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On vague α-soft continuous functions

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Abstract
In this paper, we introduce some weaker forms of vague soft continuous functions and study their characterizations. We also provide a decomposition of $V\alpha\tilde{S}$-continuous functions. Further, we introduce the notion of vague soft irresolute functions and show the concepts of vague soft continuity and vague soft irresoluteness are independent of each other.

Keywords
$VS\tilde{S}$- continuous functions, $VP\tilde{S}$-continuous functions, $V\alpha\tilde{S}$-continuous functions, $VR\tilde{S}$-continuous functions, $V\tilde{S}$-irresolute functions, $VP\tilde{S}$-irresolute functions, $V\alpha\tilde{S}$-irresolute functions.

AMS Subject Classification
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Contents

1. Introduction

In 1999, Molodtsov [13] proposed a completely new concept called soft set theory to model uncertainty, that is free from the difficulties that have troubled the usual theoretical approaches. Maji et al [11] defined and studied several operations on soft sets. After that, Shabir and Naz [15] introduced and deeply studied the concept of soft topological spaces. Hussain and Ahmad [8] continued investigating the properties of the theory of soft topological spaces. Later, C.G. Aras et al. [4] introduced the notion of soft continuous functions using soft points. Since then, many researchers [2, 3, 9, 12, 14, 16, 17] initiated and studied some weaker forms of soft continuous functions. They also discussed some basic properties and characterizations of such functions.
Further, Xu et al. [18] introduced the concept of vague soft sets by combining the notions of the vague sets and the soft sets. This theory makes descriptions of the world more realistic, practical and accurate. Chang Wang, Yaya li [6] introduced the notion of vague soft topological spaces. They also proposed several operations on vague soft sets and some basic properties of these operations have been revealed so far. In this work, we introduce some weaker forms of vague soft continuous functions and some of their properties, characterizations are studied. Also we provide a decomposition of $V\alpha\tilde{S}$- continuous functions. Finally, we introduce the notions of vague soft irresolute functions, vague soft open (closed) functions and show the concept of vague soft continuity and vague soft irresoluteness are independent of each other.

2. Preliminaries

Definition 2.1. *[7] A vague set $A = \{(x_i, [t_A(x_i), 1-f_A(x_i)]) | x_i \in X\}$ in the universe $X = \{x_1, x_2, ..., x_n\}$ is characterized by a truth-membership function $t_A : X \to [0,1]$, and a false-membership function $f_A : X \to [0,1]$, where $t_A(x_i)$ is a lower bound on the grade of membership of x_i derived from the evidence of x_i, $f_A(x_i)$ is the lower bound on the negation of x_i derived from the evidence against x_i and $0 \le t_A(x_i) + f_A(x_i) \le 1$ for any $x_i \in X$. The grade of membership of x_i in the vague set is bounded to a subinterval $[t_A(x_i), 1-f_A(x_i)]$ of $[0,1]$. The vague value $[t_A(x_i), 1-f_A(x_i)]$ indicates that the exact grade of membership $\mu_A(x_i)$ of x_i may be unknown, but it is bounded by $t_A(x_i) \le \mu_A(x_i) \le 1-f_A(x_i)$, where $0 \le t_A(x_i) + f_A(x_i) \le 1$.*

Definition 2.2. *[18] Let X be an initial universe set, $V(X)$ the set of all vague sets on X, E a set of parameters, and $A \subseteq E$.*

A pair (F,A) is called a vague soft set over X, where F is a mapping given by $F: A \to V(X)$. The set of all vague soft sets on X is denoted by $V\tilde{S}(X,E)$, called vague soft classes.

Definition 2.3. *[18] A vague soft set (F,A) over X is said to be a null vague soft set denoted by $\hat{\emptyset}$, if $\forall e \in A$, $t_{F(e)}(x) = 0$, $1 - f_{F(e)}(x) = 0$, $x \in X$.*

Definition 2.4. *[18] A vague soft set (F,A) over X is said to be an absolute vague soft set denoted by $\hat{X}$, if $\forall e \in A$, $t_{F(e)}(x) = 1$, $1 - f_{F(e)}(x) = 1$, $x \in X$.*

Definition 2.5. *[6] Let X be an initial universe set, E be the nonempty set of parameters and τ be the collection of vague soft sets over X, then τ is said to be a vague soft topology on X if*

1. *$\hat{\emptyset}_E, \hat{X}_E$ belongs to τ.*
2. *the union of any number of vague soft sets in τ belongs to τ.*
3. *the intersection of any two vague soft sets in τ belongs to τ.*

The triplet (X,τ,E) is called a vague soft topological space over X.

Definition 2.6. *[10] Let (X,τ,E) be a vague soft topological space and (F,E) be a vague soft set over X. Then vague soft interior of (F,E) is defined by,*
$v\tilde{s}int(F,E)=\bigcup \{(G,E) \ / \ (G,E) \in \tau \text{ and } (G,E) \subseteq (F,E)\}$.

Definition 2.7. *[10] Let (X,τ,E) be a vague soft topological space and (F,E) be a vague soft set over X. Then vague soft closure of (F,E) is defined by,*
$v\tilde{s}cl(F,E)=\bigcap \{(H,E) \ / \ (H,E) \in \tau^c \text{ and } (F,E) \subseteq (H,E)\}$.

Definition 2.8. *[10] A vague soft set (F,A) of a vague soft topological space (X,τ,E) is said to be*

1. *vague semi-soft open if $(F,A) \subseteq v\tilde{s}cl(v\tilde{s}int(F,A))$.*
2. *vague pre-soft open if $(F,A) \subseteq v\tilde{s}int(v\tilde{s}cl(F,A))$.*
3. *vague α-soft open if $(F,A) \subseteq v\tilde{s}int(v\tilde{s}cl(v\tilde{s}int(F,A)))$.*
4. *vague regular-soft open if $(F,A) = v\tilde{s}int(v\tilde{s}cl(F,A))$.*

The complement of vague semi-soft open (resp. vague pre-soft open, vague α-soft open, vague regular-soft open) set is called vague semi-soft closed (resp.vague pre-soft closed, vague α-soft closed, vague regular-soft closed) set. And we denote the family of all vague semi-soft open sets (resp.vague pre-soft open, vague α-soft open sets, vague regular-soft open sets) of a vague soft topological space (X,τ,E) by $VS\tilde{S}O(X)$ (resp. $VP\tilde{S}O(X)$, $V\alpha\tilde{S}O(X)$, $VR\tilde{S}O(X)$).

Theorem 2.9. *[10] A vague soft set (F,A) of a vague soft topological space (X,τ,E) is vague α-soft open set iff (F,A) is both vague semi-soft open and vague pre-soft open set.*

Definition 2.10. *[5] Let $V\tilde{S}(X,E)$ and $V\tilde{S}(Y,K)$ be two vague soft classes, and let $u: X \to Y$ and $p: E \to K$ be mappings. Then a vague soft function $g_{pu} = (u,p): V\tilde{S}(X,E) \to V\tilde{S}(Y,K)$ is defined as: for $(F,A) \in V\tilde{S}(X,E)$, the image of (F,A) under g_{pu} denoted by $g_{pu}(F,A) = (u(F), p(A))$, is a vague soft set in $V\tilde{S}(Y,K)$ given by*

$$t_{u(F)(\beta)}(y) = \begin{cases} \sup\limits_{\alpha \in p^{-1}(\beta)\cap A,\ x \in u^{-1}(y)} t_{F(\alpha)}(x) & if\ u^{-1}(y) \neq \emptyset, \\ 0 & otherwise, \end{cases}$$

and

$$1 - f_{u(F)(\beta)}(y) = \begin{cases} \sup\limits_{\alpha \in p^{-1}(\beta)\cap A,\ x \in u^{-1}(y)} 1 - f_{F(\alpha)}(x) & if\ u^{-1}(y) \neq \emptyset, \\ 0 & otherwise. \end{cases}$$

for all $\beta \in p(A)$ and $y \in Y$.

Definition 2.11. *[5] Let $V\tilde{S}(X,E)$ and $V\tilde{S}(Y,K)$ be two vague soft classes, and let $g_{pu} = (u,p): V\tilde{S}(X,E) \to V\tilde{S}(Y,K)$ be a vague soft function and (G,B) be a vague soft set in $V\tilde{S}(Y,K)$. Then the inverse image of (G,B) under g_{pu}, denoted by $g_{pu}^{-1}(G,B) = (u^{-1}(G), p^{-1}(B))$ is a vague soft set in $V\tilde{S}(X,E)$ given by*
$t_{u^{-1}(G)(\alpha)}(x) = t_{G(p(\alpha))}(u(x))$
and $1 - f_{u^{-1}(G)(\alpha)}(x) = 1 - f_{G(p(\alpha))}(u(x))$

for all $\alpha \in p^{-1}(B)$ and $x \in X$.

Theorem 2.12. *[5] Let $V\tilde{S}(X,E)$ and $V\tilde{S}(Y,K)$ be two vague soft classes. For the vague soft function $g_{pu}: V\tilde{S}(X,E) \to V\tilde{S}(Y,K)$, the following statements hold,*

1. *$g_{pu}^{-1}(\hat{\emptyset}_K) = \hat{\emptyset}_E$.*
2. *$g_{pu}^{-1}(\hat{Y}_K) = \hat{X}_E$*
3. *$(g_{pu}^{-1}(G,B))^c = g_{pu}^{-1}((G,B)^c)$*
4. *If $(G_1,B) \subseteq (G_2,B)$, then $g_{pu}^{-1}(G_1,B) \subseteq g_{pu}^{-1}(G_2,B)$, $\forall\ (G_1,B),(G_2,B) \in V\tilde{S}(Y,K)$.*

Theorem 2.13. *[5] Let $V\tilde{S}(X,E)$ and $V\tilde{S}(Y,K)$ be two vague soft classes. For the vague soft function $g_{pu}: V\tilde{S}(X,E) \to V\tilde{S}(Y,K)$, the following statements hold:*

1. *$g_{pu}(\hat{\emptyset}_E) \subseteq \hat{\emptyset}_K$ and if p is surjective, the equality holds.*
2. *$g_{pu}(\hat{X}_E) \subset \hat{Y}_K$ and if g_{pu} is surjective the equality holds.*
3. *$(g_{pu}(F,A))^c \subseteq g_{pu}((F,A)^c)$ if g_{pu} is surjective. And if g_{pu} is bijective then its equality holds.*
4. *If $(F_1,A) \subseteq (F_2,A)$, then $g_{pu}(F_1,A) \subseteq g_{pu}(F_2,A)$, $\forall\ (F_1,A),(F_2,A) \in V\tilde{S}(X,E)$.*

Theorem 2.14. *[5] Let $(F,A),(G,B)$ be two vague soft sets in $V\tilde{S}(X,E)$ and $V\tilde{S}(Y,K)$ respectively, and g_{pu}: $V\tilde{S}(X,E) \to V\tilde{S}(Y,K)$ be a vague soft function. Then*

1. $(F,A) \subseteq g_{pu}^{-1}(g_{pu}(F,A))$ *and if* g_{pu} *is injective, the equality holds.*

2. $g_{pu}(g_{pu}^{-1}(G,B)) \subseteq (G,B)$ *and if* g_{pu} *is surjective, the equality holds.*

Theorem 2.15. *[5] Let* (X,τ,E) *and* (Y,σ,K) *be two vague soft topological spaces. The vague soft function* g_{pu}*:* $V\tilde{S}(X,E) \to V\tilde{S}(Y,K)$ *is called vague soft continuous, if and only if for all* $(G,K) \in \sigma$*,* $g_{pu}^{-1}(G,K) \in \tau$*.*

Throughout this paper $(X,\tau,E),(Y,\sigma,K)$ are denote the vague soft topological spaces on X, Y respectively.

3. Characterizations of Vague α-soft continuous functions

Definition 3.1. *Let* (X,τ,E) *and* (Y,σ,K) *be two vague soft topological spaces and let* $g_{pu} : (X,\tau,E) \to (Y,\sigma,K)$ *be a vague soft function. Then* g_{pu} *is called;*

1. *vague semi-soft continuous (*$VS\tilde{S}$*-continuous in short) if* $g_{pu}^{-1}(G,K) \in VS\tilde{S}O(X)$ *for all* $(G,K) \in \sigma$*.*

2. *vague pre-soft continuous (*$VP\tilde{S}$*-continuous in short) if* $g_{pu}^{-1}(G,K) \in VP\tilde{S}O(X)$ *for all* $(G,K) \in \sigma$*.*

3. *vague* α*-soft continuous (*$V\alpha\tilde{S}$*-continuous in short) if* $g_{pu}^{-1}(G,K) \in V\alpha\tilde{S}O(X)$ *for all* $(G,K) \in \sigma$*.*

4. *vague regular-soft continuous (*$VR\tilde{S}$*-continuous in short) if* $g_{pu}^{-1}(G,K) \in VR\tilde{S}O(X)$ *for all* $(G,K) \in \sigma$*.*

Theorem 3.2. *Let* (X,τ,E) *and* (Y,σ,K) *be two vague soft topological spaces and let* $g_{pu} : (X,\tau,E) \to (Y,\sigma,K)$ *be a vague soft function. Then the following conditions are equivalent:*

1. g_{pu} *is* $VS\tilde{S}$*-continuous function.*

2. *For each vague soft closed set* (G,K) *over* (Y,σ,K)*,* $g_{pu}^{-1}(G,E) \in VS\tilde{S}C(X)$*.*

3. *For each vague soft set* (F,E) *over* (X,τ,E)*,* $g_{pu}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(F,E))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))$*.*

4. *For each vague soft set* (S,K) *over* (Y,σ,K)*,* $\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(S,K))) \subseteq g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(S,K))$*.*

Proof. **1 ⇔ 2 :** Let (G,K) be a vague soft closed set over (Y,σ,K). Then $(G,K)^c \in \sigma$. By 1, we have $g_{pu}^{-1}((G,K)^c) \in VS\tilde{S}O(X)$. Now from the Theorem 2.12 (3) we have $g_{pu}^{-1}((G,K)^c) = (g_{pu}^{-1}(G,K))^c \in VS\tilde{S}O(X)$.
And hence $g_{pu}^{-1}(G,K) \in VS\tilde{S}C(X)$.

Conversely, let (H,K) be vague soft open set in (Y,σ,K). Then $(H,K)^c$ be vague soft closed set in (Y,σ,K) and $g_{pu}^{-1}((H,K)^c) \in VS\tilde{S}C(X)$. But $g_{pu}^{-1}((H,K)^c) = (g_{pu}^{-1}(H,K))^c \in VS\tilde{S}C(X)$, $g_{pu}^{-1}(H,K) \in VS\tilde{S}O(X)$.
Hence, g_{pu} is $VS\tilde{S}$-continuous function.

2 ⇒ 3 : Let (F,E) be a vague soft set over (X,τ,E). Then $\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))$ is vague soft closed set over (Y,σ,K). By using by 2, $g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)))$ is vague semi-soft closed in (X,τ,E).Thus,
$g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))) \supseteq \text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)))))$
$\supseteq \text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(g_{pu}(F,E))))) \supseteq \text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(F,E))$.
That is, $g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))) \supseteq \text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}((F,E)))$.
$\Rightarrow$ $\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)) \supseteq g_{pu}(g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)))$
$\supseteq g_{pu}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(F,E)))$.
Hence, $g_{pu}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(F,E)))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))$.

3 ⇒ 4 : Let (S,K) be a vague soft set over (Y,σ,K) and let $g_{pu}^{-1}(S,K) = (F,E)$.
By 3, we have $g_{pu}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(F,E))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))$.
$g_{pu}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(S,K)))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}(g_{pu}^{-1}(S,K))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(S,K)))$.
$\Rightarrow$ $g_{pu}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(S,K))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(S,K))$.
$\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(S,K))) \subseteq g_{pu}^{-1}(g_{pu}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(S,K)))) \subseteq g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(S,K)))$.
Hence, $\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K))) \subseteq g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(G,K)))$.

4 ⇒ 1 : Let (G,K) be a vague soft closed set in (Y,σ,K). Using part 4 for (G,K) we obtain,
$\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K)) \subseteq g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(G,K)) = g_{pu}^{-1}(G,K)$.
Therefore $g_{pu}^{-1}(G,K) \in VS\tilde{S}C(X)$. Hence g_{pu} is $VS\tilde{S}$-continuous function. □

Theorem 3.3. *A vague soft function* $g_{pu} : (X,\tau,E) \to (Y,\sigma,K)$ *is* $VS\tilde{S}$*-continuous iff* $g_{pu}(\text{vs}\tilde{\text{s}}\text{cl}(F,E)) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))$ *for every vague soft set* (F,E) *over* (X,τ,E)*.*

Proof. Let g_{pu} be a $VS\tilde{S}$-continuous function. Now $\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))$ is a vague soft closed set of (Y,σ,K), by $VS\tilde{S}$-continuity of g_{pu}, $g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)))$ is vague semi-soft closed and $(F,E) \subseteq g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)))$. But $\text{vs}\tilde{\text{s}}\text{cl}(F,E)$ is the smallest vague semi-soft closed set containing (F,E), we have $\text{vs}\tilde{\text{s}}\text{cl}(F,E) \subseteq g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)))$.
$\Rightarrow g_{pu}(\text{vs}\tilde{\text{s}}\text{cl}(F,E)) \subseteq g_{pu}(g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))$.

Conversely, let (G,K) be vague soft closed set of (Y,σ,K). Since $g_{pu}^{-1}(G,K) \in V\tilde{S}(X,E)$, we have
$g_{pu}(\text{vs}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}(g_{pu}^{-1}(G,K))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(G,K) = (G,K)$.
i.e., $g_{pu}(\text{vs}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K)))) \subseteq (G,K)$
Thus, $\text{vs}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K)) \subseteq g_{pu}^{-1}(g_{pu}(\text{vs}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K)))) \subseteq g_{pu}^{-1}(G,K)$. This implies that $g_{pu}^{-1}(G,K)$=$\text{vs}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K))$. Therefore, $g_{pu}^{-1}(G,K)$ is vague semi-soft closed set. Hence g_{pu} is $VS\tilde{S}$-continuous function. □

Theorem 3.4. *A vague soft function* $g_{pu} : (X,\tau,E) \to (Y,\sigma,K)$ *is* $VS\tilde{S}$*-continuous iff* $g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{int}(G,K)) \subseteq \text{vs}\tilde{\text{s}}\text{int}(g_{pu}^{-1}(G,K))$ *for every vague soft set* (G,K) *over* (Y,σ,K)*.*

Proof. Let g_{pu} be a $VS\tilde{S}$-continuous function. Now v$\tilde{\text{s}}$int$((G,K)$ is a vague soft open set of (Y,σ,K), by vague semi-soft continuity of g_{pu}, g_{pu}^{-1}(v$\tilde{\text{s}}$int$((G,K))$ is vague semi-soft open in (X,τ,E) and g_{pu}^{-1}(v$\tilde{\text{s}}$int$(G,K)) \subseteq g_{pu}^{-1}(G,K)$. But vs$\tilde{\text{s}}int(g_{pu}^{-1}(G,K))$ is the largest vague semi-soft open set contained in $g_{pu}^{-1}(G,K)$,
hence g_{pu}^{-1}(v$\tilde{\text{s}}$int$(G,K)) \subseteq$vs$\tilde{\text{s}}$int$(g_{pu}^{-1}(G,K))$.

Conversely, let (G,K) be vague soft open set of (Y,σ,K). g_{pu}^{-1}(v$\tilde{\text{s}}$int$(G,K))\subseteq$vs$\tilde{\text{s}}$int$(g_{pu}^{-1}(G,K))$.
$\Rightarrow$ $g_{pu}^{-1}(G,K)$= g_{pu}^{-1}(v$\tilde{\text{s}}$int$(G,K)) \subseteq$ vs$\tilde{\text{s}}$int$(g_{pu}^{-1}(G,K)) \subseteq g_{pu}^{-1}(G,K)$. This implies that vs$\tilde{\text{s}}int(g_{pu}^{-1}(G,K)) = g_{pu}^{-1}(G,K)$. Therefore, $g_{pu}^{-1}(G,K)$ is vague semi-soft open. Hence g_{pu} is $VS\tilde{S}$-continuous function. □

Theorem 3.5. *A vague soft function $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ is $VS\tilde{S}$-continuous iff vs$\tilde{s}$cl$(g_{pu}^{-1}(G,K)) \subseteq g_{pu}^{-1}(v\tilde{s}cl(G,K))$ for every vague soft set (G,K) over (Y,σ,K).*

Proof. The proof is follows from the Theorem 3.3. □

Theorem 3.6. *If $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ is bijective $VS\tilde{S}$-continuous function then v$\tilde{s}$int$(g_{pu}(F,E)) \subseteq g_{pu}(vs\tilde{s}int(F,E))$ for every vague soft set (F,E) over (X,τ,E).*

Proof. Let g_{pu} be a $VS\tilde{S}$-continuous function. Now v$\tilde{\text{s}}$int$(g_{pu}(F,E))$ is a vague soft open set of (Y,σ,K), by $VS\tilde{S}$-continuity of g_{pu}, g_{pu}^{-1}(v$\tilde{\text{s}}$int$(g_{pu}(F,E)))$ is vague semi-soft open in (X,τ,E). Since g_{pu} is injective and v$\tilde{\text{s}}$int$(g_{pu}(F,E)) \subseteq g_{pu}(F,E)$ then the result 1 of Theorem 2.14,
g_{pu}^{-1}(v$\tilde{\text{s}}$int$(g_{pu}(F,E))) \subseteq g_{pu}^{-1}(g_{pu}((F,E))) = (F,E)$. But vs$\tilde{\text{s}}int(F,E)$ is the largest vague semi-soft open set contained in (F,E), hence g_{pu}^{-1}(v$\tilde{\text{s}}$int$(g_{pu}(F,E))) \subseteq$ vs$\tilde{\text{s}}$int(F,E). Also, since g_{pu} is surjective,
v$\tilde{\text{s}}$int$(g_{pu}(F,E)) = g_{pu}(g_{pu}^{-1}$(v$\tilde{\text{s}}int(g_{pu}(F,E)))) \subseteq g_{pu}$(vs$\tilde{\text{s}}int(F,E)))$ by 2 of Theorem 2.14.
Hence, v$\tilde{\text{s}}$int$(g_{pu}(F,E)) \subseteq g_{pu}$(vs$\tilde{\text{s}}int(F,E))$. □

Theorem 3.7. *Every $V\tilde{S}$-continuous function in vague soft topological spaces is $VS\tilde{S}$-continuous.*

Proof. It follows from the fact that every vague soft open set is vague semi-soft open [10] in vague soft topological spaces. □

Definition 3.8. *Let (F,E) be a vague soft set over (X,τ,E). Then its vague pre-soft interior and vague pre-soft closure are defined as:*

vp$\tilde{s}$int$(F,E) = \bigcup\{(G,E)$ / (G,E) is vague pre-soft open and $(G,E)\subseteq(F,E)\}$.

vp$\tilde{s}$cl$(F,E) = \bigcap\{(S,E)$ / (S,E) is vague pre-soft closed and $(F,E)\subseteq(S,E)\}$.

Remark 3.9. *Let (F,E) be a vague soft set over (X,τ,E). Then,*
1. vp$\tilde{s}$int$(F,E) = (F,E) \cap$ v$\tilde{s}$int(v$\tilde{s}$cl$(F,E))$
2. vp$\tilde{s}$cl$(F,E) = (F,E) \cup$ v$\tilde{s}$int(v$\tilde{s}$cl$(F,E))$.

Theorem 3.10. *Let (F,E) and (G,E) be two vague soft sets over (X,τ,E). Then the following properties are hold:*

1. *vp$\tilde{s}$int$(\hat{\emptyset}_E) = \hat{\emptyset}_E$ and vp$\tilde{s}$int$(\hat{X}_E) = \hat{X}_E$.*
2. *vp$\tilde{s}$cl$(\hat{\emptyset}_E) = \hat{\emptyset}_E$ and vp$\tilde{s}$cl$(\hat{X}_E) = \hat{X}_E$.*
3. *vp$\tilde{s}$int$(F,E) \subseteq (F,E) \subseteq$ vp$\tilde{s}$cl(F,E).*
4. *(F,E) is a vague pre-soft open set iff vp$\tilde{s}$int$(F,E) = (F,E)$.*
5. *(F,E) is a vague pre-soft closed set iff vp$\tilde{s}$cl$(F,E) = (F,E)$.*
6. *vp$\tilde{s}$int(vp$\tilde{s}$int$(F,E))$ = vp$\tilde{s}$int(F,E) and vp$\tilde{s}$cl(vp$\tilde{s}$cl$(F,E))$ = vp$\tilde{s}$cl(F,E).*
7. *$(F,E)\subseteq(G,E)$ implies vp$\tilde{s}$int$(F,E) \subseteq$ vp$\tilde{s}$int(G,E) and vp$\tilde{s}$cl$(F,E) \subseteq$ vp$\tilde{s}$cl(G,E).*

Theorem 3.11. *Let (X,τ,E) and (Y,σ,K) be two vague soft topological spaces, $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ be a vague soft function. Then the following conditions are equivalent:*

1. *g_{pu} is $VP\tilde{S}$-continuous function.*
2. *For each vague soft closed set (G,K) over (Y,σ,K), $g_{pu}^{-1}(G,E) \in VP\tilde{S}C(X)$.*
3. *For each vague soft set (F,E) over (X,τ,E), g_{pu}(v$\tilde{s}$cl(v$\tilde{s}$int$(F,E))) \subseteq$v$\tilde{s}$cl$(g_{pu}(F,E))$.*
4. *For each vague soft set (S,K) over (Y,σ,K), v$\tilde{s}$cl(v$\tilde{s}$int$(g_{pu}^{-1}(S,K))) \subseteq g_{pu}^{-1}$(v$\tilde{s}cl(S,K))$.*

Proof. **1** $\Leftrightarrow$ **2** : Let (G,K) be a vague soft closed set over (Y,σ,K). Then (G,K)$^c \in \sigma$. By 1, we have $g_{pu}^{-1}((G,K)^c) \in VP\tilde{S}O(X)$. Now from the Theorem 2.12 (3) we have $g_{pu}^{-1}((G,K)^c) = (g_{pu}^{-1}(G,K))^c \in VP\tilde{S}O(X)$. And hence $g_{pu}^{-1}(G,K) \in VP\tilde{S}C(X)$.

Conversely, let (H,K) be vague soft open set in (Y,σ,K). Then $(H,K)^c$ be vague soft closed set in (Y,σ,K) and $g_{pu}^{-1}((H,K)^c) \in VP\tilde{S}C(X)$.
Since $g_{pu}^{-1}((H,K)^c) = (g_{pu}^{-1}(H,K))^c \in VP\tilde{S}C(X)$, $g_{pu}^{-1}(H,K) \in VP\tilde{S}O(X)$. Hence, g_{pu} is $VP\tilde{S}$-continuous function.

2 $\Rightarrow$ **3** : Let (F,E) be a vague soft set over (X,τ,E). Then v$\tilde{\text{s}}$cl$(g_{pu}(F,E))$ is vague soft closed set over (Y,σ,K). By using by 2, g_{pu}^{-1}(v$\tilde{\text{s}}$cl$(g_{pu}(F,E)))$ is vague pre-soft closed in (X,τ,E). Thus,
g_{pu}^{-1}(v$\tilde{\text{s}}$cl$(g_{pu}(F,E))) \supseteq$v$\tilde{\text{s}}$cl(v$\tilde{\text{s}}$int$(g_{pu}^{-1}$(v$\tilde{\text{s}}$cl$(g_{pu}(F,E)))))$
$\supseteq$v$\tilde{\text{s}}$cl(v$\tilde{\text{s}}$int$(g_{pu}^{-1}(g_{pu}(F,E)))))$
$\supseteq$v$\tilde{\text{s}}$cl(v$\tilde{\text{s}}$int$(F,E))$.
That is, g_{pu}^{-1}(v$\tilde{\text{s}}$cl$(g_{pu}(F,E))) \supseteq$v$\tilde{\text{s}}$cl(v$\tilde{\text{s}}$int$((F,E)))$.
$\Rightarrow$ v$\tilde{\text{s}}$cl$(g_{pu}(F,E)) \supseteq g_{pu}(g_{pu}^{-1}$(v$\tilde{\text{s}}cl(g_{pu}(F,E))) \supseteq g_{pu}$(v$\tilde{\text{s}}$cl(v$\tilde{\text{s}}int(F,E)))$.

Hence, g_{pu}(v$\tilde{s}$cl(v$\tilde{s}$int(F,E)))) $\subseteq$v$\tilde{s}$cl$(g_{pu}(F,E))$.
3 $\Rightarrow$ 4 : Let (S,K) be a vague soft set over (Y,σ,K) and let $g_{pu}^{-1}(S,K)=(F,E)$.
By 3, we have g_{pu}(v$\tilde{s}$cl(v$\tilde{s}$int(F,E))) $\subseteq$ v$\tilde{s}$cl$(g_{pu}(F,E))$
g_{pu}(v$\tilde{s}$cl(v$\tilde{s}$int$(g_{pu}^{-1}(S,K)$))) $\subseteq$ v$\tilde{s}$cl$(g_{pu}(g_{pu}^{-1}(S,K)))$ $\subseteq$ v$\tilde{s}$cl(S,K))).
$\Rightarrow$ g_{pu}(v$\tilde{s}$cl(v$\tilde{s}$int$(g_{pu}^{-1}(S,K)$)) $\subseteq$v$\tilde{s}$cl(S,K)).
v$\tilde{s}$cl(v$\tilde{s}$int$(g_{pu}^{-1}(S,K)$)) $\subseteq g_{pu}^{-1}(g_{pu}$(v$\tilde{s}$cl(v$\tilde{s}int(g_{pu}^{-1}(S,K)$)))) $\subseteq$ g_{pu}^{-1}(v$\tilde{s}$cl(S,K))).
Hence, v$\tilde{s}$cl(v$\tilde{s}$int$(g_{pu}^{-1}(S,K)$)) $\subseteq g_{pu}^{-1}$(v$\tilde{s}$cl(S,K))).

4 $\Rightarrow$ 1 :Let (G,K) be a vague soft closed set in (Y,σ,K). Using part 4 for (G,K) we obtain,
v$\tilde{s}$cl(v$\tilde{s}$int$(g_{pu}^{-1}(G,K)$)) $\subseteq g_{pu}^{-1}$(v$\tilde{s}$cl$(G,K)) = g_{pu}^{-1}(G,K)$.
Therefore $g_{pu}^{-1}(G,K) \in VP\tilde{S}C(X)$. Hence g_{pu} is a vague pre-soft continuous function. ☐

Theorem 3.12. *A vague soft function $g_{pu} : (X,\tau,E) \to (Y,\sigma,K)$ is $VP\tilde{S}$-continuous iff g_{pu}(vp$\tilde{s}$cl(F,E)) $\subseteq$v$\tilde{s}$cl$(g_{pu}(F,E))$ for every vague soft set (F,E) in (X,τ,E).*

Proof. Let (X,τ,E) and (Y,σ,K) be vague soft topological spaces, $g_{pu} : (X,\tau,E) \to (Y,\sigma,K)$ be a $VP\tilde{S}$-continuous function and (F,E) be a vague soft set in (X,τ,E). Then v$\tilde{s}$cl$(g_{pu}(F,E))$ is a vague soft closed set in Y. Since g_{pu} is $VP\tilde{S}$-continuous, we have g_{pu}^{-1}(v$\tilde{s}$cl$(g_{pu}(F,E)$)) is vague pre-soft closed in (X,τ,E). Since $g_{pu}(F,E) \subseteq$v$\tilde{s}$cl$(g_{pu}(F,E))$,
$g_{pu}^{-1}(g_{pu}(F,E)) \subseteq g_{pu}^{-1}$(v$\tilde{s}cl(g_{pu}(F,E)$)). This implies that,
$(F,E) \subseteq g_{pu}^{-1}(g_{pu}(F,E)) \subseteq g_{pu}^{-1}$(v$\tilde{s}cl(g_{pu}(F,E)$)). But vp$\tilde{s}cl(F,E)$ is the smallest vague pre-soft closed set containing (F,E), we have vp$\tilde{s}$cl$(F,E) \subseteq g_{pu}^{-1}$(v$\tilde{s}cl(g_{pu}(F,E)$)).
Then, g_{pu}(vp$\tilde{s}$cl(F,E)) $\subseteq g_{pu}(g_{pu}^{-1}$(v$\tilde{s}cl(g_{pu}(F,E)$))) $\subseteq$ v$\tilde{s}$cl$(g_{pu}(F,E))$
$\Rightarrow$ g_{pu}(vp$\tilde{s}$cl(F,E)) $\subseteq$v$\tilde{s}$cl$(g_{pu}(F,E))$.

Conversely, we assume that g_{pu}(vp$\tilde{s}$cl(F,E)) $\subseteq$v$\tilde{s}$cl$(g_{pu}(F,E))$ for every vague soft set (F,E) in (X,τ,E). Let (G,K) be a vague soft closed set in (Y,σ,K) . Then g_{pu}^{-1}(G,K) is a vague soft set over (X,τ,E). By our assumption,
g_{pu}(vp$\tilde{s}$cl$(g_{pu}^{-1}(G,K)$)) $\subseteq$v$\tilde{s}$cl$(g_{pu}(g_{pu}^{-1}$(G,K))$\subseteq$ v$\tilde{s}$cl(G,K) =(G,K).
$\Rightarrow$ g_{pu}(vp$\tilde{s}$cl$(g_{pu}^{-1}(G,K)$)) $\subseteq (G,K)$.
$g_{pu}^{-1}(g_{pu}$(vp$\tilde{s}$cl$(g_{pu}^{-1}(G,K)$)) $\subseteq g_{pu}^{-1}(G,K)$
vp$\tilde{s}$cl$(g_{pu}^{-1}(G,K)) \subseteq g_{pu}^{-1}(g_{pu}$(vp$\tilde{s}cl(g_{pu}^{-1}(G,K)$))) $\subseteq g_{pu}^{-1}(G,K)$.
This implies that, vp$\tilde{s}$cl$(g_{pu}^{-1}(G,K)$)=$(g_{pu}^{-1}(G,K))$. Therefore, $g_{pu}^{-1}(G,K)$ is vague pre-soft closed.
Hence g_{pu} is $VP\tilde{S}$-continuous function. ☐

Theorem 3.13. *A vague soft function $g_{pu} : (X,\tau,E) \to (Y,\sigma,K)$ is $VP\tilde{S}$-continuous iff g_{pu}^{-1}(v$\tilde{s}$int(G,K)) $\subseteq$vp$\tilde{s}$int$(g_{pu}^{-1}(G,K))$ for every vague soft set (G,K) in (Y,σ,K).*

Theorem 3.14. *A vague soft function $g_{pu} : (X,\tau,E) \to (Y,\sigma,K)$ is $VP\tilde{S}$-continuous iff vp$\tilde{s}$cl$(g_{pu}^{-1}(G,K)) \subseteq g_{pu}^{-1}$(v$\tilde{s}cl(G,K)$) for every vague soft set (G,K) in (Y,σ,K).*

Theorem 3.15. *Every $V\tilde{S}$-continuous function in vague soft topological spaces is $VP\tilde{S}$-continuous.*

Proof. It is obvious from the fact that, every vague soft open set is vague pre-soft open [10] in a vague soft topological space. ☐

Definition 3.16. *Let (F,E) be a vague soft set over (X,τ,E). Then its vague α-soft interior and vague α-soft closure are defined as:*

v$\alpha\tilde{s}$int(F,E)= $\bigcup\{(G,E)$ / (G,E) is vague α-soft open and $(G,E) \subseteq (F,E)\}$.

v$\alpha\tilde{s}$cl(F,E)= $\bigcap\{(S,E)$ / (S,E) is vague α-soft closed and $(F,E) \subseteq (S,E)\}$.

Remark 3.17. *Let (F,E) be a vague soft set over (X,τ,E). Then,*
1. v$\alpha\tilde{s}$int$(F,E) = (F,E) \cap$ v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))).
2. v$\alpha\tilde{s}$cl$(F,E) = (F,E) \cup$ v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))).

Theorem 3.18. *Let (F,E) and (G,E) be two vague soft sets over (X,τ,E). Then the following properties are hold:*

1. *v$\alpha\tilde{s}$int$(\hat{0}_E) = \hat{0}_E$ and v$\alpha\tilde{s}$int$(\hat{X}_E) = \hat{X}_E$.*
2. *v$\alpha\tilde{s}$cl$(\hat{0}_E) = \hat{0}_E$ and v$\alpha\tilde{s}$cl$(\hat{X}_E) = \hat{X}_E$.*
3. *v$\alpha\tilde{s}$int$(F,E) \subseteq (F,E) \subseteq$ v$\alpha\tilde{s}$cl(F,E).*
4. *(F,E) is a vague α-soft open set if and only if v$\alpha\tilde{s}$int$(F,E) = (F,E)$.*
5. *(F,E) is a vague α-soft closed set if and only if v$\alpha\tilde{s}$cl$(F,E) = (F,E)$.*
6. *v$\alpha\tilde{s}$int(v$\alpha\tilde{s}$int(F,E)) = v$\alpha\tilde{s}$int(F,E) and v$\alpha\tilde{s}$cl(v$\alpha\tilde{s}$cl(F,E)) = v$\alpha\tilde{s}$cl(F,E).*
7. *$(F,E) \subseteq (G,E)$ implies v$\alpha\tilde{s}$int$(F,E) \subseteq$ v$\alpha\tilde{s}$int(G,E) and v$\alpha\tilde{s}$cl$(F,E) \subseteq$ v$\alpha\tilde{s}$cl(G,E).*

Theorem 3.19. *Let $g_{pu} : (X,\tau,E) \to (Y,\sigma,K)$ be a vague soft function. Then the following conditions are equivalent:*

1. *g_{pu} is $V\alpha\tilde{S}$-continuous function.*
2. *For each vague soft closed set (G,K) over (Y,σ,K), $g_{pu}^{-1}(G,E) \in V\alpha\tilde{S}C(X)$.*
3. *For each vague soft set (F,E) over (X,τ,E), g_{pu}(v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E)))) $\subseteq$v$\tilde{s}$cl$(g_{pu}(F,E))$.*
4. *For each vague soft set (S,K) over (Y,σ,K), v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl$(g_{pu}^{-1}(S,K)$))) $\subseteq g_{pu}^{-1}$(v$\tilde{s}$cl(S,K)).*

Proof. **1** $\Leftrightarrow$ **2** : Let (G,K) be a vague soft closed set over (Y,σ,K). Then $(G,K)^c \in \sigma$. By 1, we have $g_{pu}^{-1}((G,K)^c) \in V\alpha\tilde{S}O(X)$. Now from the Theorem 2.12 (3) we have $g_{pu}^{-1}((G,K)^c) = (g_{pu}^{-1}(G,K))^c \in V\alpha\tilde{S}O(X)$. And hence $g_{pu}^{-1}(G,K) \in V\alpha\tilde{S}C(X)$.

Conversely, let (H,K) be vague soft open set in (Y,σ,K). Then $(H,K)^c$ is vague soft closed set in (Y,σ,K) and $g_{pu}^{-1}((H,K)^c) \in V\alpha\tilde{S}C(X)$.
Since $g_{pu}^{-1}((H,K)^c) = (g_{pu}^{-1}(H,K))^c \in V\alpha\tilde{S}C(X)$, $g_{pu}^{-1}(H,K) \in V\alpha\tilde{S}O(X)$. Hence, g_{pu} is $V\alpha\tilde{S}$-continuous function.

2 $\Rightarrow$ **3** : Let (F,E) be a vague soft set over X.
Then $\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))$ is vague soft closed set over (Y,σ,K).
By part 2, $g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)))$ is vague α-soft closed in (X,τ,E).Thus,
$g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))) \supseteq \text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))))))$
$\supseteq \text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(g_{pu}(F,E)))))$,
$\supseteq \text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(F,E)))$.
That is, $g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))) \supseteq \text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}((F,E))))$.
$\Rightarrow$ $\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)) \supseteq g_{pu}(g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))) \supseteq g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(F,E))))$.
Hence, $g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(F,E)))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))$.

3 $\Rightarrow$ **4** : Let (S,K) be a vague soft set over (Y,σ,K) and let $g_{pu}^{-1}(S,K) = (F,E)$. By 3, we have
$g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(F,E))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))$
$g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(S,K))))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}(g_{pu}^{-1}(S,K))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(S,K)))$.
$\Rightarrow$ $g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(S,K)))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(S,K))$.
$\text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(S,K)))) \subseteq$
$g_{pu}^{-1}(g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(S,K))))) \subseteq g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(S,K))$.
Hence, $\text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(S,K)))) \subseteq g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(S,K))$.

4 $\Rightarrow$ **1** :Let (G,K) be a vague soft closed set in (Y,σ,K). Using part 4 for (G,K) we obtain, $\text{v}\tilde{\text{s}}\text{cl}(\text{v}\tilde{\text{s}}\text{int}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K))) \subseteq g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(G,K)) = g_{pu}^{-1}(G,K)$.
Therefore $g_{pu}^{-1}(G,K) \in V\alpha\tilde{S}C(X)$. Hence g_{pu} is vague α-soft continuous function. □

Theorem 3.20. *A vague soft function $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ is $V\alpha\tilde{S}$-continuous iff $g_{pu}(v\alpha\tilde{s}cl(F,E)) \subseteq v\tilde{s}cl(g_{pu}(F,E))$ for every vague soft set (F,E) in (X,τ,E).*

Theorem 3.21. *A vague soft function $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ is $V\alpha\tilde{S}$-continuous iff $g_{pu}^{-1}(v\tilde{s}int(G,K)) \subseteq v\alpha\tilde{s}int(g_{pu}^{-1}(G,K))$ for every vague soft set (G,K) in (Y,σ,K).*

Theorem 3.22. *A vague soft function $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ is $V\alpha\tilde{S}$-continuous iff $v\alpha\tilde{s}cl(g_{pu}^{-1}(G,K)) \subseteq g_{pu}^{-1}(v\tilde{s}cl(G,K))$ for every vague soft set (G,K) in (Y,σ,K).*

Theorem 3.23. *Every $V\tilde{S}$-continuous function in vague soft topological spaces is $V\alpha\tilde{S}$-continuous.*

Proof. It is obvious from the fact that, every vague soft open set is vague α-soft open [10] in a vague soft topological space. □

Theorem 3.24. *Every $V\alpha\tilde{S}$-continuous function is a $VS\tilde{S}$-continuous and $VP\tilde{S}$-continuous function.*

Proof. It is obvious from the Theorem 2.9. □

Remark 3.25. *The notion of $VS\tilde{S}$-continuous and $VP\tilde{S}$-continuous functions are independent of each other.*

Remark 3.26. *The converse of the above Theorem 3.24 need not be true and the independency of the above Remark 3.25 are shown by the following examples.*

Example 3.27. *Let $X=\{x_1,x_2\}$, $E=\{e_1,e_2,e_3\}$, $Y=\{y_1,y_2,y_3\}$ $K=\{k_1,k_2\}$ and Let (X,τ,E) and (Y,σ,K) be two vague soft topological spaces with $\tau=\{\hat{\emptyset}_E,(G,E),\hat{X}_E\}$ and $\sigma=\{\hat{\emptyset}_K,(F_1,K),(F_2,K),\hat{Y}_K\}$ where,*

$$(G,E)=\left\{\begin{array}{c}\left\langle e_1,\frac{[0.1,0.2]}{x_1},\frac{[0.1,0.3]}{x_2}\right\rangle,\\ \left\langle e_2,\frac{[0,0.1]}{x_1},\frac{[0,0.5]}{x_2}\right\rangle,\\ \left\langle e_3,\frac{[0,0.2]}{x_1},\frac{[0.2,0.4]}{x_2}\right\rangle\end{array}\right\},$$

$$(F_1,K)=\left\{\begin{array}{c}\left\langle k_1,\frac{[0.2,0.7]}{y_1},\frac{[0.1,0.9]}{y_2},\frac{[0.3,0.6]}{y_3}\right\rangle,\\ \left\langle k_2,\frac{[0.2,0.8]}{y_1},\frac{[0.4,0.5]}{y_2},\frac{[0,0.7]}{y_3}\right\rangle\end{array}\right\}$$

and

$$(F_2,K)=\left\{\begin{array}{c}\left\langle k_1,\frac{[0.5,0.9]}{y_1},\frac{[0.6,0.9]}{y_2},\frac{[0.4,0.8]}{y_3}\right\rangle,\\ \left\langle k_2,\frac{[0.8,0.8]}{y_1},\frac{[0.5,0.8]}{y_2},\frac{[0.3,0.7]}{y_3}\right\rangle\end{array}\right\}.$$

Consider vague soft function $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ where u: X→Y and p: E→K are defined as follows:

$$u(x_1)=y_1,\ u(x_2)=y_3.$$
$$p(e_1)=k_1,\quad p(e_2)=k_2,\ p(e_3)=k_1.$$

Then the vague soft function g_{pu} is $VS\tilde{S}$- continuous but neither $VP\tilde{S}$- continuous nor $V\alpha\tilde{S}$- continuous.

Example 3.28. *Let $X=\{x_1,x_2,x_3\}$, $E=\{e_1,e_2\}$, $Y=\{y_1,y_2\}$ $K=\{k_1,k_2,k_3\}$ and Let (X,τ,E) and (Y,σ,K) be two vague soft topological spaces with $\tau=\{\hat{\emptyset}_E,(G_1,E),(G_2,E),\hat{X}_E\}$ and $\sigma=\{\hat{\emptyset}_K,(F,K),\hat{Y}_K\}$, where*

$$(G_1,E)=\left\{\begin{array}{c}\left\langle e_1,\frac{[0.2,0.8]}{x_1},\frac{[0.2,0.6]}{x_2},\frac{[0.4,0.6]}{x_3}\right\rangle,\\ \left\langle e_2,\frac{[0.1,0.4]}{x_1},\frac{[0.3,0.5]}{x_2},\frac{[0.3,0.6]}{x_3}\right\rangle\end{array}\right\},$$

$$(G_2,E)=\left\{\begin{array}{c}\left\langle e_1,\frac{[0.4,0.8]}{x_1},\frac{[0.7,0.8]}{x_2},\frac{[0.6,0.7]}{x_3}\right\rangle,\\ \left\langle e_2,\frac{[0.9,1]}{x_1},\frac{[0.8,0.9]}{x_2},\frac{[0.6,0.8]}{x_3}\right\rangle\end{array}\right\}$$

and

$$(F,K)=\left\{\begin{array}{c}\left\langle k_1,\frac{[0.5,0.7]}{y_1},\frac{[0.4,0.6]}{y_2}\right\rangle,\\ \left\langle k_2,\frac{[0.7,0.8]}{y_1},\frac{[0.8,0.9]}{y_2}\right\rangle,\\ \left\langle k_3,\frac{[0.7,0.8]}{y_1},\frac{[0.6,0.8]}{y_2}\right\rangle\end{array}\right\}.$$

Consider vague soft function $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ where $u:X\to Y$ and $p:E\to K$ are defined as follows:

$$u(x_1)=y_2,\ u(x_2)=y_2,\ u(x_3)=y_2.$$
$$p(e_1)=k_1,\ p(e_2)=k_3.$$

Then the vague soft function g_{pu} is $VP\tilde{S}$- continuous but neither $VS\tilde{S}$- continuous nor $V\alpha\tilde{S}$- continuous.

Theorem 3.29. *A vague soft function $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ is $V\alpha\tilde{S}$- continuous iff it is both $VS\tilde{S}$- continuous and vague $VP\tilde{S}$-continuous.*

Proof. : **Necessity** It follows from the Theorem 3.24.

Sufficiency: Let $(G,K)\in\sigma$. Since g_{pu} is both $VS\tilde{S}$- continuous and vague $VP\tilde{S}$-continuous, $g_{pu}^{-1}(G,K)\in VS\tilde{S}O(X)$ and $g_{pu}^{-1}(G,K)\in VP\tilde{S}O(X)$. Now from the Theorem 2.9 we can have $g_{pu}^{-1}(G,K)\in V\alpha\tilde{S}O(X)$. Hence g_{pu} is $V\alpha\tilde{S}$- continuous. □

Definition 3.30. *Let $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ be a vague soft function. Then g_{pu} is called;*

1. *vague soft irresolute ($V\tilde{S}$-irresolute in short) if $g_{pu}^{-1}(G,K)\in VS\tilde{S}O(X)$ for all $(G,K)\in VS\tilde{S}O(Y)$.*
2. *vague pre-soft irresolute ($VP\tilde{S}$-irresolute in short) if $g_{pu}^{-1}(G,K)\in VP\tilde{S}O(X)$ for all $(G,K)\in VP\tilde{S}O(Y)$.*
3. *vague α-soft irresolute ($V\alpha\tilde{S}$-irresolute in short) if $g_{pu}^{-1}(G,K)\in V\alpha\tilde{S}O(X)$ for all $(G,K)\in V\alpha\tilde{S}O(Y)$.*

Remark 3.31. *The following examples shows that $V\tilde{S}$-irresolute functions are not $V\tilde{S}$-continuous. Neither are $V\tilde{S}$-continuous functions necessarily $V\tilde{S}$-irresolute.*

Example 3.32. *Let $X=\{x_1,x_2\}$, $E=\{e_1,e_2\}$, $Y=\{y_1,y_2\}$ and $K=\{k_1,k_2\}$. Let us consider $\tau=\{\hat{\emptyset}_E,(F,E),\hat{X}_E\}$ and $\sigma=\{\hat{\emptyset}_K,(G,K),\hat{Y}_K\}$ be two vague soft topological spaces on X and Y respectively where,*

$$(F,E)=\left\{\begin{array}{c}\left\langle e_1,\frac{[0.4,0.5]}{x_1},\frac{[0.4,0.5]}{x_2}\right\rangle,\\ \left\langle e_2,\frac{[0.4,0.5]}{x_1},\frac{[0.4,0.5]}{x_2}\right\rangle\end{array}\right\}$$

and

$$(G,K)=\left\{\begin{array}{c}\left\langle k_1,\frac{[0.5,0.6]}{y_1},\frac{[0.3,0.4]}{y_2}\right\rangle,\\ \left\langle k_2,\frac{[0.4,0.5]}{y_1},\frac{[0.4,0.5]}{y_2}\right\rangle\end{array}\right\}.$$

Consider vague soft function $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ where $u:X\to Y$ and $p:E\to K$ are defined as follows:

$$u(x_1)=y_1,\ u(x_2)=y_2.\qquad p(e_1)=p(e_2)=k_2.$$

Then g_{pu} is $V\tilde{S}$-continuous function but not $V\tilde{S}$-irresolute function. For

$$(H,K)=\left\{\begin{array}{c}\left\langle k_1,\frac{[0.6,0.7]}{y_1},\frac{[0.5,0.6]}{y_2}\right\rangle,\\ \left\langle k_2,\frac{[0.9,1]}{y_1},\frac{[0.7,0.8]}{y_2}\right\rangle\end{array}\right\}\in VS\tilde{S}O(Y),$$

$g_{pu}^{-1}(H,K)\notin VS\tilde{S}O(X)$. Clearly, g_{pu} is not $V\tilde{S}$ irresolute function.

Example 3.33. *Let $X=\{x_1,x_2\}$, $E=\{e_1,e_2\}$ and consider the following two vague soft topologies $\tau_1=\{\hat{\emptyset}_E,(F,E),\hat{X}_E\}$ and $\tau_2=\{\hat{\emptyset}_E,(G,E),\hat{X}_E\}$ on X where,*
$(F,E)=\left\{\left\langle e_1,\frac{[0.2,0.7]}{x_1},\frac{[0.5,0.6]}{x_2}\right\rangle,\left\langle e_2,\frac{[0.4,0.6]}{x_1},\frac{[0.4,0.7]}{x_2}\right\rangle\right\}$
and $(G,E)=\left\{\left\langle e_1,\frac{[0.4,0.7]}{x_1},\frac{[0.5,0.8]}{x_2}\right\rangle,\left\langle e_2,\frac{[0.7,0.8]}{x_1},\frac{[0.6,0.7]}{x_2}\right\rangle\right\}$.
Consider vague soft function $g_{pu}:(X,\tau_1,E)\to(X,\tau_2,E)$ where $u:X\to X$ and $p:E\to E$ are defined as follows:

$$u(x_1)=x_1,\ u(x_2)=x_2;\qquad p(e_1)=e_1,\ p(e_2)=e_2.$$

Then g_{pu} is $V\tilde{S}$-irresolute function, since $v\tilde{s}cl(G,E)=\hat{X}_E$ and any vague semi-soft open set (H,E) in (X,τ_2,E), we have $(G,E)\subseteq(H,E)$. Hence $g_{pu}^{-1}(H,E)=(H,E)\supseteq(G,E)\supseteq(F,E)$. Also $v\tilde{s}cl(F,E)=\hat{X}_E$ which implies that $g_{pu}^{-1}(H,E)$ is vague semi-soft open in (X,τ_1,E). Hence g_{pu} is $V\tilde{S}$ irresolute function. Clearly, g_{pu} is not $V\tilde{S}$-continuous function.

Theorem 3.34. *For a vague soft function $g_{pu}:V\tilde{S}(X,E)\to V\tilde{S}(Y,K)$, we have the followings:*

1. *Every $V\tilde{S}$-irresolute function is $VS\tilde{S}$-continuous.*
2. *Every $VP\tilde{S}$-irresolute function is $VP\tilde{S}$-continuous.*
3. *Every $V\alpha\tilde{S}$-irresolute function is $V\alpha\tilde{S}$-continuous.*

Proof. The proof is obvious. □

Remark 3.35. *The converse of the above Theorem 3.34 need not be true as shown in the following examples.*

Example 3.36. *Let us consider the vague semi-soft continuous function g_{pu} as in Example 3.27. Clearly it is not $V\tilde{S}$-irresolute. Because for the vague semi-soft open set*
$(F,K)=\left\{\left\langle k_1,\frac{[0.8,1]}{y_1},\frac{[1,1]}{y_2},\frac{[0.7,1]}{y_3}\right\rangle,\ \left\langle k_2,\frac{[1,1]}{y_1},\frac{[0.9,0.9]}{y_2},\frac{[1,1]}{y_3}\right\rangle\right\}$
in (Y,σ,K), $g_{pu}^{-1}(F,K)$ is not vague semi-soft open in (X,τ,E).

Example 3.37. *Let* X=$\{x_1,x_2\}$, E=$\{e_1,e_2\}$, Y=$\{y_1,y_2\}$, K=$\{k_1,k_2\}$ *and Let* (X,τ,E) *and* (Y,σ,K) *be two vague soft topological spaces with* $\tau=\{\hat{\emptyset}_E,(F,E),\hat{X}_E\}$ *and* $\sigma=\{\hat{\emptyset}_K,(G,K),\hat{Y}_K\}$, *where*
$(F,E)=\left\{\left\langle e_1,\frac{[0.7,0.9]}{x_1},\frac{[0.8,0.9]}{x_2}\right\rangle,\left\langle e_2,\frac{[0.9,1]}{x_1},\frac{[0.8,0.9]}{x_2}\right\rangle\right\}$,
$(G,K)=\left\{\left\langle k_1,\frac{[0.2,0.7]}{y_1},\frac{[0.4,0.5]}{y_2}\right\rangle,\left\langle k_2,\frac{[0.2,0.2]}{y_1},\frac{[0.3,0.4]}{y_2}\right\rangle\right\}$.
Consider vague soft function $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ *where* $u:X\to Y$ *and* $p:E\to K$ *are defined as follows:*

$$u(x_1)=y_1,\ u(x_2)=y_2;\qquad p(e_1)=k_1,\ p(e_2)=k_2.$$

Then this vague soft function g_{pu} *is vague pre-soft continuous but not* $VP\tilde{S}$*-irresolute. Because for the vague pre-soft open set* $(G_1,K)=\left\{\left\langle k_1,\frac{[0.1,0.2]}{y_1},\frac{[0,0.2]}{y_2}\right\rangle,\left\langle k_2,\frac{[0,0.1]}{y_1},\frac{[0.1,0.1]}{y_2}\right\rangle\right\}$ *in* (Y,σ,K), $g_{pu}^{-1}(G_1,K)$ *is not vague pre-soft open in* (X,τ,E).

Example 3.38. *Let* X=$\{x_1,x_2\}$, E=$\{e_1,e_2\}$, Y=$\{y_1,y_2\}$, K=$\{k_1,k_2\}$ *and let* (X,τ,E) *and* (Y,σ,K) *be two vague soft topological spaces with* $\tau=\{\hat{\emptyset}_E,(F_1,E),(F_2,E),\hat{X}_E\}$ *and* $\sigma=\{\hat{\emptyset}_K,(G,K),\hat{Y}_K\}$, *where*
$(F_1,E)=\left\{\left\langle e_1,\frac{[0.2,0.4]}{x_1},\frac{[0.1,0.4]}{x_2}\right\rangle,\left\langle e_2,\frac{[0.2,0.3]}{x_1},\frac{[0.1,0.4]}{x_2}\right\rangle\right\}$,
$(F_2,E)=\left\{\left\langle e_1,\frac{[0.5,0.7]}{x_1},\frac{[0.4,0.8]}{x_2}\right\rangle,\left\langle e_2,\frac{[0.5,0.6]}{x_1},\frac{[0.4,0.7]}{x_2}\right\rangle\right\}$ *and*
$(G,K)=\left\{\left\langle k_1,\frac{[0.5,0.7]}{y_1},\frac{[0.4,0.8]}{y_2}\right\rangle,\left\langle k_2,\frac{[0.5,0.6]}{y_1},\frac{[0.4,0.7]}{y_2}\right\rangle\right\}$.
Consider vague soft function $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ *where* $u:X\to Y$ *and* $p:E\to K$ *are defined as follows:*

$$u(x_1)=y_1,\ u(x_2)=y_2;\qquad p(e_1)=k_1,\ p(e_2)=k_2.$$

Then this vague soft function g_{pu} *is vague* α*-soft continuous but not* $V\alpha\tilde{S}$*-irresolute. Because for the vague* α*-soft open set* $(G_1,K)=\left\{\left\langle k_1,\frac{[0.6,0.8]}{y_1},\frac{[0.6,0.9]}{y_2}\right\rangle,\left\langle k_2,\frac{[0.7,0.8]}{y_1},\frac{[0.8,0.9]}{y_2}\right\rangle\right\}$ *in* (Y,σ,K), $g_{pu}^{-1}(G_1,K)$ *is not vague* α*-soft open in* (X,τ,E).

Remark 3.39. *The following figure illustrate the implications we discussed as above.*

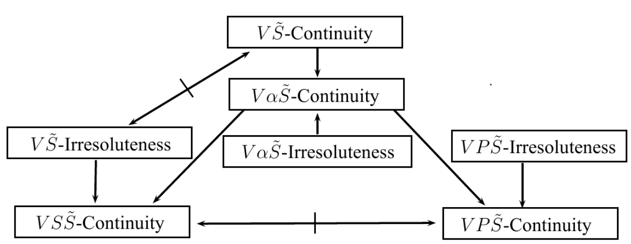

Figure 1

Definition 3.40. *Let* $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ *be a vague soft function. Then, the function* g_{pu} *is said to be*

1. *vague soft open if* $g_{pu}(F,E)$ *is vague soft open of* $V\tilde{S}(Y,K)$, *for each* (F,E) *vague soft open set of* (X,τ,E).
2. *vague soft closed if* $g_{pu}(F,E)$ *is vague soft closed of* $V\tilde{S}(Y,K)$, *for each* (F,E) *vague soft closed set of* (X,τ,E).

Theorem 3.41. *Let* $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ *be a vague soft function. Then the following statements are equivalent:*

1. g_{pu} *is a vague soft open map.*
2. $g_{pu}(v\tilde{s}int(F,E))\subseteq v\tilde{s}int(g_{pu}(F,E))$ *for each vague soft set* (F,E) *of* (X,τ,E).
3. $v\tilde{s}int(g_{pu}^{-1}(G,K))\subseteq g_{pu}^{-1}(v\tilde{s}int(G,K))$ *for each vague soft set* (G,K) *of* (Y,σ,K).

Proof. $\mathbf{1\Rightarrow 2}$: Let (F,E) be any vague soft set over (X,τ,E). Clearly v$\tilde{s}$int(F,E) is a vague soft open set over (X,τ,E). Since g_{pu} is vague soft open map, g_{pu}(v$\tilde{s}$int(F,E)) is a vague soft open set of (Y,σ,K). Thus
g_{pu}(v$\tilde{s}$int(F,E))=v$\tilde{s}$int(g_{pu}(v$\tilde{s}$int(F,E))) $\subseteq$v$\tilde{s}$int($g_{pu}(F,E)$).

$\mathbf{2\Rightarrow 3}$: Let (G,K) be any vague soft set of (Y,σ,K). Then $g_{pu}^{-1}(G,K)$ is a vague soft set of (X,τ,E). By (2), g_{pu}(v$\tilde{s}$int($g_{pu}^{-1}(G,K)$))$\subseteq$v$\tilde{s}$int($g_{pu}(g_{pu}^{-1}(G,K)$))$\subseteq$ v$\tilde{s}$int(G,K).
Thus we have,
v$\tilde{s}$int($g_{pu}^{-1}(G,K)$) $\subseteq g_{pu}^{-1}$(g_{pu}(v$\tilde{s}$int($g_{pu}^{-1}(G,K)$)))$\subseteq$
g_{pu}^{-1}(v$\tilde{s}$int(G,K)).

$\mathbf{3\Rightarrow 1}$: Let (S,E) be any vague soft open set in (X,τ,E). Then v$\tilde{s}$int$(S,E)=(S,E)$ and $g_{pu}(S,E)$ is a vague soft set of (Y,σ,K). By (3),
(S,E)=v$\tilde{s}$int(S,E) $\subseteq$v$\tilde{s}$int(g_{pu}^{-1}($g_{pu}(S,E)$)) $\subseteq$
g_{pu}^{-1}(v$\tilde{s}$int($g_{pu}(S,E)$)). Hence we have,
$g_{pu}(S,E)\subseteq g_{pu}$($g_{pu}^{-1}$(v$\tilde{s}$int($g_{pu}(S,E)$))) $\subseteq$v$\tilde{s}$int($g_{pu}(S,E)$) $\subseteq$ $g_{pu}(S,E)$. Thus $g_{pu}(S,E)$=v$\tilde{s}$int($g_{pu}(S,E)$ and hence $g_{pu}(S,E)$ is vague soft open set of (Y,σ,K). Therefore g_{pu} is vague soft open map. □

Theorem 3.42. *A vague soft function* $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ *is vague soft closed map iff* $v\tilde{s}cl(g_{pu}(F,E))\subseteq g_{pu}(v\tilde{s}cl(F,E))$ *for each vague soft set* (F,E) *of* (X,τ,E).

Proof. Let (F,E) be any vague soft set of (X,τ,E). Clearly v$\tilde{s}$cl(F,E) is a vague soft closed set of (X,τ,E). Since g_{pu} is vague soft closed map, g_{pu}(v$\tilde{s}$cl(F,E)) is a vague soft closed set over (Y,σ,K). Thus,
v$\tilde{s}$cl($g_{pu}(F,E)$) $\subseteq$v$\tilde{s}$cl(g_{pu}(v$\tilde{s}$cl(F,E)))= g_{pu}(v$\tilde{s}$cl(F,E)).

Conversely, let (F,E) be any vague soft closed set of (X,τ,E). Then v$\tilde{s}$cl$(F,E)=(F,E)$. By hypothesis,
v$\tilde{s}$cl($g_{pu}(F,E)$)$\subseteq g_{pu}$(v$\tilde{s}$cl(F,E))=g_{pu}(F,E)$\subseteq$ v$\tilde{s}$cl($g_{pu}(F,E)$).
Thus, $g_{pu}(F,E)=$ v$\tilde{s}$cl($g_{pu}(F,E)$) and hence $g_{pu}(F,E)$ is a vague soft closed set over (Y,σ,K). Therefore g_{pu} is a vague soft closed map. □

Theorem 3.43. *Let* $g_{pu}:(X,\tau,E)\to(Y,\sigma,K)$ *be a vague soft bijection mapping. Then the following statements are equivalent:*

1. g_{pu} *is a vague soft closed map.*
2. $v\tilde{s}cl(g_{pu}(F,E))\subseteq g_{pu}(v\tilde{s}cl(F,E))$ *for each vague soft set* (F,E) *over* (X,τ,E).

3. $g_{pu}^{-1}(v\tilde{s}cl(G,K)) \subseteq v\tilde{s}cl(g_{pu}^{-1}(G,K))$ *for each vague soft set* (G,K) *over* (Y,σ,K).

Proof. By the Theorem 3.42, it suffices to show that (2) is equivalent to (3).

Let (G,K) be any vague soft set of (Y,σ,K). Then $g_{pu}^{-1}(G,K)$ is a vague soft set of (X,τ,E). Since g_{pu} is onto,
$\text{v}\tilde{\text{s}}\text{cl}(G,K)=\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(g_{pu}^{-1}(G,K))) \subseteq g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K)))$
$\Rightarrow \quad g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(G,K)) \subseteq g_{pu}^{-1}(g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K))))$.
Since g_{pu} is one-to-one,
$g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(G,K)) \subseteq g_{pu}^{-1}(g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K))))$
$=\text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(G,K))$.

Conversely, let (F,E) be any vague soft set of (X,τ,E). Then $g_{pu}(F,E)$ is a vague soft set of (Y,σ,K). Since g_{pu} is one-to-one,
$g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E))) \subseteq \text{v}\tilde{\text{s}}\text{cl}(g_{pu}^{-1}(g_{pu}(F,E)))=\text{v}\tilde{\text{s}}\text{cl}(F,E)$.
$\Rightarrow \; g_{pu}(g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)))) \subseteq g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(F,E))$.
Since g_{pu} is onto,
$\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)) = g_{pu}(g_{pu}^{-1}(\text{v}\tilde{\text{s}}\text{cl}(g_{pu}(F,E)))) \subseteq g_{pu}(\text{v}\tilde{\text{s}}\text{cl}(F,E))$. □

4. Conclusion

In this paper, we have introduced and characterized some weaker forms of vague soft continuous functions. We have presented their basic properties with the help of some counterexamples. Also, we have obtained a decomposition of Vague α-soft continuity and have proved that the independency of vague soft continuity & vague soft irresoluteness. We hope that results in this paper will be helpful for further research in vague soft topological spaces.

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