



# Diffraction problem due to water wave loads on a cylinder over a coaxial bottom-mounted barrier in a channel

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## Abstract

In this work, we used the channel multipoles, eigenfunction expansion and separation of variables method to determine the diffracted potential of the wave for a floating solid cylinder in the free surface of the water over a coaxial bottom-mounted cylindrical barrier in a channel. Also, the mathematical formulation of the proposed model based on the linearized water wave theory. The forces due to diffraction acting on the floating cylinder are described here based on the Bernoulli's equation of fluid flow. The effect of various parameters of that composite structure on the exciting force and overturning moment of the floating cylinder is investigated. The result shows that a significant influence of forces acting on the floating cylinder in the presence of a bottom-mounted barrier in the channel which gives an accuracy of the analytical results.

## Keywords

Diffraction, continuity, peak value, multipoles.

## AMS Subject Classification

26A33, 30E25, 34A12, 34A34, 34A37, 37C25, 45J05.

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## 1. Introduction

The investigation of the diffraction and radiation problem

on ocean structures can give fundamental data of hydrodynamic properties of forces due to diffraction and radiation of the structure which can likewise be utilized as the premise to examine the problem of fluid-structure interaction. For the examination of structure in a channel, the main difficulties arise from the boundary conditions that occur at the walls of the channel. Therefore, the solution of the problem generally shown in literature which can be obtained by using the channel multipoles method. In [1] considered the diffraction problem of a couple of cylinders in a channel and solved the problem by using the method of image and green's function method. In [2] formulated the diffraction problem of a couple of cylinders in a channel and derived the first-order force on the cylinders in the presence of channel walls. In [3] gave the fundamental information diffraction and radiation of water waves by a cylinder in the water of finite depth. In [4, 5] studied wave generated by an array of cylinders in a channel and they used the multipoles method to compute the analytical solution. In [6, 7] presented the wave diffraction problem generated by a pair of a cylinder in water and analyzed the effect of hydrodynamics properties on the cylinders

for various parameters. In [8] derived second-order forces for the wave generated by an offshore structure in a channel. In [9, 10], introduced the channel multipoles method to solve the scattering and radiation problem of water wave by a truncated cylinder in a channel. Following the approach of [10], in [11] presented modified the scattering problem of water waves by axisymmetric bodies in a channel. In [12, 13] analysed the importance of the reflection and transmission coefficients due to the diffraction of water waves by a cylinder in a channel and compared both theoretical and experimental results. In [14] gave a fundamental analytical solution of three-dimensional incident potential due to incident waves on a cylinder in the water. In [15, 16] studied the hydrodynamic effect on a floating cylinder in the presence of a bottom obstacle. In [17] presented the wave generated by the cylinder in a channel and by using the method of image, they solved both diffraction and radiation problem due to floating buoy which is placed above a bottom-mounted cylinder. In the present paper, the separation of variables method is used to evaluate the hydrodynamic properties due to wave interaction with the cylinder under the assumption of the linearized theory of water waves i.e. we assumed the fluid is inviscid, homogenous and incompressible. Numerical results are presented with graphical representation for a range of wave number and various parameters of the proposed device.

## 2. Preliminaries

### 2.1 Formulation

Let us assume a linear water wave propagation in an ideal fluid of depth  $H$  and the fluid bounded by infinitely long channel walls of width  $d$ . Also, there is a composite structure consist of a solid cylinder which is floating in the free surface of the water and under it is a bottom-mounted cylindrical barrier and both the cylinder situated on the centreline of the walls. Let us consider the cartesian coordinate system of the model  $O - xyz$  is defined with the origin  $O$  at the floor of the sea as indicated in Fig. (1) and  $x$ -axis directed along the propagation of wave and  $z$ -axis directed vertically upward. The usual cylindrical coordinate  $O - r\theta z$  is defined with  $x = r \cos \theta$  and  $y = r \sin \theta$ . Hence the floating solid cylinder having radius  $r_1$  occupies the space is given by  $h_1 + h_2 \leq z \leq h_1 + h_2 + h_3$ ,  $r \leq r_1$ ,  $0 \leq \theta \leq 2\pi$ , after that which is denoted by Cyl.1 and the bottom-mounted cylindrical barrier having radius  $r_2 (\geq r_1)$  occupies the space is given by  $0 \leq z \leq h_1$ ,  $r \leq r_2$ ,  $0 \leq \theta \leq 2\pi$ , after that it is denoted by Cyl.2.

Since we assume that the fluid is homogeneous and irrotational, hence based on the linearity of water wave, the velocity potential  $\Phi(x, y, z, t)$  can be expressed as follow:

$$\Phi = \text{Re}[\phi(x, y, z)e^{-i\omega t}], \quad (2.1)$$

where,  $t$  and  $\omega$  define for the time and angular frequency, respectively,  $i = \sqrt{-1}$ ,  $\text{Re}$  denotes real part of complex quantity and time independent velocity potential  $\phi$  satisfies the three dimensional Laplace's equation.

With unit amplitude of the wave, the incident wave potential in the the fluid domain is given by (MacCamy and Fuchs [14])

$$\phi_i = -\frac{ig \cosh(k.z)}{\omega \cosh(k.H)} \sum_{n=0}^{\infty} \mu_n J_n(kr) \cos n\theta, \quad (2.2)$$

where  $k$  be wave number and it is determine from the following relation:  $\omega^2 = gk \tanh(kh_1)$ ,  $J_n(\cdot)$  stands for Bessel function of the first kind of order  $n$ , the gravitational acceleration denoted by  $g$  and  $\mu_n$  is given by

$$\mu_n = \begin{cases} 1, & n = 0 \\ 2i^n, & n = 1, 2, 3, \dots \end{cases} \quad (2.3)$$

Therefore, if  $\phi_s$  be the diffracted potential in the fluid domain, then total potential  $\phi$  can be expressed as follow:

$$\phi = \phi_i + \phi_s. \quad (2.4)$$

Also, the whole domain of the fluid can be separate by virtual boundary into three numbers of sub-domain as shown in Fig. (1). Let the sub-domains are denoted by  $\Omega_1$ ,  $\Omega_2$  and  $\Omega_3$  and there diffracted potential are define by  $\phi_s^1$ ,  $\phi_s^2$  and  $\phi_s^3$ , respectively.

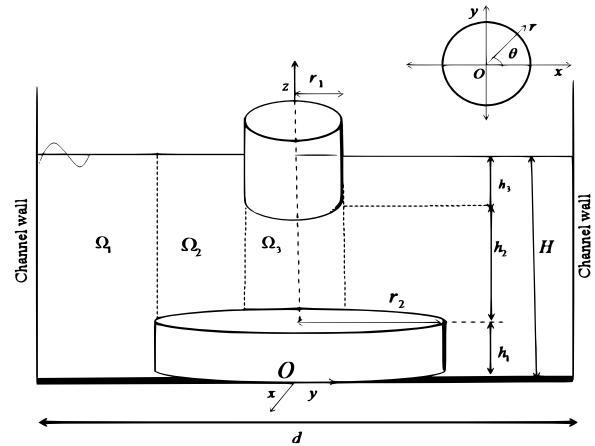


Figure 1. Structural model.

## 3. Diffraction problem

### 3.1 Boundary-value problem of diffraction

The diffracted potential  $\phi_s$  is governed by the following governing equation with boundary conditions:

$$\nabla^2 \phi_s = 0, \quad (0 < z < H, -\infty < x < \infty, -\frac{d}{2} < y < \frac{d}{2}), \quad (3.1)$$

$$\frac{\partial \phi_s}{\partial z} - \frac{\omega^2}{g} \phi_s = 0; \quad (z = H), \quad (3.2)$$

$$\frac{\partial \phi_s}{\partial z} = 0; \quad (z = 0, r \geq r_2; z = h_1, r \leq r_2), \quad (3.3)$$



$$\frac{\partial(\phi_i + \phi_s)}{\partial r} = 0; \quad (h_1 + h_2 < z < h_1 + h_2 + h_3, r = r_1; \\ 0 < z < h_1, r = r_2), \quad (3.4)$$

$$\frac{\partial \phi_s}{\partial y} = 0; \quad (y = \pm \frac{d}{2}), \quad (3.5)$$

$$\lim_{r \rightarrow \infty} \sqrt{r} \left( \frac{\partial \phi_s}{\partial r} - ik\phi_s \right) = 0. \quad (3.6)$$

Therefore, we will derived the diffracted wave potential for each subregions as indicated in structural model by using the above boundary-value problem.

### 3.2 Vertical eigenfunctions

The vertical eigenfunction i.e.  $z$ -functions can be determined by applying separation of variable method and these are define by

$$f_m(z) = \cos(\lambda_m \cdot z), \quad \text{for } \Omega_1, \quad (3.7)$$

$$g_m(z) = \cos[\alpha_m \cdot (z - h_1)], \quad \text{for } \Omega_2, \quad (3.8)$$

$$h_m(z) = \cos[\beta_m \cdot (z - h_1)], \quad \text{for } \Omega_3, \quad (3.9)$$

where the eigenvalues  $\lambda_m$ ,  $\alpha_m$  and  $\beta_m$  can be determined by applying the free surface condition given by the equation (3.2) and it will deduce the following dispersion relations:

$$\left\{ \begin{array}{l} \lambda_1 = -i.k, \quad \frac{\omega^2}{g} = g \tanh(k.H) \quad \text{for } m = 1, \\ \frac{\omega^2}{g} = -\lambda_m \tan(\lambda_m.H) \quad \text{for } m = 2, 3, \dots, \\ \alpha_1 = -ik_e, \quad \frac{\omega^2}{g} = k_e \tanh[k_e.(H - h_1)] \quad \text{for } m = 1, \\ \frac{\omega^2}{g} = -\alpha_m \tan[\alpha_m.(H - h_1)], \quad m = 2, 3, \dots, \\ \beta_m = \frac{(m-1).\pi}{h_2}, \quad m = 1, 2, 3, \dots, \end{array} \right. \quad (3.10)$$

where  $k_e$  be the wave number in the sub-domain  $\Omega_2$ . Hence we deduced the Helmholtz equation by applying the diffracted wave potential  $\phi_s^1$  in equation (3.1) for the sub-domain  $\Omega_1$  and it is given by

$$\frac{\partial^2 \phi_s^1}{\partial x^2} + \frac{\partial^2 \phi_s^1}{\partial y^2} - \lambda_m^2 \phi_s^1 = 0. \quad (3.11)$$

### 3.3 Diffraction potentials

In this section, we derived the analytical solution of the above boundary-value problem given by equations (3.1)-(3.6) based on the method of channel multipoles and separation of variables. In the sub-region  $\Omega_1$ , we apply the channel multipoles method which is developed by Linton and Evnas [10] and for the other sub-regions, we solved the problem by using separation of variables method. Therefore, the numerical solutions of diffracted potentials for each sub-regions are given as follow:

- Region  $\Omega_1$ :

$$\phi_s^1 = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \sum_{q=0}^{\infty} f_m(z) a_{q,m} [U_n(\lambda_m r) \delta_{qn} + E(n, q; m) V_n(\lambda_m r)] \times \cos n\theta. \quad (3.12)$$

- Region  $\Omega_2$ :

$$\phi_s^2 = -\phi_i + \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} [b_{n,m} S_n(\alpha_m r) + c_{n,m} T_n(\alpha_m r)] \times g_m(z) \cos n\theta. \quad (3.13)$$

- Region  $\Omega_3$ :

$$\phi_s^3 = -\phi_i + \sum_{n=0}^{\infty} \left[ d_{n,1} r^n + \sum_{m=2}^{\infty} d_{n,m} I_n(\beta_m r) h_m(z) \right] \times \cos n\theta. \quad (3.14)$$

where  $a_{n,m}$ ,  $b_{n,m}$ ,  $c_{n,m}$  and  $d_{n,m}$  are unknown constants and the parameter  $E(n, q; m)$  is define by

$$E(2n+1, 2q+1; m) = \begin{cases} -\frac{4i}{\pi} \int_0^{\infty} \frac{e^{-(k\zeta d)/2} B_{2n+1}(z) B_{2q+1}(z)}{\zeta \sinh \frac{k\zeta d}{2}} dz + \\ \frac{2}{kd} \sum_{l=0}^j \mu_l z_l^{-1} B_{2n+1}(z_l) B_{2q+1}(z_l), \quad m = 1 \\ 2 \int_1^{\infty} \frac{e^{-(\lambda_m z d)/2} A_{2n+1}(z) A_{2q+1}(z)}{\zeta \sinh \frac{\lambda_m z d}{2}} dz, \quad m \geq 2 \end{cases} \quad (3.15)$$

$$E(2n, 2q; m) = \begin{cases} -\frac{2i\mu_n}{\pi} \int_0^{\infty} \frac{e^{-(k\zeta d)/2} A_{2n}(z) A_{2q}(z)}{\zeta} \sinh \frac{k\zeta d}{2} dz + \\ \frac{\mu_n}{kd} \sum_{l=0}^j \mu_l z_l^{-1} A_{2n}(z_l) A_{2q}(z_l), \quad m = 1 \\ \mu_n \int_1^{\infty} \frac{e^{-(\lambda_m z d)/2} A_{2n}(z) A_{2q}(z)}{\zeta \sinh \frac{\lambda_m z d}{2}} dz, \quad m \geq 2 \end{cases} \quad (3.16)$$

and

$$E(2n_1, 2q; m) = E(2n, 2q+1; m) = 0, \quad m = 1, 2, 3, \dots, \quad (3.17)$$

with

$$A_{2q+1}(z) = \begin{cases} \cos(2q+1) \sin^{-1} z, & z \leq 1 \\ i(-1)^q \sinh[(2q+1) \cosh^{-1} z], & z > 1, \end{cases} \quad (3.18)$$

$$A_{2q}(z) = \begin{cases} \cos(2q \sin^{-1} z), & z \leq 1 \\ (-1)^q \cosh(2q \cosh^{-1} z), & z > 1, \end{cases} \quad (3.19)$$

$$B_{2q+1}(z) = \begin{cases} \sin[(2q+1) \sin^{-1} z], & z \leq 1 \\ (-1)^q \cosh[(2q+1) \cosh^{-1} z], & z > 1, \end{cases} \quad (3.20)$$

$$\zeta(z) = \begin{cases} -i(1-z)^{1/2}, & z \leq 1 \\ (z^2-1)^{1/2}, & z > 1 \end{cases} \quad (3.21)$$

and the integral values of equations given by (3.16) and (3.15) for  $m = 0$  are taken only the principal value of integral at their singularities and the principal integral value will satisfies the Helmholtz equation (3.11). Since the integrand of the



integral considered as a complex valued function in  $z$  which has simple poles at  $z = \pm z_l$ ,  $l = 0, 1, 2, \dots$ , where

$$z_l = (1 - (2l\pi/kd)^2)^{1/2}, \quad l = 0, 1, 2, \dots, j$$

$$z_l = i((2l\pi/kd)^2 - 1)^{1/2}, \quad l \geq j + 1$$

with  $2j\pi < kd < 2(j+1)\pi$ . Also the radial functions  $U_n(\cdot)$ ,  $V_n(\cdot)$ ,  $S_n(\cdot)$  and  $T_n(\cdot)$  appeared in velocity potentials expression are given by

$$\left\{ \begin{array}{l} U_n(\lambda_1 r) = H_n^{(1)}(i\alpha_1 r) = H_n^{(1)}(kr), \text{ for } m = 1 \\ U_n(\lambda_m r) = K_n(\lambda_m r), \text{ for } m = 2, 3, \dots \\ V_n(\lambda_1 r)(r) = J_n(\lambda_1 r) = J_n(kr), \text{ for } m = 1 \\ V_n(\lambda_m r) = I_n(\lambda_m r) \text{ for } m = 2, 3, \dots \\ S_n(\alpha_1 r) = H_n^{(1)}(\alpha_1 r) = H_n^{(1)}(k_e r), \text{ for } m = 1 \\ S_n(\alpha_m r) = K_n(\alpha_m r), \text{ for } m = 2, 3, \dots \\ T_n(\alpha_1 r) = H_n^{(2)}(\alpha_1 r) = H_n^{(2)}(k_e r), \text{ for } m = 1 \\ T_n(\alpha_m r) = I_n(\alpha_m r), \text{ for } m = 2, 3, \dots, \end{array} \right. \quad (3.22)$$

where  $H_n^{(1)}(\cdot)$  stands for the first kind of Hankel function of order  $n$ ,  $I_n(\cdot)$  and  $K_n(\cdot)$  are the first kind and second kind of modified Bessel function of order  $n$  and  $H_n^{(2)}(\cdot)$  is define for second kind of Hankel function of order  $n$ .

## 4. Main Results

### 4.1 Hydrodynamic forces and moment

The hydrodynamic forces, namely horizontal and vertical force acting on the floating cylinder i. e. Cyl.1 and overturning moment of the Cyl.1 concerning for the centroid of the bottom face of the cylinder are calculated by integrating the dynamic pressure which is given by Bernoulli's equation of continuity and it can be express in term of velocity potential as follow:

$$P = -\rho \frac{\partial \Phi(r, \theta, z, t)}{\partial t}, \quad (4.1)$$

where  $\rho$  denoted the density of the fluid Hence the time-independent dynamic pressure  $p$  based on water wave theory can be written as:

$$p = -i\omega\phi(r, \theta, z). \quad (4.2)$$

Let  $F_{s1}$ ,  $F_{v1}$  and  $M_x$  be the horizontal force, vertical force and overturning moment due to wave loads on Cyl.1 and the expression are obtained as follow:

$$\begin{aligned} F_{s1} &= -i\omega r_1 \rho \int_0^{2\pi} \int_{h_1+h_2}^{h_1+h_2+h_3} [\phi_i(r_1, \theta, z) + \phi_s^2(r_1, \theta, z)] \times \\ &\quad \cos \theta dz d\theta \\ &= -i\omega r_1 \rho \pi \sum_{m=1}^{\infty} [b_{1,m} S_1(\alpha_m r_1) + C_{1,m} T_1(\alpha_m r_1)] \times \\ &\quad \frac{\sin \alpha_m (h_2 + h_3) - \sin \alpha_m h_2}{\alpha_m}, \end{aligned} \quad (4.3)$$

$$F_{v1} = i\omega \rho \int_0^{r_1} \int_0^{2\pi} [\phi_i(r, \theta, h_1 + h_2)].r \cos \theta d\theta dr, \quad (4.4)$$

$$= 2i\omega \pi \rho \left[ \frac{d_{0,1} r_1^2}{2} + \sum_{m=2}^{\infty} [d_{0,m} r_1 I_1(\beta_m r_1) h_m(h_1)] \right], \quad (4.5)$$

$$\begin{aligned} M_x &= -i\omega \rho \int_0^{2\pi} \int_{h_1+h_2}^{h_1+h_2+h_3} [\phi_i(r_1, \theta, z) + \phi_s^2(r_1, \theta, z)] \times \\ &\quad (z - h_1 - h_2) r_1 \cos \theta dz d\theta - i\omega \rho \times \\ &\quad \int_0^{2\pi} \int_0^{r_1} [\phi_i(r, \theta, h_1 + h_2) + \phi_s^3(r, \theta, h_1 + h_2)].r^2 \cos \theta dr d\theta, \\ &= -i\omega \rho \pi r_1^2 \sum_{m=1}^{\infty} [b_{1,m} S_1(\alpha_m r_1) + c_{1,m} T_1(\alpha_m r_1)] \times \\ &\quad \left( \frac{\cos \alpha_m (h_2 + h_3)}{\alpha_m^2} - \frac{h_3 \sin \alpha_m (h_2 + h_3)}{\alpha_m} \right) - \\ &\quad i\omega \rho \pi \left[ \frac{d_{1,1} r_1^4}{4} + \sum_{m=2}^{\infty} \frac{d_{1,m} r_1^2 \cos(\beta_m h_2)}{\beta_m} .I_2(\beta_m r_1) \right]. \end{aligned} \quad (4.6)$$

And the respectively non-dimensional forces and moment are express as follow:

$$\frac{F_{s1}}{u_0} = -\frac{i\omega}{gr_1} \sum_{m=1}^{\infty} [b_{1,m} S_1(\alpha_m r_1) + C_{1,m} T_1(\alpha_m r_1)] \times \frac{\sin \alpha_m (h_2 + h_3) - \sin \alpha_m h_2}{\alpha_m}. \quad (4.7)$$

$$\frac{F_{v1}}{u_0} = \frac{2i\omega}{gr_1} \left[ \frac{d_{0,1} r_1^2}{2} + \sum_{m=2}^{\infty} [d_{0,m} r_1 I_1(\beta_m r_1) h_m(h_1)] \right], \quad (4.8)$$

$$\begin{aligned} \frac{M_x}{u_0 r_1} &= -\frac{i\omega}{gr_1} \sum_{m=1}^{\infty} [b_{1,m} S_1(\alpha_m r_1) + c_{1,m} T_1(\alpha_m r_1)] \times \\ &\quad \left( \frac{\cos \alpha_m (h_2 + h_3)}{\alpha_m^2} - \frac{h_3 \sin \alpha_m (h_2 + h_3)}{\alpha_m} \right) - \\ &\quad i\omega \rho \pi \left[ \frac{d_{1,1} r_1^4}{4} + \sum_{m=2}^{\infty} \frac{d_{1,m} r_1^2 \cos(\beta_m h_2)}{\beta_m} .I_2(\beta_m r_1) \right] \end{aligned} \quad (4.9)$$

where  $u_0 = g\rho\pi r_1^2$ .

### 4.2 Continuity conditions

Continuity conditions are also known as matching conditions and these conditions are arises between the physical and virtual boundary to preserved the flow of continuity. Therefore along the boundary  $r = r_1$  as in indicated Fig.(1), we get

$$\phi_s^1 = \phi_s^2 \quad (h_1 \leq z \leq h_1 + h_2 + h_3), \quad (4.10)$$

$$\frac{\partial \phi_s^1}{\partial r} = \begin{cases} -\frac{\partial \phi_i}{\partial r} & (0 \leq z \leq h_1), \\ \frac{\partial \phi_s^2}{\partial r} & (h_1 \leq z \leq h_1 + h_2 + h_3). \end{cases} \quad (4.11)$$

Along  $r = r_1$ , we have

$$\phi_s^2 = \phi_s^3 \quad (h_1 \leq z \leq h_1 + h_2), \quad (4.12)$$

$$\frac{\partial \phi_s^2}{\partial r} = \begin{cases} \frac{\partial \phi_s^3}{\partial r} & (h_1 \leq z \leq h_1 + h_2), \\ -\frac{\partial \phi_i}{\partial r} & (h_1 + h_2 \leq z \leq h_1 + h_2 + h_3). \end{cases} \quad (4.13)$$

Applying the velocity potential given by equations (3.12)-(3.13) into the continuity equations (4.10)-(4.13), we can calculated the unknown constant  $a_{n,m}$ ,  $b_{n,m}$ ,  $c_{n,m}$  and  $d_{n,m}$  appearing in the the expression of velocity potential as well as in hydrodynamic properties.



### 4.3 Numerical result

Now we investigate the hydrodynamic effects of the radius of the bottom-mounted barrier i.e. Cyl.2 by taking  $r_2 = 0.2H, 0.3H, 0.4H, 0.5H, 0.6H$  with fixed  $H = 3$  m (in meters). Fig. (2) illustrate the variation of non-dimensional horizontal force  $F_{s1}/u_0$  versus the non-dimensional wave number  $kr_1$  with  $r_1 = 0.2H$  and  $d = 0.8H$ . From the figure, we have seen that the curves of forces oscillating for the lower value of  $r_2$  (for example,  $r_2 \leq 0.5d$ ), but the maximum value of the horizontal force occurs for the maximum value of  $r_2$ . Therefore, we have seen that same behavior as given in Borah and Hassan [2] which gives the validation of our analytical result. Also, it is clear from the curves that the oscillating is occurs only at lower values of frequency as well as wave number and for maximum values of frequency, the forces almost vanish. Fig. (3) demonstrated the variation of non-dimensional vertical force  $F_{v1}/u_0$  versus the non-dimensional wave number  $kr_1$  with  $r_1 = 0.2H$  and  $d = 0.8H$ . From the figure, we observed that there is no distinct variation of trends of the force for all values of  $r_2$ . But due to the effect of channel walls, the incident wave will excite near trapping. Therefore, we have seen that a small spike behavior of the curves near trapping (Challan et al. [5]).

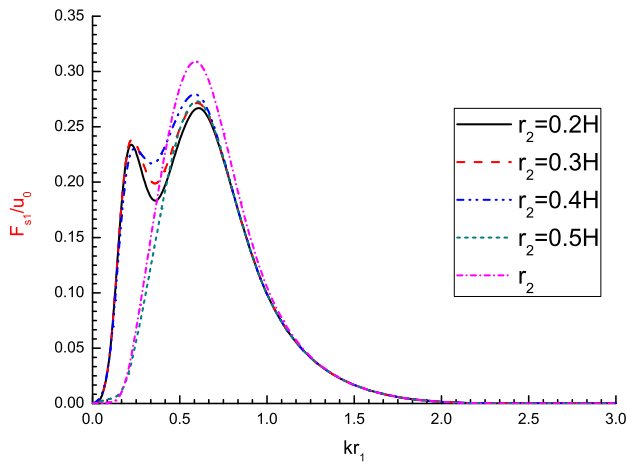


Figure 2. Variation of horizontal force against wave number for various radius of the barrier  $r_2$ .

Fig. (4) show the variation of overturning moment  $M_x/(u_0r_1)$  versus the non-dimensional wave number  $kr_1$  with  $r_1 = 0.2H$  and  $d = 0.8H$ . From the figure, we observed that the moment is less affected by the barrier than the vertical force. Also, in the first three values of  $r_2$  (for example,  $r_2 \leq 0.5d$ ), the curve of the moment have more than one flex point, as well as a small spike, occurs and for the last two situations, the curves show that no obvious oscillation occurs. But from the Figs.(2)-(4), we observed a significant effect on the forces and overturning moment in the presence of a barrier inside the channel.

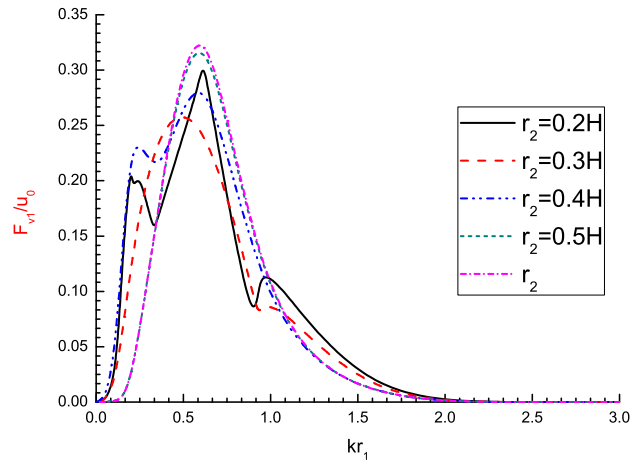


Figure 3. Variation of vertical force against wave number for various radius of the barrier  $r_2$ .

Now if we make a comparison between our present result and result given by Borah and Hassan [2], then we have seen that same observation in both results in the case of horizontal force. The difference between these two outcomes might be credited to the way that our work contains a solid cylinder in place of a hollow cylinder considered by Borah and Hassan [2]. The significant difference in our results that we observed in peak values of horizontal force for the different radius of the barrier.

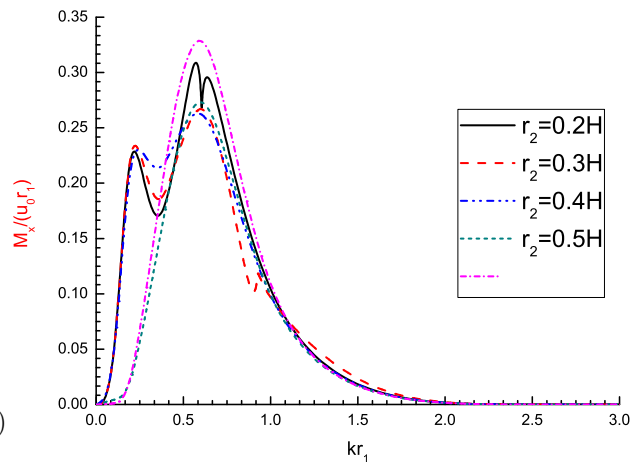


Figure 4. Variation of overturning moment against wave number for various radius of the barrier  $r_2$ .

## 5. Conclusion

In this work, the problem of water wave diffraction by a composite structure of a vertical solid cylinder coaxial over a cylindrical barrier in a channel is developed. First formu-



lated the boundary-value problem under the assumption of inviscid, ideal and homogeneous fluid. Consideration of irrotational motion allows us to present the velocity potential which satisfies Laplace's equation. To solve the boundary-value problem, we apply the method of channel multipoles as well as the separation of variables and we obtained the velocity potential in terms of Bessel's function. Analytical solution of velocity potential allows us to obtain wave forces and overturning moment due to diffraction. Numerical results for the forces and moment for various radii of the barrier are presented graphically. From the graph, we conclude that the horizontal force acting on the floating cylinder is less affected by the barrier inside the walls. Also, we observed in all three cases of the forces and moment, the curves are oscillating within the lower values of frequency and for the higher value of frequency, the curves almost tend to zero.

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