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Almost contra-continuity via topological grills

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Abstract

The purpose of this paper is, to introduce a new class of functions called almost contra-$\mathscr{G}$-precontinuous functions which is a generalization of contra-$\mathscr{G}$-precontinuous functions.

Keywords

Topological spaces, $\mathscr{G}$-preopen sets, $\mathscr{G}$-preclosed sets, almost contra-$\mathscr{G}$-precontinuous functions.

AMS Subject Classification

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Contents

1. Introduction

The idea of grills on a topological space was introduced by Choquet [5] in his classical paper. It has been found out that there is some of similarity between Choquat concept and that ideals, nets and filters. It helps to expand the topological structure which is used to measure the description rather than quantity, such as love, intelligence, beauty, quality of education and etc. Also, it expands the topological structure by using the concept of grill changes in lower approximation, upper approximation and boundary region. In 2007, Roy and Mukherjee [23] established a new form of topological structure via grills. Quite recently, Hatir and Jafari [13] have defined new classes of sets via grills and obtained a new decomposition of continuity in terms of grills. Quite recently, Hatir and Jafari [13] have defined new classes of sets in a grill topological space and obtained a new decomposition of continuity in terms of grills. In this paper, to introduced and studied a new class of functions called almost contra-$\mathscr{G}$-precontinuous functions in topological spaces.

2. Preliminaries

Throughout the paper (X,τ) and (Y,σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space (X,τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ denote the closure of A and the interior of A in X, respectively. A subset A of X is said to be regular open [26] (resp. semiopen [15], preopen [17]) if (resp. $A = \mathrm{Int}(\mathrm{Cl}(A)$, $A \subset \mathrm{Cl}(\mathrm{Int}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(A))$). The family of all regular open subsets of X is denoted by $RO(X)$. The complement of a semiopen (resp. regular open, preopen) set is called a semiclosed [7] (resp. regular closed, preclosed [17]) set. The intersection of all regular open sets containing A is called the r-kernal [10] of A and is denoted by $r\ker(A)$. The definition of grill on a topological space, as given by Choquet [5], goes as follows: A non-null collection $\mathscr{G}$ of subsets of a topological space (X,τ) is said to be a grill on X if

1. $\emptyset \notin \mathscr{G}$,
2. $A \in \mathscr{G}$ and $A \subset B$ implies that $B \in \mathscr{G}$,
3. $A,B \subset X$ and $A \cup B \in \mathscr{G}$ implies that $A \in \mathscr{G}$ or $B \in \mathscr{G}$.

Definition 2.1. *[23] Let (X,τ) be a topological space and $\mathscr{G}$ a grill on X. A mapping $\Phi : \mathscr{P}(X) \to \mathscr{P}(X)$ is defined as follows: $\Phi(A) = \Phi_{\mathscr{G}}(A,\tau) = \{x \in X : A \cap U \in \mathscr{G}$ for every open set U containing $x\}$ for each $A \in \mathscr{P}(X)$. The mapping Φ is called the operator associated with the grill $\mathscr{G}$ and the topology τ.*

Definition 2.2. *[23] Let $\mathscr{G}$ be a grill on a topological space (X,τ). Then we define a map $\Psi : \mathscr{P}(X) \to \mathscr{P}(X)$ by $\Psi(A) = A \cup \Phi(A)$ for all $A \in \mathscr{P}(X)$. The map Ψ is a Kuratowski closure axiom. Corresponding to a grill $\mathscr{G}$ on a topological space (X,τ), there exists a unique topology $\tau_{\mathscr{G}}$ on X given by $\tau_{\mathscr{G}} = \{U \subseteq X : \Psi(X\backslash U) = X\backslash U\}$, where for any $A \subset X$,*

$\Psi(A) = A \cup \Phi(A) = \tau_{\mathscr{G}}\,\mathrm{Cl}(A)$. *For any grill* $\mathscr{G}$ *on a topological space* (X,τ), $\tau \subset \tau_{\mathscr{G}}$. *If* (X,τ) *is a topological space with a grill* $\mathscr{G}$ *on* X*, then we call it a grill topological space and denote it by* $(X,\tau,\mathscr{G})$.

Definition 2.3. *[13] A subset* S *of a grill topological space* $(X,\tau,\mathscr{G})$ *is* $\mathscr{G}$*-preopen if* $S \subset \mathrm{Int}(\Psi(S))$*. The complement of a* $\mathscr{G}$*-preopen set is called a* $\mathscr{G}$*-preclosed set.*

Definition 2.4. *The intersection of all* $\mathscr{G}$*-preclosed sets containing* $S \subset X$ *is called the* $\mathscr{G}$*-preclosure of* S *and is denoted by* $p\mathrm{Cl}_{\mathscr{G}}(S)$*. The family of all* $\mathscr{G}$*-preopen (resp.* $\mathscr{G}$*-preclosed) sets of* $(X,\tau,\mathscr{G})$ *is denoted by* $\mathscr{G}PO(X)$ *(resp.* $\mathscr{G}PC(X)$*). The family of all* $\mathscr{G}$*-preopen (resp.* $\mathscr{G}$*-preclosed) sets of* $(X,\tau,\mathscr{G})$ *containing a point* $x \in X$ *is denoted by* $\mathscr{G}PO(X,x)$ *(resp.* $\mathscr{G}PC(X,x)$*).*

Definition 2.5. *A function* $f:(X,\tau,\mathscr{G}) \to (Y,\sigma)$ *is said to be* $\mathscr{G}$*-precontinuous [13] (resp. contra-*$\mathscr{G}$*-precontinuous [20]) if* $f^{-1}(V)$ *is* $\mathscr{G}$*-preopen (resp.* $\mathscr{G}$*-preclosed) set in* X *for each open set* V *of* Y*. A function* $f:(X,\tau,\mathscr{G}) \to (Y,\sigma)$ *is said to be weakly* $\mathscr{G}$*-precontinuous [21] if for every* $x \in X$ *and every open set* V *of* Y *containing* $f(x)$*, there exists* $U \in BO(Y,f(x))$ *such that* $f(U) \subset \mathrm{Cl}(V)$.

3. Almost contra-$\mathscr{G}$-precontinuous functions

Definition 3.1. *A function* $f:(X,\tau,\mathscr{G}) \to (Y,\sigma)$ *is said to be almost contra-*$\mathscr{G}$*-precontinuous if* $f^{-1}(V) \in \mathscr{G}PC(X)$ *for each* $V \in RO(Y)$.

It is clear that, every contra-$\mathscr{G}$-precontinuous function is almost contra-$\mathscr{G}$-precontinuous but the converse is not true in general.

Example 3.2. *Let* $X = \{a,b,c\}$, $\tau = \{\emptyset, \{a\}, X\}$ *and* $\mathscr{G} = \mathscr{P}(X)\backslash\{\emptyset, \{a\}\}$*. Then the identity function* $f:(X,\tau,\mathscr{G}) \to (X,\tau)$ *is almost contra-*$\mathscr{G}$*-precontinuous but not contra-*$\mathscr{G}$*-precontinuous.*

Theorem 3.3. *For a function* $f:(X,\tau,\mathscr{G}) \to (Y,\sigma)$*, the following statements are equivalent:*

(i) f *is almost contra-*$\mathscr{G}$*-precontinuous;*

(ii) $f^{-1}(F) \in \mathscr{G}PO(X)$ *for every* $F \in RC(Y)$*;*

(iii) for each $x \in X$ *and each* $F \in RC(Y,f(x))$*, there exists* $U \in \mathscr{G}PO(X,x)$ *such that* $f(U) \subset F$*;*

(iv) for each $x \in X$ *and each* $U \in RO(Y,f(x))$*, there exist* $V \in \mathscr{G}PC(X,x)$ *such that* $f(V) \subset U$*;*

(v) $f^{-1}(\mathrm{Int}(\mathrm{Cl}(G))) \in \mathscr{G}PC(X)$ *for every open subset* G *of* Y*;*

(vi) $f^{-1}(\mathrm{Cl}(\mathrm{Int}(F))) \in \mathscr{G}PO(X)$ *for every closed subset* F *of* Y*;*

(vii) $f(p\mathrm{Cl}_{\mathscr{G}}(A)) \subset r\mathrm{Ker}(f(A))$ *for every subset* A *of* X*;*

(viii) $p\mathrm{Cl}_{\mathscr{G}}(f^{-1}(A)) \subset f^{-1}(r\mathrm{Ker}(A))$ *for every subset* B *of* Y.

Proof. (i)⇔(ii): Let $F \in RC(Y)$. Then $Y\backslash F \in RO(Y)$. By (i), $f^{-1}(Y\backslash F) = X\backslash f^{-1}(F) \in \mathscr{G}PC(X)$. We have $f^{-1}(F) \in \mathscr{G}PO(X)$. The proof of the reverse in similar.
(ii)⇒(iii): Let $F \in RC(Y,f(x))$. By (ii), $f^{-1}(F) \in \mathscr{G}PO(X)$ and $x \in f^{-1}(F)$. Take $U = f^{-1}(F)$, then $f(U) \subset F$.
(ii)⇒(iii): Let $F \in RC(Y)$ and $x \in f^{-1}(F)$. From (iii), there exists a $\mathscr{G}$-preopen set U_x in X containing x such that $U \subset f^{-1}(F)$. We have $f^{-1}(F) = \bigcup_{x\in f^{-1}(F)} U_x$. Since any union of $\mathscr{G}$-preopen sets is $\mathscr{G}$-preopen, $f^{-1}(F)$ is $\mathscr{G}$-preopen in X.
(iii)⇔(iv): Let V be any regular open set of Y non-containing $f(x)$. Then, $Y\backslash V \in RC(Y,f(x))$. By (iii), there exists $U \in \mathscr{G}PO(X,x)$ such that $f(U) \subset Y\backslash V$. Hence, $U \subset f^{-1}(Y\backslash V) \subset X\backslash f^{-1}(V)$ and then $f^{-1}(V) \subset X\backslash U$. Take $H = X\backslash U$. We obtain that H is a $\mathscr{G}$-preclosed set in X non-containing x. The converse can be shown similarly.
(i)⇔(v): Let G be an open subset of Y. Since $\mathrm{Int}(\mathrm{Cl}(G))$ is regular open, then by (i), it follows that, $f^{-1}(\mathrm{Int}(\mathrm{Cl}(G))) \in \mathscr{G}PC(X)$. The converse can be shown similarly.
(i)⇔(iv): It can be obtained similar as (i)⇔(v).
(iii)⇒(vii): Let $A \subset X$ and $x \in p\mathrm{Cl}_{\mathscr{G}}(A)$ and $F \in RC(Y,f(x))$. By (iii), there exists $U \in \mathscr{G}PO(X,x)$ such that $f(U) \subset F$. Since $x \in p\mathrm{Cl}_{\mathscr{G}}(A)$, we have $U \cap A \neq \emptyset$. Hence, $f(U) \cap f(A) \neq \emptyset$ and therefore $F \cap f(A) \neq \emptyset$. It follows that $f(x) \in r\mathrm{Ker}(f(A))$ and hence $f(p\mathrm{Cl}_{\mathscr{G}}(A)) \subset r\mathrm{Ker}(f(A))$.
(vii)⇒(viii): If $B \subset Y$, then we have $f(p\mathrm{Cl}_{\mathscr{G}}(f^{-1}(B))) \subset r\mathrm{Ker}(f(f^{-1}(B)) \subset r\mathrm{Ker}(B)$. It follows that $p\mathrm{Cl}_{\mathscr{G}}(f^{-1}(A)) \subset f^{-1}(r\mathrm{Ker}(A))$.
(viii)⇒(i): Let $V \in RO(Y)$. Then by (viii), $p\mathrm{Cl}_{\mathscr{G}}(f^{-1}(V)) \subset f^{-1}(r\mathrm{Ker}(V))$. Since $V \in RO(Y)$, $r\mathrm{Ker}(V) = V$ and hence $p\mathrm{Cl}_{\mathscr{G}}(f^{-1}(V)) \subset f^{-1}(V)$, which shows that $f^{-1}(V)$ is $\mathscr{G}$-preclosed. Consequently, f is almost contra-$\mathscr{G}$-precontinuous. □

Theorem 3.4. *If* $f:(X,\tau,\mathscr{G}) \to (Y,\sigma)$ *is almost contra-*$\mathscr{G}$*-precontinuous, then it is weakly* $\mathscr{G}$*-precontinuous.*

Proof. Let $x \in X$ and V be an open set of Y containing $f(x)$. Then $\mathrm{Cl}(V) \in RC(Y,f(x))$ and by Theorem 3.3 there exists $U \in \mathscr{G}PO(X,x)$ such that $f(U) \subset \mathrm{Cl}(V)$. Hence, f is weakly $\mathscr{G}$-precontinuous. □

Theorem 3.5. *If a function* $f:(X,\tau,\mathscr{G}) \to (Y,\sigma)$ *is weakly-*$\mathscr{G}$*-precontinuous and* (Y,σ) *is a regular space, then* f *is* $\mathscr{G}$*-precontinuous.*

Proof. Clear. □

Corollary 3.6. *If* $f:(X,\tau,\mathscr{G}) \to (Y,\sigma)$ *is almost contra-*$\mathscr{G}$*-precontinuous and* (Y,σ) *is regular, then* f *is* $\mathscr{G}$*-precontinuous.*

Corollary 3.7. *If a function* $f:(X,\tau,\mathscr{G}) \to (Y,\sigma)$ *is contra-*$\mathscr{G}$*-precontinuous function and* (Y,σ) *is regular, then* f *is* $\mathscr{G}$*-precontinuous.*

Definition 3.8. *Let (X,τ) be a topological space. The collection of all regular open sets forms a base for a topology τ^*. It is called semi-regularization. In case when $\tau=\tau^*$, the topological space (X,τ) is called semi-regular [27].*

Theorem 3.9. *Let $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ be a function, where (Y,σ) is a semi-regular space. Then f is almost contra-$\mathscr{G}$-precontinuous if and only if it is contra-$\mathscr{G}$-precontinuous.*

Proof. Clear. □

Definition 3.10. *A topological space (X,τ) is said to be extremally disconnected [28] if the closure of every open set of X is open in X.*

Theorem 3.11. *Let $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ be a function, where (Y,σ) is an extremally disconnected space. Then f is almost contra-$\mathscr{G}$-precontinuous if and only if it is almost $\mathscr{G}$-precontinuous.*

Proof. Let $x\in X$ and $U\in RO(Y,f(x)$. Since (Y,σ) is extremally disconnected, U is clopen and hence it is regular closed. Then $f^{-1}(U)$ is $\mathscr{G}$-preopen in X and hence f is almost $\mathscr{G}$-precontinuous. Conversely, let $K\in RC(Y)$. Since (Y,σ) is extremally disconnected, K is regular open and $f^{-1}(K)$ is $\mathscr{G}$-preopen in X. Thus, f is almost contra-$\mathscr{G}$-precontinuous. □

Definition 3.12. *A function $f:(X,\tau,\mathscr{G})\to(Y,\sigma,\mathscr{G})$ is said to be:*

(i) $\mathscr{G}$-preopen if $f(V)\in\mathscr{G}PO(Y)$ for each $V\in\mathscr{G}PO(X)$.

(ii) $\mathscr{G}$-preirresolute if for each $x\in X$ and $V\in\mathscr{G}PO(Y,f(x))$, there exists $U\in\mathscr{G}PO(X,x)$ such that $f(U)\subset V$.

(iii) θ-irresolute [11] if for each $x\in X$ and $V\in SO(Y,f(x))$, there exists $U\in SO(X,x)$ such that $f(\mathrm{Cl}(U))\subset\mathrm{Cl}(V)$.

Theorem 3.13. *If $f:(X,\tau,\mathscr{G})\to(Y,\sigma,\mathscr{G})$ is a surjective $\mathscr{G}$-preopen and $g:(Y,\sigma,\mathscr{G})\to(Z,\gamma)$ is a function such that $g\circ f$: $(X,\tau,\mathscr{G})\to(Z,\gamma)$ is almost contra-$\mathscr{G}$-precontinuous, then g is almost contra-$\mathscr{G}$-precontinuous.*

Proof. Let V be any regular closed set in Z. Since $g\circ f$ is almost contra-$\mathscr{G}$-precontinuous, $(g\circ f)^{-1}(V)$ is $\mathscr{G}$-preopen. Since f is surjective pre-$\mathscr{G}$-preopen, $f(f^{-1}(g^{-1}(V)))$ is $\mathscr{G}$-preopen. Therefore, g is almost contra-$\mathscr{G}$-precontinuous. □

Theorem 3.14. *If $f:(X,\tau,\mathscr{G})\to(Y,\sigma,\mathscr{G})$ is $\mathscr{G}$-preirresolute and $g:(Y,\sigma,\mathscr{G})\to(Z,\gamma)$ is almost contra-$\mathscr{G}$-precontinuous, then the function $g\circ f$: $(X,\tau,\mathscr{G})\to(Z,\gamma)$ is almost contra-$\mathscr{G}$-precontinuous.*

Proof. Let $x\in X$ and W be a semiopen set in Z containing $(g\circ f)(x)$. Since g is almost contra-$\mathscr{G}$-precontinuous, there exists $V\in\mathscr{G}PO(Y,f(x))$ such that $g(V)\subset\mathrm{Cl}(W)$. Since f is $\mathscr{G}$-preirresolute, $U\in\mathscr{G}PO(X,x)$ such that $f(U)\subset V$. This shows that $(g\circ f)(U)\subset\mathrm{Cl}(W)$. Therefore, $g\circ f$ is almost contra-$\mathscr{G}$-precontinuous. □

Theorem 3.15. *If $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is almost contra-$\mathscr{G}$-precontinuous and $g:(Y,\sigma)\to(Z,\gamma)$ is θ-irresolute, then $g\circ f$: $(X,\tau,\mathscr{G})\to(Z,\gamma)$ is almost contra-$\mathscr{G}$-precontinuous.*

Proof. Similar to the proof of Theorem 3.14. □

Definition 3.16. *A filter base Λ is said to be $\mathscr{G}$-preconvergent (rsp. rc-convergent [9]) to a point $x\in X$ if for any $U\in\mathscr{G}PO(X,x)$ (resp. $U\in RC(X,x)$), there exists a $B\in\Lambda$ such that $B\subset U$.*

Theorem 3.17. *If $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is a almost contra-$\mathscr{G}$-precontinuous function, then for each point $x\in X$ and each filter base Λ in X $\mathscr{G}$-preconverging to x, the filter base $f(\Lambda)$ is rc-convergent to $f(x)$.*

Proof. Let $x\in X$ and Λ be any filter base in X $\mathscr{G}$-preconverging to x. Since f is almost contra-$\mathscr{G}$-precontinuous, then for any $V\in RC(Y,f(x))$, there exists $U\in\mathscr{G}PO(X,x)$ such that $f(U)\subset V$. Since Λ is $\mathscr{G}$-preconverging to x, there exists a $B\in\Lambda$ such that $B\subset U$. This means that $f(B)\subset V$ and therefore the filter base $f(\Lambda)$ is rc-convergent to $f(x)$. □

Definition 3.18. *A topological space (X,τ) is said to be:*

(i) P_{Σ} [30] if for any open set V of X and each $x\in V$, there exists $F\in RC(X,x)$ such that $x\in F\subset V$.

(ii) weakly P_{Σ} [19] if for any $V\in RO(X,x)$, there exists $F\in RC(X,x)$ such that $x\in F\subset V$.

Theorem 3.19. *If $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is an almost contra-$\mathscr{G}$-precontinuous and (Y,σ) is P_{Σ}, then f is $\mathscr{G}$-precontinuous.*

Proof. Let V be any open set in Y. Since Y is P_{Σ}, there exists a subfamily $\mathscr{A}$ of $RC(Y)$ such that $V=\bigcup\{F:F\in\mathscr{A}\}$. Since f is almost contra-$\mathscr{G}$-precontinuous, $f^{-1}(F)$ is $\mathscr{G}$-preopen in X for each $F\in\mathscr{A}$ and $f^{-1}(V)$ is $\mathscr{G}$-preopen in X. Therefore, f is $\mathscr{G}$-precontinuous. □

Theorem 3.20. *If a function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is an almost contra-$\mathscr{G}$-precontinuous and (Y,σ) is weakly P_{Σ}, then f is $\mathscr{G}$-precontinuous.*

Proof. Similar to the proof of Theorem 3.19. □

Definition 3.21. *A topological space (X,τ) is said to be weakly Hausdorff [25] if each element of X is an intersection of regular closed sets.*

Definition 3.22. *A grill topological space $(X,\tau,\mathscr{G})$ is said to be:*

(i) $\mathscr{G}$-pre-T_0 [22] if for each pair of distinct points in X, there exists a $\mathscr{G}$-preopen set of X containing one point but not the other.

(ii) $\mathscr{G}$-pre-T_1 [22] if for each pair of distinct points x and y of X, there exist $\mathscr{G}$-preopen sets U and V containing x and y, respectively such that $y\notin U$ and $x\notin V$.

Theorem 3.23. *If $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is an almost contra-$\mathscr{G}$-precontinuous injective and (Y,σ) is weakly Hausdorff, then $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-pre-T_1.*

Proof. Suppose that Y is weakly Hausdorff. For any two distinct points x and y in X, there exist $V,W\in RC(Y)$ such that $f(x)\in V$, $f(y)\notin V$, $f(x)\notin W$ and $f(y)\in W$. Since f is almost contra-$\mathscr{G}$-precontinuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $\mathscr{G}$-preopen subsets of X such that $x\in f^{-1}(V)$, $y\notin f^{-1}(V)$, $x\notin f^{-1}(W)$ and $y\in f^{-1}(W)$. This shows that $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-pre-T_1. □

Definition 3.24. *A topological space (X,τ) is said to be hyperconnected [28] if every open set is dense.*

Definition 3.25. *A grill topological space $(X,\tau,\mathscr{G})$ is said to be:*

(i) ultra $\mathscr{G}$-preconnected if every two non-void $\mathscr{G}$-preclosed subsets of X intersect.

(ii) $\mathscr{G}$-preconnected provided that X is not the union of two disjoint nonempty $\mathscr{G}$-preopen sets.

Theorem 3.26. *If $(X,\tau,\mathscr{G})$ is an ultra $\mathscr{G}$-preconnected space and $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is a surjective almost contra-$\mathscr{G}$-precontinuous, then (Y,σ) is hyperconnected.*

Proof. Assume that Y is not hyperconnected. Then there exists an open set V such that V is not dense in Y. Then there exist disjoint nonempty regular open subsets B_1 and B_2 in Y, namely $\mathrm{Int}(\mathrm{Cl}(V))$ and $Y\setminus\mathrm{Cl}(V)$. Since f is almost contra-$\mathscr{G}$-precontinuous surjection, $A_1=f^{-1}(B_1)$ and $A_2=f^{-1}(B_2)$ are disjoint nonempty $\mathscr{G}$-preclosed subsets of X. By assumption, the ultra-$\mathscr{G}$-preconnectedness of X implies that A_1 and A_2 must intersect. By contradiction, Y is hyperconnected. □

Theorem 3.27. *[21] If $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is weakly $\mathscr{G}$-precontinuous surjection and $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-preconnected, then (Y,σ) is connected.*

Corollary 3.28. *If $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is a almost contra-$\mathscr{G}$-precontinuous surjection and $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-preconnected, then (Y,σ) is connected.*

Corollary 3.29. *[20] If $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is a contra-$\mathscr{G}$-precontinuous surjection and $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-preconnected, then (Y,σ) is connected.*

Theorem 3.30. *[21] If $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is weakly $\mathscr{G}$-precontinuous injection and (Y,σ) is Urysohn, then $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-pre-T_2.*

Corollary 3.31. *If $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is a almost contra-$\mathscr{G}$-precontinuous injection and (Y,σ) is an Urysohn space, then $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-pre-T_2.*

Corollary 3.32. *[20] If $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is a contra-$\mathscr{G}$-precontinuous injection and Y is Urysohn, then $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-pre-T_2.*

Definition 3.33. *A topological space (X,τ) is said to be θ-irreducible [14] if every pair of nonempty regular closed sets of X has a nonempty intersection.*

Theorem 3.34. *If $(X,\tau,\mathscr{G})$ is a $\mathscr{G}$-preconnected space and the function $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is surjective almost contra-$\mathscr{G}$-precontinuous, then (Y,σ) is θ-irreducible.*

Proof. Similar to that proof of Theorem 3.26 □

Definition 3.35. *A topological space (X,τ) is said to be $\mathscr{G}$-prenormal provided that every pair of nonempty disjoint closed sets can be separated by disjoint $\mathscr{G}$-preopen sets.*

Theorem 3.36. *If (Y,σ) is a normal space and $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is almost contra-$\mathscr{G}$-precontinuous closed injection, then $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-prenormal.*

Proof. Let F_1 and F_2 be disjoint nonempty closed sets of X. Since f is injective and closed, $f(F_1)$ and $f(F_1)$ are disjoint closed sets of Y. Since Y is normal, there exist open sets V_1 and V_2 of Y such that $f(F_1)\subset V_1$, $f(F_2)\subset V_2$ and $\mathrm{Cl}(V_1)\cap\mathrm{Cl}(V_2)=\emptyset$. Then, since $\mathrm{Cl}(V_1),\mathrm{Cl}(V_2)\in RC(Y)$ and f is almost contra-$\mathscr{G}$-precontinuous, $f^{-1}(\mathrm{Cl}(V_1)),f^{-1}(\mathrm{Cl}(V_2))\in BO(X)$. Since $F_1\subset f^{-1}(V_1)$, $F_2\subset f^{-1}(V_2)$ and $f^{-1}(\mathrm{Cl}(V_1))$ and $f^{-1}(\mathrm{Cl}(V_2))$ are disjoint, X is $\mathscr{G}$-prenormal. □

Corollary 3.37. *[20] If (Y,σ) is a normal space and $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ is contra-$\mathscr{G}$-precontinuous closed injection, then $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-prenormal.*

Definition 3.38. *A topological space (X,τ) is said to be:*

(i) S-closed [29] if every regular closed cover of X has a finite subcover;

(ii) countably S-closed [8] if every countable cover of X by regular closed sets has a finite subcover;

(iii) S-Lindelof [16] if every regular closed cover of X has a countable subcover.

Definition 3.39. *A grill topological space $(X,\tau,\mathscr{G})$ is said to be:*

(i) $\mathscr{G}$-precompact if every $\mathscr{G}$-preopen cover of X has a finite subcover;

(ii) countably $\mathscr{G}$-precompact if every countable cover of X by $\mathscr{G}$-preopen sets has a finite subcover;

(iii) $\mathscr{G}$-preLindelof if every $\mathscr{G}$-preopen cover of X has a countable subcover.

Theorem 3.40. *Let $f:(X,\tau,\mathscr{G})\to(Y,\sigma)$ be an almost contra-$\mathscr{G}$-precontinuous surjection. Then the following statements hold:*

(i) If $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-precompact, then Y is S-closed;

(ii) If $(X,\tau,\mathscr{G})$ is $\mathscr{G}$-preLindelof, then Y is S-Lindelof;

(iii) If $(X,\tau,\mathcal{G})$ is countably $\mathcal{G}$-precompact, then Y is countably S-closed.

Proof. We prove only (i), the proofs of (ii) and (iii) being entirely analogous. Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra-$\mathcal{G}$-precontinuous, there $\{f^{-1}(V_\alpha) \colon \alpha \in I\}$ is a $\mathcal{G}$-preopen cover of X and hence there exists a a finite subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$ and Y is S-closed. □

Definition 3.41. *A topological space (X,τ) is said to be:*

(i) nearly compact [24] regular open cover of X has a finite subcover;

(ii) nearly countably compact [12] if every countable cover of X by regular open sets has a finite subcover;

(iii) nearly Lindelof [12] if every regular open cover of X has a countable subcover.

Definition 3.42. *A grill topological space $(X,\tau,\mathcal{G})$ is said to be:*

(i) $\mathcal{G}$-preclosed compact if every $\mathcal{G}$-preclosed cover of X has a finite subcover;

(ii) countably $\mathcal{G}$-preclosed compact if every countable cover of X by $\mathcal{G}$-preclosed sets has a finite subcover;

(iii) $\mathcal{G}$-preclosed Lindelof if every $\mathcal{G}$-preclosed cover of X has a countable subcover.

Theorem 3.43. *Let $f:(X,\tau,\mathcal{G}) \to (Y,\sigma)$ be an almost contra-$\mathcal{G}$-precontinuous surjection. Then the following statements hold:*

(i) If $(X,\tau,\mathcal{G})$ is $\mathcal{G}$-preclosed compact, then (Y,σ) is nearly compact;

(ii) If $(X,\tau,\mathcal{G})$ is $\mathcal{G}$-preclosed Lindelof, then (Y,σ) is nearly Lindelof;

(iii) If $(X,\tau,\mathcal{G})$ is countably $\mathcal{G}$-preclosed compact, then (Y,σ) is nearly countably closed.

Proof. Similar proof to the Theorem 3.40 □

Definition 3.44. *A topological space (X,τ) is said to be s-Urysohn [2] if for each pair of distinct oints x and y in X, there exist $U \in SO(X,x)$ and $V \in SO(X,y)$ such that* $\mathrm{Cl}(U) \cap \mathrm{Cl}(V) = \emptyset$.

Theorem 3.45. *If (Y,σ) is s-Urysohn and $f:(X,\tau,\mathcal{G}) \to (Y,\sigma)$ is almost contra-$\mathcal{G}$-precontinuous, then $(X,\tau,\mathcal{G})$ is $\mathcal{G}$-pre-T_2.*

Proof. It is similar to the Proof of Theorem 3.23. □

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