Malaya Journal of Matematik, Vol. 8, No. 1, 230-234, 2020
https://doi.org/10.26637/MJM0801/0039

A new integrated approach of combined FCM and CODAS method in interval valued intuitionistic fuzzy cognitive map for multi criteria decision making to evaluate and prioritize the branded mobile phones

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Abstract
In highly complex linear problems dealing with uncertainty, FCM are used to aid decision making. In multi criteria decision making (MCDM), intuitionistic fuzzy sets have been employed whereas interval valued intuitionistic fuzzy cognitive Maps are used in business decision making because of the increasing complexity of business environment. But IFS are mainly employed in MCDM. A new MCDM technique called combinative distance based assessment (CODAS) helps us to choose the alternative having the largest Euclidean and hamming distances from the negative ideal point. In this paper, a new integrated approach of combined FCM and multi criteria decision making -CODAS with IVIFCM. In order to find the effectiveness of the developed model, it is applied in the consumers perception of choice towards the selection of branded mobile phones.

Keywords
Intuitionistic Fuzzy Cognitive Map, Ranking method (CODAS), Weighted vector, Mobile phone selection.

AMS Subject Classification
03B52.

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Article History: Received **25** November **2019**; Accepted **18** February **2020**

Contents

1. Introduction

In FCM, fuzzy weights are given for causal relationships as w_{ij}. These are used to assess the edge from concept j to concept i. In FCM's, the concepts j and i in FCMs are connected by edges to represent the positive and negative relationships between the concepts [5].

An FCM is a fuzzy directed graph whose nodes represent fuzzy concepts within an application domain that occur to certain degree. Causal relations between the concepts are represented by directed edges. The strength of the relation between two concepts is weighted by the real values from $E \in [-1,1]$ [6]. For modeling and simulation of dynamic systems, FCMs are powerful tools, based on domain basic knowledge and experience.

In FCM, concepts can be causally interrelated and through fuzzy logic uncertain and imprecise knowledge is represented. It represents a number of advantages over conventional fuzzy approaches to reasoning namely handling of conflicting information, easy construction and parameterization and mental models are compared rapidly with reality.

In real life situations, many problems are encountered in decision making. In order to help the DMs elaborated suitable decisions, to rank alternative decision MCDM is found [4]. IFS are gene [6].

Realized fuzzy sets, in which their elements are characterized by both membership and non-membership value. The membership value indicates how much the degree to which an element belongs to the set whereas the non-membership value indicates to which degree it does not belong to the set.

The main advantage of IFS is considering the degree of hesitancy in the belongingness of an element to the set. DM's express their preferences and evaluate the alternatives with crisp values in the decision making process. The evaluations are uncertain in real life situations. In MCDM the vague and imprecise types of situations can be handled by fuzzy set theory proposed by Zadeh (1965) [3].

The problems are solved in fuzzy environment by combining MCDM methods with fuzzy theory under uncertain environments, FCMs and their generalizations are used effectively to solve MCDM problems with interdependent criteria. The extended version of FCM, IVIFCM uses IVIFSS to represent its concepts and weights.

Hence IVIFCM considered to be applicable to model interactions among criteria expressed by IVIFSS. IVIFCMs are suitable for solving single decision making problem but not MCGDM [7].

2. Preliminaries

Intuitionistic fuzzy sets [1]

An Intuitionistic fuzzy set (IFS) A in E is defined as an object of the following form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in E\}$ where the functions: $\mu_A : E \to [0,1]$ and $\gamma_A : E \to [0,1]$. Define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E : 0 \leq \mu_A + \gamma_A \leq 1$.

Interval-valued intuitionistic fuzzy set [2]

An interval-valued intuitionistic fuzzy set A on X can be represented as follows

$$A = \{\langle x, [\mu_A^L(x), \mu_A^U(x)]|[\gamma_A^L(x), \gamma_A^U(x)]\rangle, |x \in X\}$$

, Where $\{\mu_A^L(x), \mu_A^U(x)\}$ denotes the interval membership degree of element x belonging to the interval-valued intuitionistic fuzzy set, $\{\gamma_A^L(x), \gamma_A^U(x)\}$ denotes the interval non-membership degree of element x belonging to the interval-valued Intuitionistic fuzzy set.

Fuzzy Cognitive Map [5]

In FCMs, causal relationships of concepts can be represented by fuzzy weights which indicating to edge from a concept i to concept j. (ie) negative (Positive) relationships among the concepts. Likewise, the fuzzy values of node C_i^k is assigned to the i^{th} concept, where k indicates the iteration index and N concepts included in the FCM. The value of C_i^{k+1} for the next iteration $k+1$ can defined as,

$$C_i^{K+1} = f(C_i^k + \sum_{j=1}^{N} C_j^k \times w_{ij}) \tag{2.1}$$

Interval valued intuitionistic Fuzzy Cognitive Map [5]

To reformulate the consequence in conventional FCMs, the operators of Addition and Multiplication for interval valued Intuitionistic Fuzzy Cognitive Map are defined as

$$A \oplus B = \Big\langle x, [\mu_A^L(x) + \mu_B^L(x) - \mu_A^L(x).\mu_B^L(x), \mu_A^U(x) + \mu_B^U(x) - \mu_A^U(x).\mu_B^U(x)], [\gamma_A^L(x).\gamma_B^L(x), \gamma_A^U(x)\gamma_B^U(x)]\Big\rangle |x \in X$$

$$A \otimes B = \Big\langle x, [\mu_A^L(x) + \mu_B^L(x) - \mu_A^U(x).\mu_B^U(x), \gamma_A^L(x) + \mu_B^L(x) - \gamma_A^U(x).\gamma_B^L(x)], [\gamma_A^U(x).\gamma_B^U(x), \gamma_A^U(x)\gamma_B^U(x)]\Big\rangle |x \in X$$

Define the inference in IVIFCM by applying the above operators (i.e.,) addition and multiplication operator for IVIFS.

$$\begin{aligned} C_i^{k+1} &= \{[\mu_A^L(c), \mu_A^u(c)], [v_A^L(c), v_A^u(c)]\}_i^{k+1} \\ &= f\Big(\{[\mu_A^L(c), \mu_A^u(c)], [v_A^L(c), v_A^u(c)]\}_i^k \oplus \\ &\Big(\oplus_{j=1}^N \Big(\{[\mu_A^L(c), \mu_A^u(c)], [v_A^L(c), v_A^u(c)]\}_i^k \\ &\otimes \{[\mu_A^L(c), \mu_A^u(c)], [v_A^L(c), v_A^u(c)]\}_{ji}\Big)\Big)\Big) \end{aligned} \tag{2.2}$$

A new integrated approach for IVIFCM [5, 7]

Step 1: Obtain the decision matrix from the expert using linguistic variable to evaluate the alternative depends on the attribute.

Step 2: Convert the linguistic decision matrix into IVIFCM.

$$(i.e)\ F = (\langle \mu_{ij}^L, \mu_{ij}^U\rangle, \langle \gamma_{ij}^L, \gamma_{ij}^U\rangle).$$

Step 3: The expert was asked to indicate the strength of influence among the criteria using IVIFS.

Step 4: Determine the final value of the alternative depend on the criteria using the initial values (ie.) decision matrix and the influence values (i.e) the influence of the one criteria over the other one through the equation (2.1) and (2.2)

Step 5: Determine IVAIF NIS as follows

$$NS = [ns_{ij}]_{1\times m}$$

$$ns_j = minr_{ij}$$

where $minr_{ij} = ([min\mu_{ij}^L, min\mu_{ij}^U][max v_{ij}^L, max v_{ij}^U])$.

Step 6: The Normalized ED & NHD of alternatives from the negative ideal solution

$$ED_{AB} = \sqrt{\frac{1}{4}\sum_{i=1}^n w_j\Big((\mu_{AL}(x_i) - \mu_{BL}(x_i))^2 + (\mu_{AU}(x_i) - \mu_{BU}(x_i))^2 + (v_{AL}(x_i) - v_{BL}(x_i))^2 + (v_{AU}(x_i) - v_{BU}(x_i))^2\Big)}$$

$$HD_{AB} = \frac{1}{4}\sum_{i=1}^{n} w_j[|\mu_{AL}(x_i) - \mu_{BL}(x_i)| + |\mu_{AU}(x_i) - \mu_{BU}(x_i)| + |\nu_{AL}(x_i) - \nu_{BL}(x_i)| + |\nu_{AU}(x_i) - \nu_{BU}(x_i)|$$

Step 7: Determine the relative assessment matrix (RA) given as follows:

$$RA = [P_{ik}]_{n\times n}$$
$$P_{ik} = (ED_i - ED_k) + t(ED_i - ED_k)x(HD_i - HD_k)$$

where $K = \{1,2,..,n\}$ and t is a threshold function

$$t(x) = \begin{cases} 1\ if\ |x| \geq \theta \\ 0\ if\ |x| < \theta \end{cases}$$

Step 8: Calculate the assessment score (ASi) of each alternative

$$AS_i = \sum_{k=1}^{n} P_{ik}$$

Step 9: Rank the alternative according to the decreasing order of the AS. The alternative with the highest score is the best one.

Problem description

Today electronic gadgets have become part of adults and even children. In the selection of mobiles, people's choice for Android mobiles is at times difficult. Suppose that among Samsung, Xiaomi, One Plus, Vivo and Oppo, a consumer wants to choose the choice of theirs. These phones have some common features like innovative features, image, clarity, price, personal recommendation, durability and portable aspects, media influence and regarding post sales service facilities, which would serve as attributes.

The interval valued intuitionistic fuzzy values of the alternatives regarding the attributes are obtained from the decision maker in the form of linguistic variables as represented in table 1.The influence of the attributes which is obtained from the expert(s) in linguistic values as represented table 2. The weights of the attributes are obtained by the decision maker which is expressed in the form of fuzzy values given below.

Table 1: Fuzzy values of the alternatives regarding the attributes.

1	Very low	[0.2,0.5][0.3,0.4]
2	Low	[0.3,0.4][0.4,0.6]
3	Medium	[0.4,0.6][0.2,0.4]
4	High	[0.5,0.6][0.2,0.3]
5	Very High	[0.6,0.7][0.1,0.3]

Table 2: Influence of the attributes.

1	Very low	[0.1,0.1][0.9,0.9]
2	Low	[0.1,0.36][0.4,0.6]
3	Medium	[0.15,0.51][0.25,0.46]
4	High	[0.4,0.7][0,0.2]
5	Very High	[0.9,0.9][0.1,0.1]

Methodology

Step 1: Obtain the decision matrix from the expert using linguistic variable to evaluate the alternative depends on the attribute.

Alternatives/ Attributes	A1	A2	A3	A4	A5
C1	5	5	4	5	5
C2	5	5	5	4	4
C3	2	5	5	3	3
C4	1	5	4	4	3
C5	1	2	4	4	4
C6	5	5	5	5	5
C7	2	5	5	4	4

Step 2: Convert the linguistic decision matrix into IVIFS

Alternatives/ Attributes	A1	A2	A3
C1	[0.60, 0.70] [0.10, 0.30]	[0.60, 0.70] [0.10, 0.30]	[0.50, 0.60] [0.20, 0.30]
C2	[0.60, 0.70] [0.10, 0.30]	[0.60, 0.70] [0.10, 0.30]	[0.60, 0.70] [0.10, 0.30]
C3	[0.30, 0.40] [0.40, 0.60]	[0.60, 0.70] [0.10, 0.30]	[0.60, 0.70] [0.10, 0.30]
C4	[0.20, 0.50] [0.30, 0.40]	[0.60, 0.70] [0.10, 0.30]	[0.50, 0.60] [0.20, 0.30]
C5	[0.20, 0.50] [0.30, 0.40]	[0.30, 0.40] [0.40, 0.60]	[0.50, 0.60] [0.20, 0.30]
C6	[0.60, 0.70] [0.10, 0.30]	[0.60, 0.70] [0.10, 0.30]	[0.60, 0.70] [0.10, 0.30]
C7	[0.30, 0.40] [0.40, 0.60]	[0.60, 0.70] [0.10, 0.30]	[0.60, 0.70] [0.10, 0.30]

Alternatives/ Attributes	A4	A5
C1	[0.60, 0.70] [0.10, 0.30]	[0.60, 0.70] [0.10, 0.30]
C2	[0.50, 0.60] [0.20, 0.30]	[0.50, 0.60] [0.20, 0.30]
C3	[0.40, 0.60] [0.20, 0.40]	[0.40, 0.60] [0.20, 0.40]
C4	[0.50, 0.60] [0.20, 0.30]	[0.40, 0.60] [0.20, 0.40]
C5	[0.50, 0.60] [0.20, 0.30]	[0.50, 0.60] [0.20, 0.30]
C6	[0.60, 0.70] [0.10, 0.30]	[0.60, 0.70] [0.10, 0.30]
C7	[0.50, 0.60] [0.20, 0.30]	[0.50, 0.60] [0.20, 0.30]

Step 3: The expert was asked to indicate the strength of influence among the criteria using IVIFCM

	C1	C2	C3	C4	C5	C6	C7
C1	-	[0.48, 0.70] [0.12, 0.25]	[0.57, 0.77] [0.03, 0.17]	[0.57, 0.77] [0.03, 0.17]	[0.48, 0.70] [0.12, 0.25]	[0.73, 0.83] [0.07, 0.13]	[0.48, 0.70] [0.12, 0.25]
C2	[0.48, 0.70] [0.12, 0.25]	-	[0.57, 0.77] [0.03, 0.17]	[0.48, 0.70] [0.12, 0.25]	[0.22, 0.52] [0.22, 0.42]	[0.73, 0.83] [0.07, 0.13]	[0.22, 0.52] [0.22, 0.42]
C3	[0.73, 0.83] [0.07, 0.13]	[0.73, 0.83] [0.07, 0.13]	-	[0.47, 0.65] [0.17, 0.30]	[0.47, 0.57] [0.33, 0.40]	[0.40, 0.64] [0.20, 0.34]	[0.47, 0.65] [0.17, 0.30]
C4	[0.47, 0.57] [0.33, 0.40]	[0.47, 0.65] [0.17, 0.30]	[0.48, 0.70] [0.12, 0.25]	-	[0.40, 0.70] [0.00, 0.20]	[0.48, 0.70] [0.12, 0.25]	[0.30, 0.59] [0.40, 0.30]
C5	[0.47, 0.65] [0.17, 0.30]	[0.37, 0.54] [0.30, 0.43]	[0.40, 0.64] [0.20, 0.34]	[0.48, 0.70] [0.12, 0.25]	-	[0.22, 0.52] [0.22, 0.42]	[0.38, 0.58] [0.25, 0.38]
C6	[0.48, 0.70] [0.12, 0.25]	[0.40, 0.64] [0.20, 0.34]	[0.22, 0.52] [0.22, 0.42]	[0.40, 0.64] [0.20, 0.34]	[0.20, 0.38] [0.43, 0.56]	-	[0.10, 0.18] [0.70, 0.80]
C7	[0.23, 0.57] [0.17, 0.37]	[0.32, 0.63] [0.08, 0.28]	[0.38, 0.58] [0.25, 0.38]	[0.57, 0.77] [0.03, 0.17]	[0.22, 0.52] [0.22, 0.42]	[0.10, 0.18] [0.70, 0.80]	-

Step 4: Determine the final value of the alternative depend on the criteria using the initial values (ie.) decision matrix and the influence values (i.e) the influence of the one criteria over the other one through the equation (2.2)

	A1	A2	A3	A4	A5
C1	[0.91,0.98] [0.00009,0.008]	[0.96,0.99] [0.000006,0.006]	[0.95,0.99] [0.000012,0.003]	[0.95,0.99] [0.000025,0.00285]	[0.95,0.99] [0.000025,0.00285]
C2	[0.96,0.99] [0.0001,0.005]	[0.94,0.99] [0.00001,0.005]	[0.98,0.99] [0.00000017,0.0114]	[0.92,0.99] [0.00004,0.0048]	[0.92,0.98] [0.00002,0.005]
C3	[0.89,0.98] [0.0003,0.013]	[0.96,0.99] [0.00005,0.005]	[0.95,0.99] [0.00011,0.0036]	[0.93,0.99] [0.0002,0.0048]	[0.92,0.99] [0.00018,0.0056]
C4	[0.78,0.97] [0.0003,0.02]	[0.92,0.99] [0.00007,0.007]	[0.91,0.99] [0.0008,0.0015]	[0.89,0.98] [0.00018,0.0051]	[0.87,0.98] [0.0002,0.007]
C5	[0.78,0.96] [0.0009,0.02]	[0.86,0.98] [0.0002,0.016]	[0.86,0.97] [0.0003,0.004]	[0.90,0.99] [0.0002,0.002]	[0.89,0.97] [0.00012,0.002]
C6	[0.83,0.97] [0.0006,0.23]	[0.86,0.97] [0.0003,0.019]	[0.86,0.97] [0.0003,0.002]	[0.86,0.97] [0.0003,0.004]	[0.85,0.97] [0.0003,0.005]
C7	[0.65,0.92] [0.0008,0.03]	[0.89,0.98] [0.00008,0.02]	[0.87,0.98] [0.0001,0.012]	[0.84,0.90] [0.0004,0.01]	[0.81,0.97] [0.0004,0.02]

Step 5: Determined IVAIF NIS as follows.

C1	C2	C3
[0.91,0.98] [0.00009,0.008]	[0.92,0.98] [0.0001, 0.0114]	[0.89,0.98] [0.0003,0.013]

C4	C5	C6
[0.78,0.97] [0.0008,0.02]	[0.78,0.96] [0.0009,0.02]	[0.83,0.97] [0.0006,0.23]

C7
[0.65,0.90] [0.0008,0.03]

Step 6: The normalized ED & NHD of alternatives from the negative ideal solution

	A1	A2	A3	A4	A5
HD_i	0.0138	0.1108	0.1092	0.2844	0.0951
ED_i	0.0067	0.1096	0.1104	0.1012	0.0901

Step 7 and 8: Calculated the assessment score (AS_i)

	A1	A2	A3	A4	A5	ASi	Rank
A1	-	-0.1999	-0.1991	-0.3651	-0.3651	-0.9288	5
A2	0.1999	-	0.0016	-0.1652	0.0157	0.052	2
A3	0.1987	-0.0016	-	-0.166	0.0141	0.0484	3
A4	0.3651	0.1652	0.166	-	0.2004	0.8967	1
A5	0.1647	-0.0157	-0.0141	-0.2004	-	-0.0655	4

Step 9: Rank the alternative according to the decreasing order of the AS_i. The alternative with the highest score is the best one.

$$A4 > A2 > A3 > A5 > A1$$

(i.e) $A4$ is the best one.

3. Conclusion

In the classical decision making process, DMs express their preferences and evaluate the alternatives with crisp values. However, especially in real-life problems, theevaluations of DMs can be uncertain. The study about applying Combined FCM and Multi Criteria Decision Making - CODAS with IVIFCM on order to determine the choice of consumers in choosing the android mobile FCM that copes with complex relationships among decision concepts under a highly uncertain consumer's choice in this consumerist world. IVIFCMs are based on a concept of interval valued intuitionistic fuzzy set, providing an effective tool to deal with strong uncertainty in the values of criteria and their causal relationships. It is obvious that the CODAS method produces more sensitive results out of using two different distance measures namely Euclidean distance and Hamming distance. Since CODAS is used in ranking the alternatives, it is applied in finding the ranking of consumers' choice in buying the mobile phones among Samsung, Xiaomi, Vivo, One Plus and Oppo. The findings show that people prefer to buy Vivo in using Combined FCM Multi Criteria Decision Making-CODAS with IVIFCM.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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