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Analytic even mean labeling of some graphs

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Abstract

Let $G(V,E)$ be a graph with p vertices and q edges. A (p,q) - graph G is called an analytic even mean graph if there exist an injective function $f : V \to \{0,2,4,6,\dots,2q\}$ with an induced edge labeling $f^* : E \to Z$ such that when each edge $e = uv$ with $f(u) < f(v)$ is labeled with $f^*(uv) = \left\lceil \frac{f(v)^2 - (f(u)+1)^2}{2} \right\rceil$ if $f(u) \neq 0$ and $f^*(uv) = \left\lceil \frac{f(v)^2}{2} \right\rceil$ if $f(u) = 0$, all the edge labels are even and distinct. We prove Jewel graph, Jelly Fish graph, Triangular Book graph, Triangular Book with Book Mark admits analytic even mean labeling.

Keywords

Analytic even mean labeling, Jewel graph, Jelly Fish graph, Triangular Book graph.

AMS Subject Classification

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Contents

1. Introduction

By a graph $G = (V,E)$ with p vertices and q edges we mean a simple and undirected graph. The idea of graph labeling was bring in by Rosa in 1967[1]. Somasundaram and Ponraj [2] have set up the conception of mean labeling of graphs. A detailed survey of graph labeling can be found in [3]. Jeyanthi et al. [4] called a graph G is analytic odd mean if there exist an injective function $f : V \to \{0,1,3,5,\dots,2q-1\}$ with an induce edge labeling $f^* : E \to Z$ such that for every edge uv with $f(u) < f(v)$, $f^*(uv) = \begin{cases} \left\lceil \frac{f(v)^2 - (f(u)+1)^2}{2} \right\rceil & \text{if } f(u) \neq 0 \\ \left\lceil \frac{f(v)^2}{2} \right\rceil & \text{if } f(u) = 0 \end{cases}$ is injective.

$A(p,q)$ - graph G is called an analytic even mean graph if there exist an injective function $f : V \to \{0,2,4,6,\dots,2q\}$ with an induced edge labeling $f^* : E \to Z$ such that when each edge $e = uv$ with $f(u) < f(v)$ is labeled with $f^*(uv) = \left\lceil \frac{f(v)^2 - (f(u)+1)^2}{2} \right\rceil$ if $f(u) \neq 0$ and $f^*(uv) = \left\lceil \frac{f(v)^2}{2} \right\rceil$ if $f(u) = 0$, all the edge labels are even and distinct. This labeling f is named, an analytic even mean labeling [5]. The Jewel J_n is the graph with vertex set $V(J_n) = \{u,v,x,y,u_i; 1 \leq i \leq n\}$ and edge set $E(J_n) = \{ux,uy,xy,xv,yv,uu_i,vu_i; 1 \leq i \leq n\}$ [6]. The Jelly Fish graph $J(m,n)$ is obtained from a 4-cycle u,v,s and t by joining s and t with an edge and appending m pendent edges to u and n pendent edges to v [7]. The Triangular Book with n-pages is defined as n copies of cycle C_3 sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by $B(3,n)$. In other words it is the complete tripartite graph $K_{1,1,n}$ [8]. The Triangular Book with Book Mark is a Triangular Book $B(3,n)$ with a pendent edge attached at any one end vertices of the spine. This graph is denoted by $TB_n(u,v)(v,w)$ [8].

2. Main Results

In this section, we present and prove the main results.

Theorem 2.1. *The Jewel Graph J_n admits an analytic even mean labeling.*

Proof. Let $G = J_n$ be the graph with

$V(G) = \{u, v, x, y, v_i; 1 \leq i \leq n\}$ and
$E(G) = \{xu, yu, xy, xv, yv, uv_i, vv_i; 1 \leq i \leq n\}$.
Then $|V(G)| = n+4$, $|E(G)| = 2n+5$.
Define $f : V(G) \to \{0, 2, 4, \ldots, 2(2n+5)\}$ by
$f(x) = 0,\ f(y) = 2,\ f(u) = 4,\ f(v) = 6,\ f(v_i) = 8+2i; 1 \leq i \leq n$.
Let f^* be the generated edge labeling of f.
Now $f^*(xu) = 8,\ f^*(yu) = 4,\ f^*(xv) = 18,\ f^*(yv) = 14,\ f^*(xy) = 2$.

$$f^*(uv_i) = \left\lceil \frac{4i^2 + 32i + 39}{2} \right\rceil \ ; \ 1 \leqslant i \leqslant n$$

$$f^*(vu_i) = \left\lceil \frac{4i^2 + 32i + 15}{2} \right\rceil \ ; \ 1 \leqslant i \leqslant n$$

According to this, all the edge labels are even and distinct. For the edges uv_i and vv_i the edge labels increased by $4i+18$, as i increases. Hence the graph J_n is an analytic even mean graph. □

Example 2.2. *Analytic even mean labeling of J_4 is exposed in the following figure.*

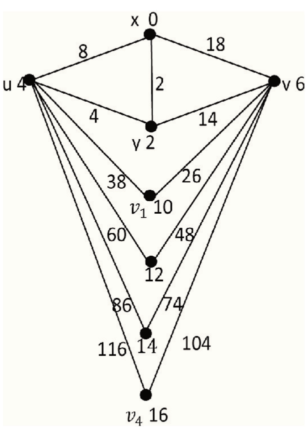

Figure 1

Theorem 2.3. *The Jelly Fish Graph $J(m,n)$ admits an analytic even mean labeling.*

Proof. Let $G = J(m,n)$ be the graph with $V(G) = \{u, v, s, t, u_i, v_j; 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(G) = \{tu, tv, ts, su, sv, uu_i, vv_j; 1 \leq i \leq m, 1 \leq j \leq n\}$.
Then $|V(G)| = m+n+4$, $|E(G)| = m+n+5$.
Define $f : V \to \{0, 2, 4, \ldots, 2(m+n+5)\}$ by
$f(t) = 0,\ f(s) = 2,\ f(u) = 4,\ f(v) = 6,\ f(u_i) = 6+2i; 1 \leq i \leq m$. $f(v_j) = 6+2m+2j; 1 \leq j \leq n$.
Let f^* be the generated edge labeling of f.
$f^*(ts) = 2, f^*(tu) = 8, f^*(tv) = 18, f*(sv) = 14, f^*(su) = 4$.

$$f^*(uu_i) = \left\lceil \frac{4i^2 + 24i + 11}{2} \right\rceil \ ; \ 1 \leqslant i \leqslant m$$

$$f^*(vv_i) = \left\lceil \frac{4m^2 + 4j^2 + 24m + 8mj + 24j - 13}{2} \right\rceil$$
$; \ 1 \leqslant j \leqslant n$

We observe that, for the edges uu_i, the edge labels increased by $4i+14$ as i increases and for the edges vv_j, the edge labels increased by $4i+34$ as j increases. According to this, all the edge labels are even and distinct. Hence the Jelly Fish graph $J(m,n)$ is an analytic even mean graph. □

Example 2.4. *Analytic even mean labeling of $J(5,3)$ is exposed in the following figure.*

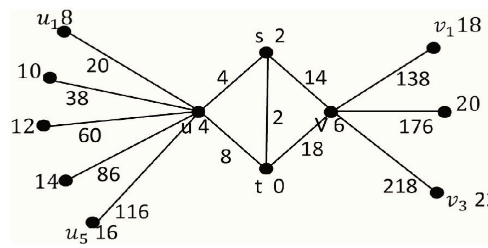

Figure 2

Theorem 2.5. *The Triangular Book Graph $B(3,n)$ admits an analytic even mean labeling.*

Proof. Let $G = B(3,n)$ be the Triangular Book graph with $V(G) = \{x, y, v_1, v_2, \ldots, v_n\}$ and $E(G) = \{xy, xv_i, yv_i; 1 \leq i \leq n\}$.
Then $|V(G)| = n+2$, $|E(G)| = 2n+1$.
Define $f : V(G) \to \{0, 2, 4, \ldots, 2(2n+1)\}$ by
$f(x) = 0, f(y) = 2, f(v_i) = 2+2i; 1 \leq i \leq n$.
Let f^* be the generated edge labeling of f.

$$f^*(xy) = 2,\ f^*(xv_i) = \lceil 2i^2 + 4i + 2 \rceil \ ; \ 1 \leqslant i \leqslant n$$

$$f^*(yv_i) = \left\lceil \frac{4i^2 + 8i - 5}{2} \right\rceil \ ; \ 1 \leqslant i \leqslant n$$

Here, the edge labels of the edges xv_i and yv_i are increased by $4i+6$ as i increases. Hence all the edge labels are even and distinct. Therefore the Triangular Book graph $B(3,n)$ is an analytic even mean graph. □

Example 2.6. *Analytic even mean labeling of $B(3,5)$ is exposed in the following figure.*

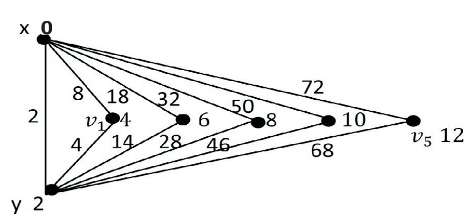

Figure 3

Theorem 2.7. *The Triangular Book with Book Mark $TB_n(u,v)(v,w)$ admits an analytic even mean labeling.*

Proof. Let $G = TB_n(u,v)(v,w)$ be the Triangular Book with Book Mark.
Let $V(G) = \{u, v, w, v_1, v_2, \ldots, v_n\}$ and u, v be the spine vertices and let the pendent vertex w be attached to the vertex v. Then
$E(G) = \{wv, uv, vv_i, uv_i; 1 \leq i \leq n\}$ and $|V(G)| = n+3$, $|E(G)| = 2(n+1)$.
Define $f : V \to \{0, 2, 4, \ldots, 4(n+1)\}$ by
$f(u) = 0$, $f(w) = 2$, $f(v) = 4$, $f(v_i) = 4 + 2i; 1 \leq i \leq n$.
Let f^* be the generated edge labeling of f.
$f^*(uv) = 8$. $f^*(wv) = 4$.

$$f^*(uv_i) = \lceil 2i^2 + 8i + 8 \rceil \ ; \ 1 \leqslant i \leqslant n$$
$$f^*(vv_i) = \left\lceil \frac{4i^2 + 16i - 9}{2} \right\rceil \ ; \ 1 \leqslant i \leqslant n$$

According to this, all the edge labels are even and distinct. Here the edge labels of uv_i and vv_i are increased by $4i + 10$ as i increases. Hence the Triangular Book with Book Mark $TB_n(u,v)(v,w)$ admits an analytic even mean labeling. □

Example 2.8. *Analytic even mean labeling of $TB_5(u,v)(v,w)$ is shown in the following figure.*

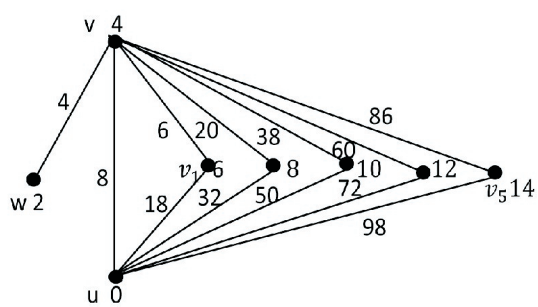

Figure 4

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