. $(\varepsilon_n), (\mu_n) \in \triangle \Rightarrow (\varepsilon_n * \mu_n) \in \triangle$

Elements of $\triangle$ are called $\triangle$ sequences.

Now consider the class U of pairs of sequences defined by $U = \{((f_n), (\varepsilon_n)) : (f_n) \subseteq G^{\mathbb{N}}, \varepsilon_n \in \triangle\}$ for each n.

The pair $((f_n), (\varepsilon_n) \in U$ is said to be quotient of sequences denoted by $f_n/\varepsilon_n \, \chi$ if $f_n * \varepsilon_m = f_m * \varepsilon_n \quad for \quad \forall m, n \in \mathbb{N}$.

Two quotients of sequences f_n/ϕ_n and g_n/ψ_n are equivalent $f_n/\phi_n \sim g_n/\psi_n$ if $f_n * \psi_m = g_m * \phi_n \quad for \quad \forall m, n \in \mathbb{N}$.

The relation $\sim$ is an equivalence relation on U and hence splits U into equivalence classes.The equivalence class containing f_n/ϕ_n is denoted by [f_n/ϕ_n]. These equivalence classes are called Boehmians and is denoted by $B(G, F, \triangle, *)$

The sum and multiplication by a scalar of two Boehmians can be defined as,

$$[f_n/\phi_n] + [g_n/\psi_n] = [((f_n * \psi_n) + (g_n * \phi_n))/(\phi_n * \psi_n)]$$

and

$$\gamma[f_n/\phi_n] = [\gamma f_n/\phi_n] \qquad \gamma \in \mathbb{C}$$

The operation $*$ and differentiation are given by ,

$$[f_n/\phi_n] * [g_n/\psi_n] = [(f_n * g_n)/(\phi_n * \psi_n)]$$

and

$$D^\alpha [f_n/\phi_n] = [D^\alpha f_n/\phi_n]$$

G is equipped with notion of convergence.

The intrinsic relationship between the notion of convergence and the product $*$ is given by,

1. If $f_n \to f$ as $n \to \infty$ in G and $\phi \in F$ is any fixed element then $f_n * \phi \to f * \phi$ in G as $n \to \infty$
2. If $f_n \to f$ as $n \to \infty$ in G and $(\delta_n) \in \triangle$ then $f_n * \delta_n \to f$ in G as $n \to \infty$
   This operation $*$ can be extended to $B(G, F, \triangle, *) \times F$
3. If $[f_n/\delta_n] \in B(G, F, \triangle, *)$ and $\phi \in F$ then $[f_n/\delta_n] * \phi = [(f_n * \phi)/\delta_n]$

In $B(G, F, \triangle, *)$ two types of convergences, δ-convergence and $\triangle$-convergence are defined as:

(δ-convergence) A sequence of Boehmians(γ_n) in $B(G, F, \triangle, *)$ is said to be δ convergent to a Boehmian γ in

$B(G,F,\triangle,*)$ denoted by $\gamma_n \xrightarrow{\delta} \gamma$ if there exist a delta sequence (δ_n) such that

$$(\gamma_n * \delta_n), (\gamma * \delta_n) \in G \quad \forall k, n \in \mathbb{N}$$

and

$$(\gamma_n * \delta_k) \to \gamma * \delta_k \quad n \to \infty \quad in\ G \quad \text{for every } k \in \mathbb{N}$$

The following is equivalent statement of δ-convergent. The sequences $\gamma_n \xrightarrow{\delta} \gamma\ (n \to \infty)$ in $B(G,F,\triangle,*)$ if and only if there is $g_{n,k}, g_k \in G$ and $\delta_k \in \triangle$ such that

$$\gamma_n = [g_{n,k}/\delta_k] \qquad \gamma = [g_k/\delta_k] \quad \text{and for each} \quad k \in \mathbb{N}$$

$$g_{n,k} \to g_k \quad as \quad n \to \infty \quad \text{in G.}$$

($\triangle$-convergence) A sequence of Boehmians (γ_n) in $B(G,F,\triangle,*)$ is said to be $\triangle$ convergent to a Boehmian γ in $B(G,F,\triangle,*)$ denoted by $\gamma_n \xrightarrow{\delta} \gamma$ if there exist a $(\delta_n) \in \triangle$ such that

$$(\gamma_n - \gamma) * \delta_n \in G \quad \forall n \in \mathbb{N}$$

and

$$(\gamma_n - \gamma) * \delta_n \to 0 \quad as \quad n \to \infty \quad in \quad G.$$

Now we consider a space of Boehmians $B(H,D,\triangle,*)$ with $H(\mathbb{R})$ as a group $D(\mathbb{R})$ Schwartz space of test functions as a subgroup of $H(\mathbb{R})$.$*$ is an operation for $H(\mathbb{R})$ and $D(\mathbb{R})$ and $\triangle$ as set of delta sequence[9]. Also we denote $D'(\mathbb{R})$ strong dual of $D(\mathbb{R})$ of Schwartz distribution.

Theorem 5.1. *Let $[h_n/\psi_n] \in B(H,D,\triangle,*)$ then the sequence $M(f_n)$ converges in $D'(\mathbb{R})$ Moreover if $[h_n/\psi_n] = [g_n/\phi_n]$ in the sense of $B(H,D,\triangle,*)$ then $M(f_n)$ and $M(g_n)$ converge to the same limit.*

Proof. For delta sequence (δ_n) we have,

$$M(\delta_n) \to M(\delta)(v)$$

$$\Rightarrow M(\delta_n) \to v \quad as \quad n \to \infty \quad by (2.6)$$

Let $\phi \in D(\mathbb{R})$ be such that $M(\phi_k) > 0$ on the support of ϕ, $k \in \mathbb{N}$.

f_n/ψ_n is quotient of sequences implies that $f_n * \psi_m = f_m * \psi_n$

$$M(f_n) \cdot M(\psi_m) = M(f_m) \cdot M(\psi_n) \tag{5.1}$$

$$\begin{aligned} therefore \quad M(f_n)(\phi) &= M(f_n)\,\phi \cdot \frac{M(\psi_k)}{M(\psi_k)} \\ &= M(f_n)\,\frac{M(\psi_k)}{M(\psi_k)} \cdot \phi \\ &= M(f_n)\,(M(\psi_k)) \cdot \frac{\phi}{M(\psi_k)} \\ &= M(f_k)\,(M(\psi_n)) \cdot \frac{\phi}{M(\psi_k)} \\ &= M(f_k) \cdot (\phi \frac{M(\psi_n)}{M(\psi_k)}) \end{aligned}$$

$$Now \quad (\frac{\phi\, M(\psi_n)}{M(\psi_k)}) \to \frac{\phi \cdot v}{M(\psi_k)} \quad in \quad D(\mathbb{R})$$

Hence the sequence $M(f_n)$ converges in $D'(\mathbb{R})$

$$Let \quad [\frac{f_n}{\psi_n}] = [\frac{g_n}{\phi_n}] \quad in \quad B(H,D,\triangle,*) \quad \text{and define}$$

$$h_n = \begin{cases} f_{\frac{n+1}{2}} * \psi_{\frac{n+1}{2}} & \text{n is odd} \\ g_{\frac{n}{2}} * \psi_{\frac{n}{2}} & \text{n is even} \end{cases}$$

$$\delta_n = \begin{cases} \psi_{\frac{n+1}{2}} * \phi_{\frac{n+1}{2}} & \text{n is odd} \\ \psi_{\frac{n}{2}} * \phi_{\frac{n}{2}} & \text{n is even} \end{cases}$$

$$then \quad [\frac{h_n}{\delta_n}] = [\frac{f_n}{\psi_n}] = [\frac{g_n}{\phi_n}]$$

The sequence $M(h_n)$ converges in D' Moreover.

$$\lim_{n\to\infty} M(h_{2n-1}) = \lim_{n\to\infty} M(f_n)$$

therefore $M[(h_n)_{n=1}^{\infty}]$ and $M(f_n)$ converge to the same limit.

Similarly $M[(h_n)_{n=1}^{\infty}]$ and $M(g_n)$ converge to the same limit.

So M maps $B(H,D,\triangle,*)$ into $D'(\mathbb{R})$

We define Mahgoub transform of Boehmians

$$\gamma = [f_n/\phi_n] \in B(H,D,\triangle,*) \quad by$$

$$\breve{M}(\gamma) = \lim_{n\to\infty} M(f_n) \tag{5.2}$$

Theorem 5.2. *The Mahgoub transform $\breve{M} : B(H,D,\triangle,*) \to D'(\mathbb{R})$ is linear.*

Proof. Let $\beta_1, \beta_2 \in B(H,D,\triangle,*)$ such that $\beta_1 = [f_n/\phi_n], \beta_2 = [g_n/\psi_n]$ then

$$\begin{aligned} (\beta_1 + \beta_2) &= [((f_n * \psi_n) + (g_n * \phi_n))/(\phi_n * \psi_n)] \\ \breve{M}(\beta_1 + \beta_2) &= lim M(f_n * \psi_n) + lim M(g_n * \phi_n) \\ &= \lim_{n\to\infty} M(f_n) + \lim_{n\to\infty} M(g_n) \\ &= \breve{M}\beta_1 + \breve{M}\beta_2. \end{aligned}$$

Also

$$\breve{M}(\alpha\beta) = \breve{M}(\alpha f_n/\phi_n) = \alpha \lim_{n\to\infty} M f_n = \alpha \breve{M}\beta$$

where $\alpha \in \mathbb{C}, \beta = [f_n/\phi_n]$. □

□

Theorem 5.3. *If $\beta = [f_n/\phi_n]$ in $B(H,D,\triangle,*)$ and $\breve{M}\beta = 0$ in D' then β=0 in $B(H,D,\triangle,*)$.*

Proof. Assume that $\beta = [f_n/\phi_n]$ and $\breve{M}\beta = 0$, then by using the definition of $\breve{M}$ we get $\lim_{n\to\infty} M(f_n) = 0$ with the help of definition of Mahgoub transform $f_n \to 0$ almost every where in $H'(\mathbb{R})$ and f_n/ϕ_n is a zero quotient of functions or equivalently $\beta = [f_n/\phi_n]$ is zero equivalence class in $B(H,D,\triangle,*)$. □

Theorem 5.4. *$\breve{M}$ is one-one mapping from $B(H,D,\triangle,*)$ to $D'(\mathbb{R})$*

Proof. Let $\breve{M}\beta_1 = \breve{M}\beta_2$, $\beta_1, \beta_2 \in B(H,D,\triangle,*)$ then by above theorem (4.2) and (4.3)

$$\breve{M}(\beta_1 - \beta_2) = 0 \quad in \quad D'(\mathbb{R})$$

$$\beta_1 - \beta_2 = 0 \quad \Rightarrow \beta_1 = \beta_2$$

□

Theorem 5.5 (Convolution Theorem)**.** *Let $\beta_1 = [f_n/\phi_n]$, $\beta_2 = [g_n/\psi_n] \in \beta$ then we have*

$$\breve{M}(\beta_1 * \beta_2) = \frac{1}{v}\breve{M}(\beta_1)\cdot\breve{M}(\beta_2).$$

Proof.

$$\begin{aligned}
\breve{M}([f_n/\phi_n]*[g_n/\psi_n]) &= \breve{M}[(f_n*g_n)/(\phi_n*\psi_n)] \\
&= \lim_{n\to\infty} M(f_n*g_n) \\
&= \lim_{n\to\infty} \frac{1}{v} M(f_n)\cdot(g_n) \\
&= \frac{1}{v}\lim_{n\to\infty} M(f_n)\cdot\lim_{n\to\infty} M(g_n) \\
&= \frac{1}{v}\breve{M}(\beta_1)\cdot\breve{M}(\beta_2)
\end{aligned}$$

□

Theorem 5.6. *The Mahgoub transform $\breve{M}(\beta)$ is infinitely smooth.*

Proof. Let $\beta = [f_n/\psi_n] \in B(H,D,\triangle,*)$ and let J be bounded set in $\mathbb{R}$ then for $m \in \mathbb{N}$ we have $M(\psi_n) > 0$ on J.

$$\begin{aligned}
\breve{M}(\beta) &= \lim_{n\to\infty} M(f_n) \\
&= \lim_{n\to\infty} \frac{M(f_n)\cdot M(\psi_m)}{M(\psi_m)} \\
&= v\lim_{n\to\infty} \frac{M(f_n*\psi_m)}{M(\psi_m)} \\
&= v\lim_{n\to\infty} \frac{M(f_m*\psi_n)}{M(\psi_m)} \\
&= \lim_{n\to\infty} \frac{M(f_m)\cdot M(\psi_n)}{M(\psi_m)} \\
&= \frac{M(f_m)}{M(\psi_m)}\lim_{n\to\infty} M(\psi_n) \\
&= \frac{M(f_m)}{M(\psi_m)}\cdot v \qquad \text{on } J
\end{aligned}$$

but $M(f_n)\cdot M(\psi_m) \in H(\mathbb{R})$ and $M(\psi_m) > 0$ on J

$$\Rightarrow \breve{M}(\beta) \quad \text{is infinitely smooth.}$$

□

6. Conclusion

In this paper we have obtained the relation between Mahgoub transform and other integral transforms. We defined the Mahgoub transform in distributional sense. Also convolution and Boehmian space for Mahgoub transform is defined.

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