Malaya Journal of Matematik, Vol. 8, No. 1, 125-128, 2020
https://doi.org/10.26637/MJM0801/0021

Faint continuity via topological grill

P. Maragatha Meenakshi[1]* and S. Chandran[2]

Abstract
The aim of this paper is to introduce and characterize a new class of functions called almost $\mathscr{G}$-semicontinuous functions in grill topological spaces by using $\mathscr{G}$-semiopen sets.

Keywords
Grill topological spaces, $\mathscr{G}$-semiopen sets, faint $\mathscr{G}$-semicontinuity.

AMS Subject Classification
54D10.

[1] *Department of Mathematics, Periyar E.V.R. College, Affiliated to Bharathidasan University, Tiruchirappalli-620023, Tamil Nadu, India.*
[2] *Department of Mathematics, Valluvar College of Science and Management, Affiliated to Bharathidasan University, Karur-639003, Tamil Nadu, India.*
***Corresponding author**: [1]maragathameenakship@gmail.com; [2]surachandranva@gmail.com
Article History: Received **17** September **2019**; Accepted **28** December **2019**

Contents

1. Introduction

The idea of grills on a topological space was first introduced by Choquet [4]. The concept of grills has shown to be a powerful supporting and useful tool like nets and filters, for getting a deeper insight into further studying some topological notions such as proximity spaces, closure spaces and the theory of compactifications and extension problems of different kinds (see [2], [3], [13] for details). In [11], Roy and Mukherjee defined and studied a typical topology associated rather naturally to the existing topology and a grill on a given topological space. Quite recently, Al-Omari and Noiri [1] have defined new classes of sets in a grill topological space and obtained a new decomposition of continuity in terms of grills. The purpose of this paper, we introduce and study a new class of functions called faintly $\mathscr{G}$-semicontinuous functions in grill topological space. Some characterizations and several basic properties of this class of functions are obtained.

2. Preliminaries

Let A be a subset of a topological space (X,τ). We denote the closure of A and the interior of A by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. The θ-closure [14] of A, denoted by $\mathrm{Cl}_\theta(A)$, is defined to be the set of all $x \in X$ such that $A \cap \mathrm{Cl}(U) \neq \emptyset$ for every open neighbourhood U of X. If $A = \mathrm{Cl}_\theta(A)$, then A is called θ-closed [14]. The complement of a θ-closed set is called a θ-open set [14]. It follows from [14] that the collection of θ-open sets in a topological space (X,τ) forms a topology τ_θ on X. The θ-interior of A is defined by the union of all θ-open sets contained in A and is denoted by $\mathrm{Int}_\theta(A)$. The definition of grill on a topological space, as given by Choquet [4], goes as follows: A non-null collection $\mathscr{G}$ of subsets of a topological space (X,τ) is said to be a grill on X if

1. $\emptyset \notin \mathscr{G}$,
2. $A \in \mathscr{G}$ and $A \subset B$ implies that $B \in \mathscr{G}$,
3. $A, B \subset X$ and $A \cup B \in \mathscr{G}$ implies that $A \in \mathscr{G}$ or $B \in \mathscr{G}$.

Definition 2.1. *[11] Let (X,τ) be a topological space and $\mathscr{G}$ a grill on X. A mapping $\Phi : \mathscr{P}(X) \to \mathscr{P}(X)$ is defined as follows: $\Phi(A) = \Phi_{\mathscr{G}}(A,\tau) = \{x \in X : A \cap U \in \mathscr{G}$ for every open set U containing $x\}$ for each $A \in \mathscr{P}(X)$. The mapping Φ is called the operator associated with the grill $\mathscr{G}$ and the topology τ.*

Definition 2.2. *[11] Let $\mathscr{G}$ be a grill on a topological space (X,τ). Then we define a map $\Psi : \mathscr{P}(X) \to \mathscr{P}(X)$ by $\Psi(A) = A \cup \Phi(A)$ for all $A \in \mathscr{P}(X)$. The map Ψ is a Kuratowski closure axiom. Corresponding to a grill $\mathscr{G}$ on a topological space (X,τ), there exists a unique topology $\tau_{\mathscr{G}}$ on X given by $\tau_{\mathscr{G}} = \{U \subseteq X : \Psi(X \backslash U) = X \backslash U\}$, where for any $A \subset X$,*

$\Psi(A) = A \cup \Phi(A) = \tau_{\mathcal{G}}\,\mathrm{Cl}(A)$. *For any grill* $\mathcal{G}$ *on a topological space* (X,τ), $\tau \subset \tau_{\mathcal{G}}$. *If* (X,τ) *is a topological space with a grill* $\mathcal{G}$ *on* X, *then we call it a grill topological space and denote it by* $(X,\tau,\mathcal{G})$.

Definition 2.3. *[1] A subset* S *of a grill topological space* $(X,\tau,\mathcal{G})$ *is said to be* $\mathcal{G}$*-semiopen if* $S \subset \Psi(\mathrm{Int}(S))$. *The complement of a* $\mathcal{G}$*-semiopen set is called a* $\mathcal{G}$*-semiclosed set.*

Definition 2.4. *The intersection of all* $\mathcal{G}$*-semiclosed sets containing* $S \subset X$ *is called the* $\mathcal{G}$*-semiclosure of* S *and is denoted by* $s\mathrm{Cl}_{\mathcal{G}}(S)$. *The union of all* $\mathcal{G}$*-semiopen sets contained in* S *is called the* $\mathcal{G}$*-semiinterior of* S *and is denoted by* $s\mathrm{Int}_{\mathcal{G}}(S)$. *The family of all* $\mathcal{G}$*-semiopen (resp.* $\mathcal{G}$*-semiclosed) sets of* $(X,\tau,\mathcal{G})$ *is denoted by* $\mathcal{G}SO(X)$ *(resp.* $\mathcal{G}SC(X)$*). The family of all* $\mathcal{G}$*-semiopen (resp.* $\mathcal{G}$*-semiclosed) sets of* $(X,\tau,\mathcal{G})$ *containing a point* $x \in X$ *is denoted by* $\mathcal{G}SO(X,x)$ *(resp.* $\mathcal{G}SC(X,x)$*).*

Definition 2.5. *[7] A subset* S *of a topological space* (X,τ) *is semiopen if* $S \subset \mathrm{Cl}(\mathrm{Int}(S))$. *The complement of a semiopen set is called a semiclosed set.*

Definition 2.6. *A function* $f:(X,\tau)\to(Y,\sigma)$ *is said to be faintly continuous [8] (resp. faintly semicontinuous [10]) if* $f^{-1}(V)$ *is open (resp. semiopen) in* X *for every* θ*-open set* V *of* Y.

Definition 2.7. *[1] A function* $f:(X,\tau,\mathcal{G})\to(Y,\sigma)$ *is said to be* $\mathcal{G}$*-semicontinuous at a point* $x\in X$ *if for each open set* V *of* Y *containing* $f(x)$*, there exists* $U\in\mathcal{G}SO(X,x)$ *such that* $f(U)\subset V$. *If* f *has this property at each point of* X*, then it is said to be* $\mathcal{G}$*-semicontinuous.*

3. Faintly $\mathcal{G}$-semicontinuous functions

Definition 3.1. *A function* $f:(X,\tau,\mathcal{G})\to(Y,\sigma)$ *is said to be faintly* $\mathcal{G}$*-semicontinuous at a point* $x\in X$ *if for each* θ*-open set* V *of* Y *containing* $f(x)$*, there exists* $U\in\mathcal{G}SO(X,x)$ *such that* $f(U)\subset V$. *If* f *has this property at each point of* X*, then it is said to be faintly* $\mathcal{G}$*-semicontinuous.*

Theorem 3.2. *For a function* $f:(X,\tau,\mathcal{G})\to(Y,\sigma)$*, the following statements are equivalent:*

1. f *is faintly* $\mathcal{G}$*-semicontinuous;*
2. $f^{-1}(V)$ *is* $\mathcal{G}$*-semiopen in* X *for every* θ*-open set* V *of* Y*;*
3. $f^{-1}(F)$ *is* $\mathcal{G}$*-semiclosed in* X *for every* θ*-closed subset* F *of* Y*;*
4. $f:(X,\tau,\mathcal{G})\to(Y,\sigma_\theta)$ *is* $\mathcal{G}$*-semicontinuous.*
5. $s\mathrm{Cl}_{\mathcal{G}}(f^{-1}(B))\subseteq f^{-1}(\mathrm{Cl}_\theta(B))$ *for every subset* B *of* Y*;*
6. $f^{-1}(\mathrm{Int}_\theta(G))\subseteq s\mathrm{Int}_{\mathcal{G}}(f^{-1}(G))$ *for every subset* G *of* Y.

Proof. (1)⇒(2): Let V be an θ-open subset of Y and $x\in f^{-1}(V)$. Since $f(x)\in V$ and f is faintly $\mathcal{G}$-semicontinuous, there exists $U_x\in\mathcal{G}SO(X,x)$ such that $f(U_x)\subset V$. It follows that $x\in U_x\subset f^{-1}(V)$. We obtain $f^{-1}(V)=\bigcup_{x\in f^{-1}(V)} U_x$. Since any union of $\mathcal{G}$-semiopen sets is $\mathcal{G}$-semiopen, $f^{-1}(V)$ is $\mathcal{G}$-semiopen in X.
(2)⇒(1): Let $x\in X$ and V be a θ-open set of Y containing $f(x)$. By (2), $f^{-1}(V)$ is a $\mathcal{G}$-semiopen set of X containing x. Take $U=f^{-1}(V)$. Then $f(U)\subset V$. This shows that f is faintly $\mathcal{G}$-semicontinuous.
(2)⇒(3): Let V be any θ-closed set of Y. Since $Y\setminus V$ is θ-open, by (2), it follows that $f^{-1}(Y\setminus V)=X\setminus f^{-1}(V)$ is $\mathcal{G}$-semiopen. Consequently, $f^{-1}(V)$ is $\mathcal{G}$-semiclosed in X.
(3)⇒(2): Let V be a θ-open set of Y. Then $Y\setminus V$ is θ-closed in Y. By (3), $f^{-1}(Y\setminus V)=X\setminus f^{-1}(V)$ is $\mathcal{G}$-semiclosed and $f^{-1}(V)$ is $\mathcal{G}$-semiopen. The other implications are obvious. □

Theorem 3.3. *Every* $\mathcal{G}$*-semicontinuous function is faintly* $\mathcal{G}$*-semicontinuous.*

Proof. Clear. □

The converse of Theorem 3.3 is not true in general as can be seen from the following example.

Example 3.4. *Let* $X=\{a,b,c\}$, $\tau=\{\emptyset,\{b\},X\}$, $\sigma=\{\emptyset,\{a\},X\}$ *and* $\mathcal{G}=\mathcal{P}(X)\setminus\{\emptyset,\{a\}\}$. *Then the identity function* $f:(X,\tau,\mathcal{G})\to(X,\sigma)$ *is faintly* $\mathcal{G}$*-semicontinuous but not* $\mathcal{G}$*-semicontinuous.*

Corollary 3.5. *Every faintly continuous function is faintly* $\mathcal{G}$*-semicontinuous.*

Theorem 3.6. *Every faintly* $\mathcal{G}$*-semicontinuous function is faintly semicontinuous.*

Proof. Clear. □

The converse of Theorem 3.6 is not true in general as can be seen from the following example.

Example 3.7. *Let* $X=\{a,b,c\}$, $\tau=\{\emptyset,\{b\},\{a,c\},X\}$, $\sigma=\{\emptyset,\{a\},X\}$ *and* $\mathcal{G}=\mathcal{P}(X)\setminus\{\emptyset,\{a\}\}$. *Then the identity function* $f:(X,\tau,\mathcal{G})\to(X,\sigma)$ *is faintly semicontinuous but not faintly* $\mathcal{G}$*-semicontinuous.*

Remark 3.8. *The composition of faintly* $\mathcal{G}$*-semicontinuous functions is not faintly* $\mathcal{G}$*-semicontinuous as it can be seen from the following example.*

Example 3.9. *Let* $X=\{a,b,c\}$, $\tau=\{\emptyset,\{a\},\{b,c\},X\}$, $\mathcal{G}=\mathcal{P}(X)\setminus\{\emptyset,\{c\}\}$ *and* $\mathcal{J}=\mathcal{P}(X)\setminus\{\emptyset,\{a\}\}$. *Then the identity function* $f:(X,\tau,\mathcal{G})\to(X,\tau,\mathcal{J})$ *is faintly* $\mathcal{G}$*-semicontinuous and the function* $g:(X,\tau,\mathcal{J})\to(X,\tau)$ *defined by* $g(a)=b$, $g(b)=c$ *and* $g(c)=a$ *is faintly semi-*$\mathcal{J}$*-continuous but their composition is not faintly* $\mathcal{G}$*-semicontinuous.*

It is well known that a function $f:(X,\tau)\to(Y,\sigma)$ is said to be quasi-θ-continuous [10] if $f^{-1}(V)$ is θ-open in (X,τ) for every θ-open set V of (Y,σ).

Theorem 3.10. *If a function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is faintly $\mathscr{G}$-semicontinuous and the function $g:(Y,\sigma)\to(Z,\gamma)$ is quasi-θ-continuous, then $g\circ f:(X,\tau,\mathscr{G})\to(Z,\gamma)$ is faintly $\mathscr{G}$-semicontinuous.*

Proof. Let W be any θ-open set of (Z,γ). Then $g^{-1}(W)$ is θ-open in (Y,σ) and hence $(g\circ f)^{-1}(W)=f^{-1}(g^{-1}(W))$ is $\mathscr{G}$-semiopen in $(X,\tau,\mathscr{G})$. This shows that $g\circ f$ is faintly $\mathscr{G}$-semicontinuous. □

Theorem 3.11.
1. *$f:(X,\tau,\mathscr{P}(X)\backslash\{\emptyset\})\to(Y,\sigma)$ is a faintly $\mathscr{G}$-semicontinuous function if, and only if it is faintly semicontinuous.*
2. *$f:(X,\tau,\{X\})\to(Y,\sigma)$ is a faintly $\mathscr{G}$-semicontinuous function if, and only if it is faintly continuous.*

Proof. The proof follows from Corollaries 3.3 and 3.4 of [1]. □

Theorem 3.12. *If a function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is faintly $\mathscr{G}$-semicontinuous and (Y,σ) is a regular space, then f is $\mathscr{G}$-semicontinuous.*

Proof. Let V be any open set of Y. Since Y is regular, V is θ-open in Y. Since f is faintly $\mathscr{G}$-semicontinuous, by Theorem 3.2, we have $f^{-1}(V)$ is $\mathscr{G}$-semiopen and hence f is $\mathscr{G}$-semicontinuous. □

Definition 3.13. *A $\mathscr{G}$-semifrontier of a subset A of $(X,\tau,\mathscr{G})$ is defined as $sFr_{\mathscr{G}}(A)=s\mathrm{Cl}_{\mathscr{G}}(A)\cap s\mathrm{Cl}_{\mathscr{G}}(X\backslash A)$.*

Theorem 3.14. *The set of all points $x\in X$ in which a function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is not faintly $\mathscr{G}$-semicontinuous is the union of $\mathscr{G}$-semifrontier of the inverse images of θ-open sets containing $f(x)$.*

Proof. Suppose that f is not faintly $\mathscr{G}$-semicontinuous at $x\in X$. Then there exists a θ-open set V of Y containing $f(x)$ such that $f(U)$ is not contained in V for each $U\in\mathscr{G}SO(X,x)$ and hence $x\in\mathrm{Cl}_\theta(X\backslash f^{-1}(V))$. On the otherhand, $x\in f^{-1}(V)\subset s\mathrm{Cl}_{\mathscr{G}}(f^{-1}(V))$ and hence $x\in sFr_{\mathscr{G}}(f^{-1}(U))$. Conversely, suppose that f is faintly $\mathscr{G}$-semicontinuous at $x\in X$ and let V be a θ-open set of Y containing $f(x)$. Then there exists $U\in\mathscr{G}SO(X,x)$ such that $U\subset f^{-1}(V)$. Hence $x\in\mathrm{Int}_\theta(f^{-1}(V))$. Therefore, $x\in sFr_{\mathscr{G}}(f^{-1}(V))$ for each open set V of Y containing $f(x)$. □

Theorem 3.15. *Let $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ be a function and $g:(X,\tau)\to(X\times Y,\tau\times\sigma)$ the graph function of f. If g is faintly $\mathscr{G}$-semicontinuous, then f is faintly $\mathscr{G}$-semicontinuous.*

Proof. Let $x\in X$ and let V be a θ-open set of Y containing $f(x)$. Then $X\times V$ is θ-open in $X\times Y$ [[8], Theorem 5] and contains $g(x)=(x,f(x))$. Therefore, there exists $U\in\mathscr{G}SO(X,x)$ such that $g(U)\subset X\times V$. This implies that $f(U)\subset V$. Hence f is faintly $\mathscr{G}$-semicontinuous. □

Definition 3.16. *A grill topological space $(X,\tau,\mathscr{G})$ is said to be $\mathscr{G}$-semiconnected if X cannot be written as a disjoint union of two nonempty $\mathscr{G}$-semiopen sets.*

Theorem 3.17. *If a function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is a faintly $\mathscr{G}$-semicontinuous function and $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-semiconnected, then (Y,σ) is a connected space.*

Proof. Assume that (Y,σ) is not connected. Then there exist nonempty open sets V_1 and V_2 of (Y,σ) such that $V_1\cap V_2=\emptyset$ and $V_1\cup V_2=Y$. Hence we have $f^{-1}(V_1)\cap f^{-1}(V_2)=\emptyset$ and $f^{-1}(V_1)\cup f^{-1}(V_2)=X$. Since f is surjective, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty subsets of X. Since V_i is open and closed, V_i is θ-open for each $i=1,2$. Since f is faintly $\mathscr{G}$-semicontinuous, $f^{-1}(V_i)\in\mathscr{G}SO(X)$. Therefore, $(X,\tau,\mathscr{G})$ is not $\mathscr{G}$-semiconnected. This is a contradiction and hence (Y,σ) is connected. □

Definition 3.18. *A grill topological space $(X,\tau,\mathscr{G})$ is said to be:*

1. *$\mathscr{G}$-semi-T_1 [9] (resp. θ-T_1) if for each pair of distinct points x and y of X, there exists $\mathscr{G}$-semiopen (resp. θ-open) sets U and V containing x and y, respectively such that $y\notin U$ and $x\notin V$.*
2. *$\mathscr{G}$-semi-T_2 [9] (resp. θ-T_2 [12]) if for each pair of distinct points x and y in X, there exists disjoint $\mathscr{G}$-semiopen (resp. θ-open) sets U and V in X such that $x\in U$ and $y\in V$.*

Theorem 3.19. *If a function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is faintly $\mathscr{G}$-semicontinuous injection and (Y,σ) is a θ-T_1 space, then $(X,\tau,\mathscr{G})$ is a $\mathscr{G}$-semi-T_1 space.*

Proof. Suppose that (Y,σ) is a θ-T_1 space. For any distinct points x and y in X, there exist $V,W\in\sigma_\theta$ such that $f(x)\in V$, $f(y)\notin V$, $f(x)\notin W$ and $f(y)\in W$. Since f is faintly $\mathscr{G}$-semicontinuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $\mathscr{G}$-semiopen subsets of $(X,\tau,\mathscr{G})$ such that $x\in f^{-1}(V)$, $y\notin f^{-1}(V)$, $x\notin f^{-1}(W)$ and $y\in f^{-1}(W)$. This shows that $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-semi-T_1. □

Theorem 3.20. *If a function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is faintly $\mathscr{G}$-semicontinuous injection and (Y,σ) is a θ-T_2 space, then $(X,\tau,\mathscr{G})$ is a $\mathscr{G}$-semi-T_2 space.*

Proof. Suppose that (Y,σ) is θ-T_2. For any pair of distinct points x and y in X, there exist disjoint θ-open sets U and V in Y such that $f(x)\in U$ and $f(y)\in V$. Since f is faintly $\mathscr{G}$-semicontinuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $\mathscr{G}$-semiopen in X containing x and y, respectively. Therefore, $f^{-1}(U)\cap f^{-1}(V)=\emptyset$ because $U\cap V=\emptyset$. This shows that X is $\mathscr{G}$-semi-T_2. □

Definition 3.21. *A graph $G(f)$ of a function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is said to be θ-$\mathscr{G}$-closed if for each $(x,y)\in(X\times Y)\backslash G(f)$, there exist $U\in\mathscr{G}SO(X,x)$ and $V\in\sigma_\theta$ containing y such that $(U\times V)\cap G(f)=\emptyset$.*

Lemma 3.22. *A graph $G(f)$ of a function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is θ-$\mathscr{G}$-closed in $X\times Y$ if and only if for each $(x,y)\in(X\times Y)\setminus G(f)$, there exist $U\in\mathscr{G}SO(X,x)$ and $V\in\sigma_\theta$ containing y such that $f(U)\cap V=\emptyset$.*

Proof. It is an immediate consequence of Definition 3.21. □

Theorem 3.23. *If a function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is faintly $\mathscr{G}$-semicontinuous and (Y,σ) is a θ-T_2 space, then $G(f)$ is θ-$\mathscr{G}$-closed.*

Proof. Let $(x,y)\in(X\times Y)\setminus G(f)$, then $f(x)\neq y$. Since (Y,σ) is θ-T_2, there exist θ-open sets V and W in Y such that $f(x)\in V$, $y\in W$ and $V\cap W=\emptyset$. Since f is faintly $\mathscr{G}$-semicontinuous, $f^{-1}(V)\in\mathscr{G}SO(X,x)$. Take $U=f^{-1}(V)$. We have $f(U)\subset V$. Therefore, we obtain $f(U)\cap V=\emptyset$. This shows that $G(f)$ is θ-$\mathscr{G}$-closed. □

Theorem 3.24. *Let $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ has θ-$\mathscr{G}$-closed graph $G(f)$. If f is a faintly $\mathscr{G}$-semicontinuous injection, then $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-semi-T_2.*

Proof. Let x and y be any two distinct points of X. Then since f is injective, we have $f(x)\neq f(y)$. Then, we have $(x,f(y))\in(X\times Y)\setminus G(f)$. By Lemma 3.22, $U\in\mathscr{G}SO(X)$ and $V\in\sigma_\theta$ such that $(x,f(y))\in U\times V$ and $f(U)\cap V=\emptyset$. Hence $U\cap f^{-1}(V)=\emptyset$ and $y\notin U$. Since f is faintly $\mathscr{G}$-semicontinuous, there exists $W\in\mathscr{G}SO(X,y)$ such that $f(W)\subset V$. Therefore, we have $f(U)\cap f(W)=\emptyset$. Since f is injective, we obtain $U\cap W=\emptyset$. This implies that $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-semi-T_2. □

References

[1] A. Al-Omari and T. Noiri, Decomposition of continuity via grills, *Jordan J. Math and Stat.*, 4(1) (2011), 33-46.

[2] K. C. Chattopadhyay, O. Njastad and W. J. Thron, Merotopic spaces and extensions of closure spaces, Can. J. Math., 35 (4) (1983), 613-629.

[3] K. C. Chattopadhyay and W. J. Thron, Extensions of closure spaces, Can. J. Math., 29 (6) (1977), 1277-1286.

[4] G. Choqet, Sur les notions de filter et grill, Comptes Rendus Acad. Sci. Paris, 224 (1947), 171-173.

[5] E. Hatir and S. Jafari, On some new calsses of sets and a new decomposition of continuity via grills, *J. Adv. Math. Studies*, 3 (1) (2010), 33-40.

[6] S. Jafari, Some properties of quasi θ-continuous functions, *Far East J. Math. Soc.*, 6(1998), 689-696.

[7] N. Levine, Semi-open sets and sem-continuity in topological spaces, Amer. Math. Monthly, 70 (1963) 36?41.

[8] P. E. Long and L. L. Herrington, The T_θ-topology and faintly continuous functions, *Kyungpook Math. J.*, 22(1982), 7-14.

[9] P. Maragatha Meenakshi and S. Chandran, New separation axioms via $\mathscr{G}$-semiopen sets (submitted).

[10] T. Noiri and V. Popa, Weak forms of faint continuity, *Bull. Math. Soc. Sci. Math. Roumanie*, 34(82)(1990), 263-270.

[11] B. Roy and M. N. Mukherjee, On a typical topology induced by a grill, Soochow J. Math., 33 (4) (2007), 771-786.

[12] S. Sinharoy and S. Bandyopadhyay, On θ-completely regular and locally θ-H-closed spaces, *Bull. Cal. Math. Soc.*, 87(1995), 19-28.

[13] W. J. Thron, Proximity structure and grills, Math. Ann., 206 (1973), 35-62.

[14] N. V. Velicko, H-closed topological spaces, *Trans. Amer. Math. Soc.*, 78,2(1968), 103-118.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆