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Vague positive implicative filter of BL- algebras

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Abstract

In this paper, the concept of vague positive implicative filter (VPIF) of BL-algebra is introduced. Investigate some important properties of vague positive implicative filter (VPIF) of BL-algebras with illustrations. Further, we discuss some equivalent conditions of vague filter (VF) of BL-algebras. Finally, we obtain the necessary condition of vague Boolean filter (VBF) is a vague positive implicative filter (VPIF).

Keywords

BL-algebra; Filter; Vague set (VS); Vague Filter (VF); Vague Boolean Filter (VBF); Vague positive implicative filter (VPIF).

AMS Subject Classification

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1. Introduction

L.A. Zadeh [12] introduced the notion of fuzzy set (FS) theory in 1965. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1, 2] in 1986 as an extension of fuzzy set(FS). Hajek [4] introduced the concept of BL-algebras as the structures for Basic Logic. Gau and Buehrer [3] proposed the concept of vague set(VS) in 1993, by replacing the value of an element in a set with a subinterval of $[0,1]$. Thus, the grade of membership in vague set S is subinterval $[t_\delta(x), 1-f_\delta(x)]$ of $[0,1]$. The authors [8], [9], and [10] introduced the notions of vague filter (VF), vague prime (VP), Boolean filters (BFs) and vague implicative filter (VIF) of BL-algebras and investigate some of their related properties with exemplifications. The aim of this paper, we introduce the definition of vague positive implicative filter(VPIF) of BL-algebras, and investigate some important properties with exemplifications.

2. Preliminaries

In this section, we recall some basic knowledge of BL-algebras, vague sets and vague filters and their properties which are helpful to develop the main results.

Definition 2.1. *[4] The A BL-algebras is an algebra $(A,\vee,\wedge,*,0,1)$ of type $(2,2,2,2,0,0)$ such that*

(i) $(A,\vee,\wedge,0,1)$ *is a bounded lattice,*

(ii) $(A,*,1)$ *is a commutative monoid,*

(iii) $*$ *and* $\mapsto$ *form an adjoint pair, that is,* $w \le u \mapsto v$ *if and only if* $u * w \le v$,

(iv) $u \wedge v = u * (u \mapsto v)$,

(v) $(uv) \vee (v \mapsto u) = 1$ *for all* $u,v,w \in A$.

Definition 2.2. *[4] In a BL-algebra A, the following properties hold for all $u,v,w \in A$,*

(i) $v \mapsto (u \mapsto w) = u \mapsto (v \mapsto w) = (u*v) \mapsto w$,

(ii) $1 \mapsto u = u, u \mapsto u = 1, u \mapsto (v \mapsto u) = 1,$
$u \mapsto 1 = 0 \mapsto u = 1,$

(iii) $u \le v$ *if and only if* $u \mapsto v = 1$,

(iv) $u \vee v = ((u \mapsto v) \mapsto v) \wedge ((v \mapsto u) \mapsto u)$,

(v) $u \leq v$ *implies* $v \mapsto w \leq u \mapsto w$,

(vi) $u \leq v$ *implies* $w \mapsto u \leq w \mapsto v$,

(vii) $u \mapsto v \leq (w \mapsto u) \mapsto (w \mapsto v)$,

(viii) $u \mapsto v \leq (v \mapsto w) \mapsto (u \mapsto w)$,

Definition 2.3. *[3] Let $D[0,1]$ denote the family of all closed subintervals of $[0,1]$. Now we define refined maximum $(rmax)$ and $'' \geq ''$ on elements $D_1 = [p_1, q_1]$ and $D_2[p_2, q_2]$ of $D[0,1]$ as $rmax(D_1, D_2) = [max\{p_1, p_2\}, max\{q_1, q_2\}]$. Similarly, we can define $'' \leq '', =$ and rmin.*

Definition 2.4. *[8] Let S be VS of a BL-algebra A is called a vague filter(VF) of A if it satisfies the following axioms.*

(i) $V_S(1) \geq V_S(u)$,

(ii) $V_S(v) \geq rmin\{V_S(u \mapsto v), V_S(u)\}$ *for all* $u, v \in A$.

Proposition 2.5. *[8] Every VF Sof BL-algebra A is order preserving.*

Proposition 2.6. *[8] Let S bea vague set of BL-algebra A. Let S be a VF of A. Then the following hold if for all $u, v, w \in A$,*

(i) *If* $V_S(u \mapsto v) = V_S(1)$ *then* $V_S(u) \leq V_S(v)$,

(ii) $V_S(u \wedge v) = rmin\{V_S(u), V_S(v)\}$,

(iii) $V_S(u * v) = rmin\{V_S(u), V_S(v)\}$,

(iv) $V_S(0) = rmin\{V_S(u), V_S(u^-)\}$,

(v) $V_S(u \mapsto w) \geq rmin\{V_S(u \mapsto v), V_S(v \mapsto w)\}$,

(vi) $V_S(u \mapsto v) \leq V_S(u * w \mapsto v * w)$,

(vii) $V_S(u \mapsto v) \leq V_S((v \mapsto w) \mapsto (u \mapsto w))$,

(viii) $V_S(u \mapsto v) \leq V_S((w \mapsto u) \mapsto (w \mapsto v))$,

(ix) $u \mapsto v^- = v \mapsto u^- = u^{--} \mapsto v^- = (u * v)^-$.

3. Vague positive implicative filter

In this part, we introduce a notion of VPIF and investigate some related properties with exemplifications.

Definition 3.1. *Let S be a VF of BL-algebra A. S is called a VPIF, if it satisfies,*

(i) $V_S(1) \geq V_S(u)$,

(ii) $V_S(u \mapsto w) \geq rmin\{V_S(u \mapsto (v \mapsto w), V_S(u \mapsto v)\}$ *for all* $u, v, w \in A$.

Example 3.2. *Let $A = \{0, p, q, r, 1\}$. Define $u \wedge v = min\{u, v\}$, $u \vee v = max\{u, v\}$ and $'' * ''$ and $'' \mapsto ''$ given by the following tables I and II.*

$*$	0	p	q	1
0	0	0	0	0
p	0	p	p	p
q	0	p	q	q
1	0	p	q	1

Table I:$'' * ''$ operator

$\mapsto$	0	p	q	1
0	1	1	1	1
p	0	1	1	1
q	0	p	1	1
1	0	p	q	1

Table II:$'' \mapsto ''$ operator

Then $(A, \vee, \wedge, *, \mapsto, 0, 1)$ is a BL-algebra. Define VSS of A as follows:

$$S = \{(0, [0.2, 0.5]), (p, [0.2, 0.5]), (q, [0.2, 0.5]),$$

$$(r, [0.4, 0.7]), (1, [0.7, 0.7])\}.$$

It is easily verify that S is a VPIF of A.

Proposition 3.3. *Every VPIF is a vague filter.*

Proof. Let S be a VPIF of A. Then taking $u = 1$ in (ii) of definition 3.1, we have $V_S(1 \mapsto w) \geq rmin\{V_S(1 \mapsto (v \mapsto w), V_S(1 \mapsto v)\}$ for all $u, v, w \in A$.

$$V_S(w) \geq rmin\{V_S((v \mapsto w), V_S(v)\}$$

From (ii) proposition 2.6 and (i) of definition 3.1 is exist. Thus, S be a VF of A.

□

Converse of the proposition 3.3 may not be true. We prove this by the example as shown below

Example 3.4. *Let $A = \{0, p, q, r, 1\}$. Define $u \wedge v = min\{u, v\}$, $u \vee v = max\{u, v\}$ and $'' * ''$ and $'' \mapsto ''$ given by the following tables III and IV*

$*$	0	p	q	r	1
0	0	0	0	0	0
p	0	0	0	p	p
q	0	p	q	p	q
r	0	0	0	r	r
1	0	p	q	r	1

Table III:$'' * ''$ operator

$\mapsto$	0	p	q	r	1
0	1	1	1	1	1
p	r	1	1	1	1
q	r	r	1	r	1
r	0	q	q	1	1
1	0	p	q	r	1

Table IV:$'' \mapsto ''$ operator

Then, $(A, \vee, \wedge, *, \mapsto, 0, 1)$ *is a BL-algebra. Define a VSS of A as follows:*

$$S = \{(0, [0.1, 0.2]), (p, [0.3, 0.4]), (q, [0.3, 0.4]), (1, [0.5, 0.9])\}$$

. It is easily verify that S is a VF, but S is not a VPIF of A. Since

$$\begin{aligned} V_S(q \mapsto p) &= V_S(q) \\ &= [0.3, 0.4] \\ &< rmin\{V_S(q \mapsto (q \mapsto p)), V_S(q \mapsto q)\} \\ &= V_S(1) = [0.5, 0.9]. \end{aligned}$$

Next, we obtain some characteristics of VPIFs as follows.

Proposition 3.5. *Let S be VF of A. The following are equivalent for all* $u, v, w \in A$*.*

(i) *S is a VPIF,*

(ii) $V_S(u \mapsto v) \geq V_S(u \mapsto (u \mapsto v))$,

(iii) $V_S(u \mapsto v) = V_S(u \mapsto (u \mapsto v))$,

(iv) $V_S(u \mapsto (v \mapsto w)) \leq V_S((u \mapsto v) \mapsto (u \mapsto w))$,

(v) $V_S(u \mapsto (v \mapsto w)) = V_S((u \mapsto v) \mapsto (u \mapsto w))$,

(vi) $V_S((u * v) \mapsto w) = V_S((u \wedge v) \mapsto w)$.

Proof. $(i) \Rightarrow (ii)$

Let S be a VPIF of A.
Then from (ii) definition 3.1, we have

$$V_S(u \mapsto w) \geq rmin\{V_S(u \mapsto (v \mapsto w)), V_S(u \mapsto v)\} \tag{3.1}$$

for all $u, v, w \in A$.
Put $w = v$ and $v = u$ in (3.1), we get

$$\begin{aligned} V_S(u \mapsto v) &\geq rmin\{V_S(u \mapsto (u \mapsto v), V_S(u \mapsto u)\} \\ &\quad \text{[From (ii) of Proposition 2.5]} \\ &= rmin\{V_S(u \mapsto (u \mapsto v), V_S(1)\} \\ &\quad \text{[From (ii) of Proposition 2.5]} \\ &= V_S(u \mapsto (u \mapsto v) \\ &\quad \text{[From Definition 2.2]} \end{aligned}$$

Thus, we have

$$V_S(u \mapsto v) \geq V_S(u \mapsto (u \mapsto v)).$$

$(ii) \Rightarrow (iii)$
Since $u \mapsto v \leq u \mapsto (u \mapsto v)$, from proposition 2.6, we have $V_S(u \mapsto v) \leq V_S(u \mapsto (u \mapsto v))$ for all $u, v \in A$, and from (ii), we get

$$V_S(u \mapsto v) = V_S(u \mapsto (u \mapsto v)).$$

$(iii) \Rightarrow (i)$
If S is a VF of A.
The from (v) of proposition 2.6 and (i) of Prop. 2.5, we have

$$\begin{aligned} V_S(u \mapsto (u \mapsto w)) &\geq rmin\{V_S(u \mapsto v), V_S(v \mapsto (u \mapsto w)) \\ &= rmin\{V_S(u \mapsto v), V_S(u \mapsto (v \mapsto w))\}. \end{aligned}$$

Then from (iii), we have $V_S(u \mapsto w) \geq rmin\{V_S(u \mapsto v), V_S(u \mapsto (v \mapsto w))\}$ and from (i) of definition 3.1, we get S is a VPIF.
$(i) \Rightarrow (iv)$
If S is a VPIF of A. Then from (ii) of definition 3.1, we have

$$\begin{aligned} &V_S(u \mapsto ((u \mapsto v) \mapsto w)) \\ &\quad \geq rmin\{(wV_S(u \mapsto (v \mapsto w) \mapsto (((u \mapsto v) \mapsto w))), \\ &\quad V_S((u \mapsto (v \mapsto w)))\}. \end{aligned}$$

From (i), (vii) and $(viii)$ of the proposition 2.5, we have

$$\begin{aligned} &V_S(u \mapsto ((u \mapsto v) \mapsto w)) \\ &\quad = V_S((u \mapsto v) \mapsto (u \mapsto w)) \text{and} \\ &V_S(u \mapsto (v \mapsto w) \mapsto ((u \mapsto v) \mapsto w)))) \\ &\quad = V_S((u \mapsto v) \mapsto ((u \mapsto v) \mapsto (u \mapsto w))) \\ &\quad = V_S(1) \end{aligned}$$

It follows that,

$$\begin{aligned} &V_S((u \mapsto v) \mapsto (u \mapsto w)) \\ &\quad \geq rmin\{V_S(1), V_S(u \mapsto (v \mapsto w))\} \\ &\quad = V_S(u \mapsto (v \mapsto w)) \\ &\quad \text{[From the Definition 2.3]} \end{aligned}$$

$(iv) \Rightarrow (v)$ Since

$$\begin{aligned} u \mapsto (v \mapsto w) &= v \mapsto (u \mapsto w) \\ &= (1 \mapsto v) \mapsto (u \mapsto w) \\ &\geq (u \mapsto v) \mapsto (u \mapsto w), \end{aligned}$$

From the proposition 2.6, we have

$$V_S(u \mapsto (v \mapsto w)) \geq V_S((u \mapsto v) \mapsto (u \mapsto w))$$

From (iv), we get

$$V_S(u \mapsto (v \mapsto w)) = V_S((u \mapsto v) \mapsto (u \mapsto w)).$$

$(v) \Rightarrow (vi)$
Since

$$\begin{aligned} u \mapsto (v \mapsto w) &= u * v \mapsto w \text{ and} \\ (u \wedge v) \mapsto w &= (u * (u \mapsto v)) \mapsto w \\ &= (u \mapsto v) \mapsto (u \mapsto w) \end{aligned}$$

From (v) we have

$$V_S((u * v) \mapsto w) = V_S((u \wedge v) \mapsto w).$$

$(vi) \Rightarrow (i)$

If S is a VF of A, then $V_S(1) \geq V_S(u)$. From (v) of proposition 3.3 and (i) of proposition 2.5, we have

$$\begin{aligned}&V_S(u \mapsto (u \mapsto w))\\ &\geq rmin\{V_S(u \mapsto v), V_S(v \mapsto (u \mapsto w))\}\\ &= rmin\{V_S(u \mapsto v), V_S(u \mapsto (v \mapsto w))\}\end{aligned}$$

Since

$$\begin{aligned}&V_S(u \mapsto (u \mapsto w)) = V_S(u * u \mapsto w),\\ &V_S((u * u) \mapsto w) = V_S((u \wedge u) \mapsto w) \text{ [From (vi)]}\\ &= V_S(u \mapsto w).\end{aligned}$$

Thus,

$$V_S(u \mapsto w) \geq rmin\{V_S(u \mapsto (v \mapsto w), V_S(u \mapsto v)\}.$$

Hence, S is a VPIF of A. □

Proposition 3.6. *Let S be a VF of A. Then S is a VPIFA if and only if $V_S(v) \geq rmin\{V_S((v \mapsto w) \mapsto (u \mapsto v)), V_S(u)\}$ for all $u, v, w \in A$.*

Proof. Let S be a VPIFA. Then, from (i) of proposition 2.5, we have,

$$\begin{aligned}&rmin\{V_S((v \mapsto w) \mapsto (u \mapsto v)), V_S(u)\}\\ &\quad = rmin\{V_S(u \mapsto ((v \mapsto w) \mapsto v)), V_S(u)\}\\ &\quad \leq V_S((v \mapsto w) \mapsto v)).\end{aligned}$$

From (vii) of proposition 2.5, we have

$$(v \mapsto w) \mapsto v \leq w \mapsto v \leq (v \mapsto w) \mapsto ((w \mapsto v) \mapsto v).$$

Then, From proposition 2.6, we have

$$\begin{aligned}&V_S((v \mapsto w) \mapsto v) \leq V_S(w \mapsto v)\\ &\quad \leq V_S((v \mapsto w) \mapsto ((w \mapsto v) \mapsto v))\\ &\quad \leq V_S((w \mapsto v) \mapsto v).\end{aligned}$$

Thus,

$$\begin{aligned}&rmin\{V_S((v \mapsto w) \mapsto v), V_S(w \mapsto v)\}\\ &\quad \leq rmin\{V_S((w \mapsto v) \mapsto v), V_S(w \mapsto v)\}\\ &\quad \leq V_S(v).\end{aligned}$$

Therefore, we have

$$V_S(v) \geq rmin\{V_S((v \mapsto w) \mapsto (u \mapsto v)), V_S(u)\}.$$

Conversely, let S satisfies

$$V_S(v) \geq rmin\{V_S((v \mapsto w) \mapsto (u \mapsto v)), V_S(u)\}.$$

Then we easily prove that,

$$V_S(u \mapsto w) \geq rmin\{V_S(u \mapsto (v \mapsto w), V_S(u \mapsto v)\}.$$

Since S is VF,

$$V_S(1) \geq V_S(u).$$

Hence, S is VPIF of A. □

Proposition 3.7. *Let S_1 and S_2 be two VFs, of $S_1 \subseteq S_2, V_{S_1}(1) = V_{S_2}(1)$. If S_1 is a VPIF, so is S_2.*

Proof. From the proposition 3.5, we only prove that $V_{S_2}(u \mapsto w) \geq V_{S_2}(u \mapsto (u \mapsto w))$ for all $u, w \in A$.

Let $t = u \mapsto (u \mapsto w)$, then

$$\begin{aligned}u \mapsto (u \mapsto (t \mapsto w)) &= t \mapsto (u \mapsto (u \mapsto w))\\ &= t \mapsto t = 1.\end{aligned}$$

If S_1 is a VPIF, and from (iii) of the proposition 3.5, then

$$V_{S_1}(u \mapsto (t \mapsto w)) = V_{S_1}(u \mapsto (u \mapsto (t \mapsto w))) = V_{S_1}(1).$$

That is

$$V_{S_1}(t \mapsto (u \mapsto w)) = V_{S_1}(1) = V_{S_2}(1).$$

From $S_1 \subseteq S_2$, we get

$$\begin{aligned}V_{S_2}(t \mapsto (u \mapsto w)) &\geq V_{S_1}(t \mapsto (u \mapsto w))\\ &= V_{S_2}(1),\end{aligned}$$

from (i) of definition 2.3, we have,

$$V_{S_2}(t \mapsto (u \mapsto w)) = V_{S_2}(1).$$

Since S_2 is a VF,

$$V_{S_2}(u \mapsto w) \geq rmin\{V_{S_2}(t \mapsto (u \mapsto w)), V_{S_2}(t)\}.$$

Thus

$$\begin{aligned}V_{S_2}(u \mapsto w) &\geq rminV_{S_2}(1), V_{S_2}(t)\\ &= V_{S_2}(t) = V_{S_2}(u \mapsto (u \mapsto w)).\end{aligned}$$

Hence,S_2 is a VPIF. □

Proposition 3.8. *Every VBF is a VPIF, the converse may not be true.*

Proof. Let S be a VBF. Then

$$\begin{aligned}&V_S(u \mapsto w)\\ &\geq rmin\{V_S((u \vee u^-) \mapsto (u \mapsto w)), V_S(u \vee u^-)\}\\ &= rmin\{V_S((u \vee u^-) \mapsto (u \mapsto w)), V_S(1)\}\\ &= V_S((u \vee u^-) \mapsto (u \mapsto w)).\end{aligned}$$

Since

$$\begin{aligned}&(u \vee u^-) \mapsto (u \mapsto w)\\ &\quad = (u \mapsto (u \mapsto w)) \wedge (u^- \mapsto (u \mapsto w))\\ &\quad = u \mapsto (u \mapsto w),\end{aligned}$$

and from the proposition 2.5, we have

$$V_S((u \vee u^-) \mapsto (u \mapsto w)) = V_S(u \mapsto (u \mapsto w)).$$

Thus, we have

$$V_S(u \mapsto w) \geq V_S(u \mapsto (u \mapsto w)).$$

We consider proposition 3.5, we get S is a VPIF. □

We prove converse is not true from the following example.

Example 3.9. *We consider the example 3.2, Sis a VPIF, but S is not a VBF, since $V_S(q \vee q^-) = V_S(q) \neq V_S(1)$.*

Proposition 3.10. *Let S be a VPIF of A. S is a VBF if and only if*

$$V_S((u \mapsto v) \mapsto v) = V_S((v \mapsto u) \mapsto u) \text{ for all } u, v \in A.$$

Proof. Let S be a VPIF of A. We know that

$$u = 1 \mapsto u \leq (v \mapsto u) \mapsto u$$

and

$$v \leq (v \mapsto u) \mapsto u,$$

it follows that

$$((v \mapsto u) \mapsto u)^- \leq u^- \leq u \mapsto v$$

and

$$\begin{aligned}(u \mapsto v) \mapsto v &\leq ((v \mapsto u) \mapsto u) \mapsto v \\ &\leq ((v \mapsto u) \mapsto u)^- \mapsto ((v \mapsto u) \mapsto u).\end{aligned}$$

Then, we have

$$V_S(((v \mapsto u) \mapsto u)^- \mapsto ((v \mapsto u) \mapsto u)) \geq V_S((u \mapsto v) \mapsto v).$$

Since S is a VBF, from $V_S(u) = V_S(u^- \mapsto u)$, and (ix) of proposition 2.6, we get

$$V_S((v \mapsto u) \mapsto u) = V_S(((v \mapsto u) \mapsto u)^- \mapsto ((v \mapsto u) \mapsto u))$$

Thus, we have

$$V_S((v \mapsto u) \mapsto u) \geq V_S((u \mapsto v) \mapsto v) \tag{3.2}$$

Same method to prove

$$V_S((v \mapsto u) \mapsto u) \leq V_S((u \mapsto v) \mapsto v) \tag{3.3}$$

From (3.2) and (3.3), we get

$$V_S((u \mapsto v) \mapsto v) = V_S((v \mapsto u) \mapsto u)$$

Conversely, if S be a VPIF of A, and satisfies $V_S((u \mapsto v) \mapsto v) = V_S((v \mapsto u) \mapsto u)$.

Replace y by u^-, we have

$$V_S((u \mapsto u^-) \mapsto u^-) = V_S((u^- \mapsto u) \mapsto u).$$

Then, we get

$$V_S(u \vee u^-) = V_S((u \mapsto u^-) \mapsto u^-) \tag{3.4}$$

To Prove: S is a VBF.

It is enough to prove $V_S((u \mapsto u^-) \mapsto u^-) = V_S(1)$. Since S is a VPIF, from (v) of proposition 3.5, we have

$$\begin{aligned}V_S((u \mapsto u^-) \mapsto u^-) &= V_S((u \mapsto u^-) \mapsto (u \mapsto 0) \\ &= V_S(u \mapsto (u^- 0)) \\ &= V_S(u \mapsto u^{--}) = V_S(1) \end{aligned} \tag{3.5}$$

From (3.4) and (3.5), we get

$$V_S(u \vee u^-) = V_S(1).$$

Thus, S is a VBF. □

4. Conclusion

In the present paper, we have introduced the notion of a VPIF of BL-algebra, and investigate some related properties. Moreover, we have obtained some necessary and sufficient condition between VPIF and BF of BL-algebra.

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