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Exact 2-distance b-coloring of some classes of graphs

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Abstract

Given a graph G, the exact distance-p (or p-distance) graph $G^{[ep]}$ has $V(G)$ as its vertex set and two vertices are adjacent whenever the distance between them in G equals p. An exact 2-distance coloring of a graph G is a proper coloring of vertices of G such that any two vertices which are at distance exactly 2 receive distinct colors. An exact 2-distance chromatic number of G is the minimum k for which G admits an exact 2-distance coloring with k colors. A b-coloring of a graph G by k colors is a proper k-vertex coloring such that in each color class, there exists a vertex adjacent to at least one vertex in every other color class. In this paper we introduce a new coloring called exact 2-distance b-coloring. It is a b-coloring of G such that any two vertices at distance exactly 2 receive distinct colors and a graph G is called exact 2-distance b-colorable graph if it admits such a coloring. An exact 2-distance b-chromatic number $\chi_{e2b}(G)$ of G is the largest integer k such that G has an exact 2-distance b-coloring with k-colors. If each color class contains a vertex that has a 2-neighbour in all other color classes, such a vertex is called an exact 2-distance color dominating vertex. Some results based on exact 2-distance b-coloring are obtained. Exact 2-distance b-chromatic number of some classes of graphs are obtained.

Keywords

Exact 2-distance coloring ($e2$-coloring), exact 2-distance chromatic number ($e2$-number), b-coloring, b-chromatic number, exact 2-distance b-coloring($e2$-b-coloring), exact 2-distance b-chromatic number ($e2$-b-number), exact 2-distance b-colorable graph($e2$-b-colorable graph), exact 2-distance color dominating vertex($e2$-b-cdv).

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Contents

1. Introduction

All graphs considered in this paper are finite, simple and undirected. For those terminologies not defined in this paper, the reader may refer to [[1]]. A proper k-coloring of a graph G is an assignment of k-colors to the vertices of G such that no two adjacent vertices are assigned the same color. The chromatic number $\chi(G)$ is the minimum k for which G admits a proper k-coloring. Based on this proper coloring of vertices, various types of coloring were defined. The distance coloring was introduced by F. Kramer and H. Kramer [[4]],[[5]] in 1969. As the name suggests it is based on distance between two vertices. A 2-distance coloring of a graph G is an assignment of colors to the vertices of G such that every two vertices at distance at most 2 receive distinct colors. The 2-distance chromatic number of G, denoted $\chi_2(G)$ is the smallest integer k for which G admits a 2-distance k-coloring. One should be careful while defining exact distance coloring. Since k-distance coloring is a coloring of G in which two vertices u, v receive distinct colors if $d(u,v) \leq k$, while in exact k-distance coloring u, v receive distinct colors if $d(u,v) = k$. Hence if $ux_1x_2\cdots x_{r-1}v$ is a $u-v$ path, then in a k-distance coloring, $r \leq k, r-1+2 \leq k$(i.e., $r+1 \leq k$), each vertex receive distinct colors while in exact k-distance coloring, all the x_i's may receive same color. If u and v are vertices such that $d(u,v) = 2$, then u is said to be a 2-neighbour of v and vice versa. The set of all 2-neighbours of u is denoted by $N_2(u)$ and is called open 2-neighbournood of u and $N_2[u] = N_2(u) \cup \{u\}$ is called the closed 2-neighbourhood of u. In this paper, we consider only exact 2-distance coloring. Exact k-distance coloring of G can also be analyzed from the exact k-distance graph.

The concept of the exact p-distance (or distance-p) graph, where p is a positive integer, was introduced by Simi'c [[7]] in the 1980s and was recently rediscovered by Ne'set'ril and Ossona De Mendez[[6]]. If G is a graph, then the exact p-distance graph $G^{[ep]}$ of G is the graph with $V(G^{[ep]}) = V(G)$ and two vertices in $G^{[ep]}$ are adjacent if and only if they are at distance exactly p in G. In particular, the exact 2-distance graph $G^{[e2]}$ of G is the graph with $V(G^{[e2]}) = V(G)$ and two vertices in $G^{[e2]}$ are adjacent if and only if they are at distance exactly 2 in G. Note that $G^{[e1]} = G$. Clearly, $\chi(G^{[e2]}) = \chi_{e2}(G)$. The other coloring of interest is b-coloring. The concept of b-coloring was introduced by Irving and Manlove[[2]] in 1991. A b-coloring of G by k-colors is a proper k-coloring such that in each color class, there exists a vertex adjacent to at least one vertex in every other color class. Such a vertex is called a color dominating vertex. The b-chromatic number $\chi_b(G)$ of G is the largest integer k such that G has a b-coloring by k-colors. The m-degree $m(G)$ of a graph was defined as $m(G) = \max\{i : 1 \leq i \leq |V(G)|, G$ has at least i vertices of degree at least $i-1\}$. In this paper, we have defined a new coloring based on two types of coloring viz. (i) exact p-distance coloring and (ii) b-coloring. In this paper, an attempt is made to combine the concept of exact 2-distance coloring and b-coloring. Difficulty arose as b-coloring tries for maximum coloring and exact 2-distance for minimum coloring. Hence to support the definition exact 2-distance b-coloring or e2b-coloring, the terms color dominating vertex, b-spectrum and b-continuity which are the fundamental terminologies of b-coloring are redefined based on distance. Consequently chromatic parameter exact 2-distance b-chromatic number $\chi_{e2b}(G)$ of G is introduced. Results are obtained for some well known classes of graphs.

2. Definitions and some prior results related to exact 2-distance coloring

In this section, graphs are constructed from given graph G based on exact distance between two vertices of G. Here, we discuss exact 2-distance coloring and exact 2-distance graph. We give the general definition first. In this section, the particular case i.e, $p = 2$ of exact p-distance coloring and the corresponding graphs are studied. Hence we give the definition for the particular case. Also 2-distance chromatic number of some well known graphs are given. Further exact 2-distance chromatic number of some graph families which are not studied ealier are discussed.

Definition 2.1. *An exact p-distance coloring (or an ep-coloring) of a graph G is defined as a coloring of vertices of G which are at distance exactly p receive distinct colors.*

An exact p-distance chromatic number (or an ep- number) $\chi_{ep}(G)$ of G is the minimum k for which G admits an ep-coloring with k-colors.

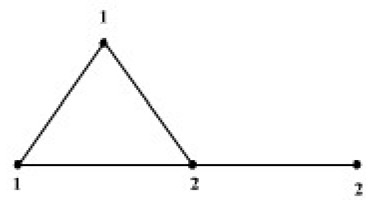
Definition 2.2. *An exact 2-distance coloring (or an e2-coloring) of a graph G is defined as a coloring of vertices of G which are at distance exactly 2 receive distint colors.*

An exact 2-distance chromatic number (or an e2- number) $\chi_{e2}(G)$ of G is the minimum k for which G admits an e2-coloring with k-colors.
i.e., $\chi_{e2}(G) = \min\{k : G$ has an e2 coloring with k colors $\}$.

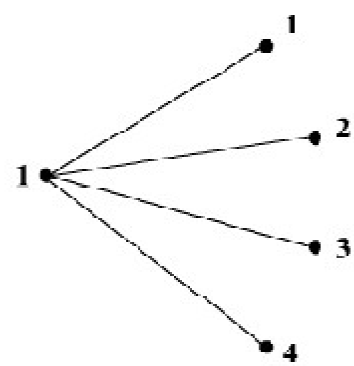
Definition 2.3. *The color classes of an e2-coloring are called e2-color classes.*

Definition 2.4. *For a vertex $u, d_2(u)$ is called the d_2-degree of u.*

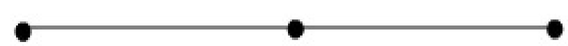
Example 2.5.



(a) G with $\chi(G) = 3$ and $\chi_{e2}(G) = 2$.



(b) G with $\chi(G) = 2$ and $\chi_{e2}(G) = 4$.



(c) G with $\chi(G) = \chi_{e2}(G) = 2$.

Figure 1

Notation 2.6. *Let u be a vertex of a graph G. Then*

i) $d_2(u) = |N_2(u)|$

ii) $\Delta_2(G) = \max\{d_2(u) : \forall u \in V(G)\}$, *where $\Delta_2(G)$ the maximum degree with respect to 2-neighbours.*

iii) $\delta_2(G) = \min\{d_2(u) : \forall u \in V(G)\}$, *where $\delta_2(G)$ the minimum degree with respect to 2-neighbours.*

observation 2.7. *Let $G = (V, E)$ be a connected graph of order $n \geq 3$. Then the followings hold.*

(i) If $u \in V$, then $d_2(u) \leq n - 1 - d(u)$.

(ii) $d_2(u) \leq n - 2$.

Proof. (i) $d_2(u) = |N_2(u)| \leq |V(G)| - |N[u]| = n - (d(u) + 1)$

$$d_2(u) \leq n - 1 - d(u).$$

(ii) Since $d(u) \geq 1, d_2(u) \leq n - 2$. □

observation 2.8 ([8]). *Let G be a bipartite graph, then $G^{[e2]}$ is not connected.*

observation 2.9 ([6], [7]). *For any graph G, $\chi(G^{[e2]}) = \chi_{e2}(G)$*

observation 2.10. *i) An e2-coloring is not a proper vertex coloring. i.e., adjacent vertices may receive the same color.*

ii) A given class of graphs need not to have unique χ_{e2}.

iii) If H is any induced sub graph of G, then $\chi_{e2}(H) \leq \chi_{e2}(G)$.

iv) e2-color classes need not be independent.

v) For any incomplete graph of order at least 3, $\chi_{e2}(G) \geq 2$.

Proof. i) By the definition of $e2$-coloring, adjacent vertices may receive the same color.

ii) Refer example 2.5, figure 1(b) and 1(c). Though $K_{1,4}$ and P_3 are bipartite graphs, $\chi_{e2}(K_{1,4}) = 4$ and $\chi_{e2}(P_3) = 2$.

iii) Trivial.

iv) Follows from (i).

v) Since P_3 is an induced sub graph of G, from (iii), $\chi_{e2}(G) \geq 2$. □

The following proposition gives exact bounds of χ_{ek} for some standard graphs.

Proposition 2.11 ([3]). *i) For $n \geq 3, \chi_{ek}(P_n) = 2$, for $2 \leq k \leq n-1$.*

ii) For $n \geq 2, \chi_{e2}(K_{1,n}) = n$.

iii) For $n \geq 5, \chi_{e2}(W_n) = \lfloor \frac{n}{2} \rfloor$.

iv) For $m \geq 1$ and $n \geq 1, \chi_{e2}(K_{m,n}) = \max\{m,n\}$.

v) For $m \geq 1$ and $n \geq 1$, $\chi_{ek}(B_{m,n}) = \begin{cases} \max\{m,n\}+1, if\ k=2 \\ 2, if\ k = 1\ or 3 \end{cases}$

Lemma 2.12. *If G is a connected incomplete graph of order $n \geq 3$, then $2 \leq \chi_{e2}(G) \leq n-1$ and the bounds are sharp.*

Proof. Lower inequality follows from observation 2.10 (v). Let u be an arbitrary vertex of G. Since G is connected, u is adjacent to at least one vertex say v. Assign color 1 to u and v, and distinct $n-2$ colors to the remaining $n-2$ vertices. This give an $e2$-coloring of G. Hence the upper inequality follows. From observation 2.11, $\chi_{e2}(P_4) = 2$ and $\chi_{e2}(K_{1,n-1}) = n-1$. Hence, the bounds are sharp. □

Lemma 2.13. *If a graph G has a vertex u such that $N(u)$ is independent, then $\chi_{e2}(G) \geq d(u) \geq \delta$.*

Proof. Let $d(u) = r$. Then $N[u]$ induces $K_{1,r}$. From observation 2.10 (iii), $\chi_{e2}(G) \geq \chi_{e2}(K_{1,r}) = r = d(u) \geq \delta$. □

Proposition 2.14. *For $n \geq 4$,*

$$\chi_{e2}(C_n) = \begin{cases} 2, if\ n \equiv 0(mod 4) \\ 3, otherwise \end{cases}$$

Proof. Let C_n be $u_1, u_2, \cdots, u_n$. Based on n, there are four cases to consider.
Let $c : V(C_n) \to \{1,2\}$ be a coloring defined as follows.

$$c(u_i) = \begin{cases} 1, if\ i \equiv 1(mod\ 4) \\ \quad\ \ i \equiv 2(mod\ 4) \\ 2, if\ i \equiv 0(mod\ 4) \\ \quad\ \ i \equiv 3(mod\ 4) \end{cases}$$

From the above it is clear that any u_i and u_{i+1} receive distinct colors. Hence it is an $e2$-coloring of P_n. Since $C_n, d(u_1, u_{n-1}) = d(u_2, u_n) = 2$, it is enough to verify the colors for the pair of vertices u_1 and u_{n-1}, and u_2 and u_n. Clearly, $c(u_1) = c(u_2) = 1$.
Case (i): $n \equiv 0(mod\ 4)$
Since $n \equiv 0(mod\ 4), n-1 \equiv 3(mod\ 4)$. Therefore, $c(u_n) = c(u_{n-1}) = 2$. As $c(u_1) = c(u_2) = 1, c$ is an $e2$-coloring of C_n.
Case (ii): $n \equiv 1(mod\ 4)$
As in case (i), color C_n such that all u_i's are colored except the vertex u_n. Since $n \equiv 1(mod\ 4), n-1 \equiv 0(mod\ 4)$ and $n-2 \equiv 3(mod\ 4)$. Hence, $c(u_{n-1}) = c(u_{n-2}) = 2$. Thus u_1 and u_{n-1} receive distinct colors. Since $c(u_2) = 1$ and $c(u_{n-2}) = 2, u_n$ cannot be assigned either the color 1 or the color 2. Assign color 3 to u_n. This gives a minimum $e2$-coloring of C_n.
Case (iii): $n \equiv 2(mod\ 4)$
Follow the same color scheme as in case (i) up to the vertex u_{n-2}. Since $n \equiv 2(mod\ 4), n-3 \equiv 3(mod\ 4)$ and $n-2 \equiv 0(mod\ 4)$. Hence, $c(u_{n-3}) = c(u_{n-2}) = 2$. Since $c(u_1) = 1$ and $d(u_{n-1}, u_1) = d(u_{n-1}, u_{n-3}) = 2, u_{n-1}$ can't be given either color 1 or 2. Hence $\chi_{e2}(C_n) > 2$. Now let $c(u_{n-1}) = c(u_n) = 3$. Then c is an $e2$-coloring.
Case (iv): $n \equiv 3(mod\ 4)$
Color up to the vertex u_{n-2}. Clearly $n-3 \equiv 0(mod\ 4)$ and hence, $c(u_{n-3}) = 2$. Now $d(u_{n-1}, u_1) = d(u_{n-1}, u_{n-3}) = 2$. Since $c(u_1) = 1, u_{n-1}$ can't be given the colors 1 as well as 2. Hence $\chi_{e2}(C_n) > 2$. Now let $c(u_{n-1}) = c(u_n) = 3$. Then c is an $e2$-coloring. □

3. Exact 2-distance b-coloring

In this section some definitions and results of an exact 2-distance b-coloring are discussed. Also exact 2-distance b-chromatic number of some graph families are obtained. Further exact 2-distance b-discontinuity properties of some graph families are discussed.

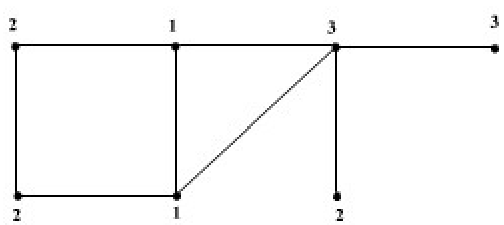
Definition 3.1. *An exact 2-distance b-coloring (or an e2-b coloring)) of a graph G is an e2-coloring of G such that each color class contains a vertex that has a 2-neighboour in all other color classes.*

An exact 2-distance b-chromatic number (or an e2-b-number) $\chi_{e2b}(G)$ of G is the largest integer k such that G has e2-b-coloring with k-colors.

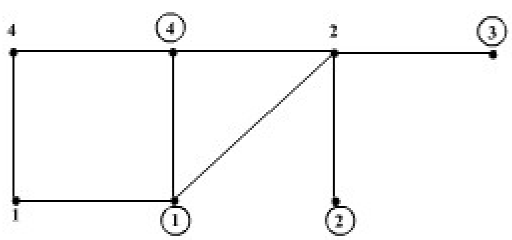
A vertex which has a 2-neighbour in all other classes is called an exact 2-distance color dominating vertex (or an e2-cdv). If u is an e2-cdv of color i, then it is called an e2-i-cdv. The color classes of an e2-b-coloring is called e2-color classes.

Definition 3.2. *The term m_2 degree of $m_2(G)$ of G is defined as the largest integer m such that G has at least m vertices having at least $(m-1)$ 2- neighbours.*

Example 3.3.



(a)$\chi_{e2}(G) = 3$



(b) $\chi_{e2b}(G) = 4$

Figure 2

Proposition 3.4. *Any minimum e2-coloring of a graph is an e2-b-coloring.*

Proof. Let G be a graph with $\chi_{e2}(G) = l$. Then there exists an $e2$-coloring say 'c' with l colors. Then its vertex set V can be partitioned into l color classes namely $V_1, V_2, \cdots, V_l$. Suppose c is not an $e2$-b-coloring. Then atleast one color class V_i does not have any $e2$-cdv. Therefore each u does not have any 2-neighbour in atleast some $V_j, j \neq i$. Put u is the respective V_j. Hence V_i becomes $\emptyset$. Therefore there exists an $e2$-coloring with $l-1$ colors, a contradiction. □

Definition 3.5. *If for each integer k satisfying $\chi_{e2}(G) \leq k \leq \chi_{e2b}(G)$, G has an e2b-coloring by k-colors, then G is said to be an exact 2-distance b-continuous (or e2-b-continuous) graph.*

Definition 3.6. *The exact 2-distance b-spectrum (or e2-b-spectrum) $S_{e2b}(G)$of G is defined by the set of all k such that G has an e2-b-coloring by k-colors. In other words, $S_{e2b}(G) = \{k : G$ has an e2-b-coloring with k-colors $\}$.*
Thus, if $S_{e2b}(G)$ contains all the integers from $\chi_{e2}(G)$ to $\chi_{e2b}(G)$, then G is e2-b-continuous.

observation 3.7. *For a graph G,*

i) $\chi_{e2}(G) \leq \chi_{e2b}(G) \leq m_2(G) \leq \Delta_2(G) + 1$.

ii) If $\chi_{e2b}(G) = \chi_{e2}(G)$ or $\chi_{e2b}(G) = \chi_{e2}(G) + 1$, then G is e2-b-continuous.

observation 3.8. *(i) For* $n \geq 3, m_2(P_n) = \begin{cases} 2, if\ 3 \leq n \leq 6 \\ 3, if\ n \geq 7 \end{cases}$

(ii) For $n \geq 4, m_2(C_n) = \begin{cases} 2, if\ n = 4 \\ 3, if\ n \geq 5 \end{cases}$

(iii) For $n \geq 2, m_2(K_{1,n}) = n$.

(iv) For $m, n \geq 2, m_2(K_{m,n}) = \max\{m, n\}$.

(v) For $n \geq 5, m_2(W_n) = n - 3$

(vi) For $m \geq 1$ *and* $n \geq 1, m_2(B_{m,n}) = \max\{m, n\} + 1$

Proposition 3.9. *(i) For* $n \geq 2, \chi_{e2b}(K_{1,n}) = n$.

(ii) For $m \geq 1$ *and* $n \geq 1, \chi_{e2b}(K_{m,n}) = \max\{m, n\}$.

(iii) For $m \geq 1$ *and* $n \geq 1, \chi_{e2b}(B_{m,n}) = \max\{m, n\} + 1$.

Proof. (i) to (iii) follow from observations 3.7,3.8 and Proposition 2.11. □

Proposition 3.10. *For* $n \geq 3$,

$$\chi_{e2b}(P_n) = \begin{cases} 2, if\ 3 \leq n \leq 6 \\ 3, if\ n \geq 7 \end{cases}$$

Proof. From proposition 2.11, $\chi_{e2}(P_n) = 2$, for all $n \geq 3$. Then there are two cases to consider.
Case(i): $3 \leq n \leq 6$
From observations 3.7 (i) and 3.8 (i), $\chi_{e2b}(P_n) = 2$.
Case (ii): $n \geq 7$
From observations 3.7(i) and 3.8 (i),

$$2 \leq \chi_{e2b}(P_n) \leq 3 \tag{3.1}$$

There are at least three vertices namely, v_3, v_4, v_5 having two 2-neighbours. Therefore these three vertices must receive three distinct colors namely 1, 2 and 3. Let $c(v_3) = 1; c(v_4) = 2, c(v_5) = 3$. Since $d(v_3, v_1) = d(v_3, v_5) = 2$, assign color 2 to the vertices v_1 and v_7.

$\therefore v_3$ is e2-1- cdv and v_5 is e2-3-cdv. Since $d(v_3,v_2) = d(v_5,v_6) = 1$ assign 1 to v_2 and 3 to v_6. Hence v_4 is e2-2-cdv. For the remaining vertices $v_i, i \geq 8$.

$$c(v_i) = \begin{cases} 2, if\ i \equiv 0(mod4) \\ \qquad i \equiv 3(mod4) \\ 1, if\ i \equiv 1(mod4) \\ \qquad i \equiv 2(mod4) \end{cases}$$

Clearly, this coloring is an $e2b$-coloring by 3-colors. Therefore from (1), $\chi_{e2b}(P_n) = 3$. ☐

Corollary 3.11. *P_n is e2b-continuous.*

Proof. From observation 3.7(ii) and the above proposition 3.10, P_n is $e2b$-continuous. ☐

Theorem 3.12. *For $n \geq 4$,*

$$\chi_{e2b}(C_n) = \begin{cases} 2, if\ n = 4, 8 \\ 3, if\ n \geq 5\ and\ n \neq 8 \end{cases}$$

Proof. There are 4 cases.
Case(i): $n = 4$
From observation 3.7(i) and 3.8(iii,)$\chi_{e2b}(C_n) = 2$.
Case (ii): $n = 8$

By the similar argument as in case (i), $2 \leq \chi_{e2b}(C_n) \leq 3, n = 8$. Assign three distinct colors say 1,2 and 3 to the vertices v_{i-2}, v_i and $v_{i+2}, i = 3$ or 4 or 5 in any manner. Let $c(v_1) = 1; c(v_3) = 2; c(v_5) = 3$. Then v_3 is e2-2-cdv. Since $c(v_5) = 3; c(v_1) = 1, d(v_5,v_7) = 2$ and $d(v_1,v_7) = 2$,colors 1 or 3 cannot be assigned to the vertex v_7. Hence assign color 2 to v_7.
$\therefore v_1$ and v_5 cannot be e2-1 cdv and e2-3 cdv. Since $c(v_1) = 1, c(v_3) = 2, c(v_5) = 3$ and $c(v_7) = 2$, assign colors 1 or 3 to any two of the remaining vertices $v_i, i = 2,4,6,8$ which are not yet colored. Clearly, $d(v_i,v_j) = 2$ or 4, $i, j = 2,4,6,8, i \neq j$.
Subcase(a): Suppose $d(v_i,v_j) = 2, i, j = 2,4,6,8$ and $i \neq j$. Assign colors 1 and 3 to any two of these vertices say v_2 and v_4. (ie), Let $c(v_2) = 1; c(v_4) = 3$. Then color 1 cannot be assigned to v_8 and 3 cannot be assigned to v_6. To get e2-1 cdv and e2-3 cdv, color 2 should be given to both of the vertices v_6 and v_8. But $d(v_2,v_8) = 2$. Therefore assign color 2 to one of the vertices, namely v_6. Hence v_4 is e2-3 cdv. Clearly e2-1-cdv cannot be obtained.
Subcase(b): Suppose $d(v_i,v_j) = 4, i, j = 2,4,6,8$ and $i \neq j$ proceed as in subcase (a).
$\chi_{e2b}(C_n) \neq 3, n = 8$.
Case (iii): $n \not\equiv 0(mod3)$
From proposition 2.13 and observations 3.7(i) and 3.8(iii), $\chi_{e2b}(C_n) = 3$.
Case (iv): $n \equiv 0(mod4)$ and $n \geq 12$.

By similar argument as in case (i), $2 \leq \chi_{e2b}(C_n) \leq 3$. Assign colors 1,2,3 to the vertices $v_i, i = 1$ to 12 in cyclic order. Since C_n contains P_7 as an induced subgraph, C_n has e2-1-cdv, e2-2-cdv, e2-3-cdv.

For the remaining vertices, assign color 1,1,3,3 in cyclic order. Since $d(v_{n-1},v_1) = d(v_n,v_2) = 2, 1 = c(v_1) \neq c(v_{n-1}) = 3$ and $3 = c(v_n) \neq c(v_2) = 2$, an e2-b-coloring by 3 colors is obtained.

$$\therefore \chi_{e2b}(C_n) = 3$$

. ☐

Corollary 3.13. *C_n is e2b-continuous.*

Proof. From observation 3.7(ii) and the above proposition 3.10, C_n is $e2b$-continuous. ☐

Proposition 3.14. *For $n \geq 5$*

$$\chi_{e2b}(W_n) = \lfloor \frac{n}{2} \rfloor$$

Proof. From observation 3.7(i) and 3.8, $\lfloor \frac{n}{2} \rfloor \leq \chi_{e2b}(W_n) \leq n-3$.
The color schemes slightly varies according as n is even or odd. Suppose c is an e2-b-coloring.
Case(i): n is even

Let v be the central vertex and $v_1, v_2, \cdots v_{n-1}$ are the vertices of $C_{n-i}, v_1, v_3, v_5, \cdots, v_{n-3}$ are mutually at distance 2 and hence must receive distinct colors. Assign to them respectively the colors $1,2,3,\cdots,\lceil \frac{n-3}{2} \rceil$.
Suppose v_{n-1} is given a new color
$\lceil \frac{n-3}{2} \rceil + 1 = \lceil \frac{n-1}{2} \rceil = \lceil \frac{n}{2} \rceil = \frac{n}{2} = \lfloor \frac{n}{2} \rfloor$. Then this color cannot be given to any of the vertices $v_2, v_4, \cdots, v_{n-4}$. But v_{n-2}. Since v_1 need to have 2-neighbour of color $\lfloor \frac{n}{2} \rfloor$ and v_{n-1} is not a 2-neighbour of $v_1, \lfloor \frac{n}{2} \rfloor$ should be given to v_{n-2}.

If v_2 is given a new color, then this color cannot be assigned to $v_4, v_6, \cdots, v_{n-4}$. Further no vertex other than v_1 can receive color 1. Hence v_1 is the only dominating vertex of color 1. But v_1 cannot dominate the color $c(v_2)$, contradiction. Hence v_2 cannot be assigned any new color.

By similar argument $v_2, v_4, \cdots, v_{n-4}$ cannot be assigned any new color. For the vertices $v_{2i}, i = 1,2,\cdots,\frac{n-4}{2}$, assign colors $i+1$ respectively. Also assign any one of these colors $1,2,\cdots,\lfloor \frac{n}{2} \rfloor$ to the central vertex. Clearly the vertices $v_1, v_3, \cdots, v_{n-3}$ and v_{n-2} are e2-color dominating vertices of colors $1,2,3,\cdots,\lfloor \frac{n}{2} \rfloor$ respectively. Hence an e2-b-coloring by $\lfloor \frac{n}{2} \rfloor$ colors.
$\therefore \chi_{e2b}(W_n) = \lfloor \frac{n}{2} \rfloor, n\ is\ even.$
Case(ii): n is odd.

$v_1, v_3, v_5, \cdots, v_{n-2}$ are mutually at distance 2 and hence must receive distinct colors. Assign to them respectively the colors $1,2,3,\cdots,\lceil \frac{n-2}{2} \rceil$. Suppose v_2 is given a new color $\lceil \frac{n-2}{2} \rceil + 1$. This color cannot be given to the any one of the vertices $v_4, v_6, \cdots, v_{n-1}$. Since v_1 need to have 2-neighbour of color $\lceil \frac{n-2}{2} \rceil + 1$. $\therefore v_1$ cannot dominate the color $c(v_2)$, a contradiction. Hence v_2 cannot be assigned any new color.
By similar argument $v_4, v_6, \cdots, v_{n-2}$ cannot be assigned any new color. Therefore for the vertices, $v_{2i}, i = 1,2,\cdots,\frac{n-1}{2}$ assign colors i respectively.

Also assign any one of the colors $1,2,\cdots,\lceil \frac{n-2}{2} \rceil$ to

u. Clearly, $v_1, v_3, v_5, \cdots, v_{n-2}$ are $e2$-color dominating vertices of colors $1, 2, \cdots, \lceil \frac{n-2}{2} \rceil$. Hence an $e2$-b-coloring by $\lceil \frac{n-2}{2} \rceil = \lfloor \frac{n}{2} \rfloor$ colors is obtained. □

4. Conclusion

In this paper, a new type of coloring called exact 2-distance b-coloring and it's chromatic parameters were introduced. Some results based on exact 2-distance b-coloring were obtained. Also exact 2-distance b-chromatic number of some classes of graphs were obtained.

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