Malaya Journal of Matematik, Vol. 8, No. 1, 254-257, 2020

https://doi.org/10.26637/MJM0801/0043

Domination and s-path domination in some brick product graphs

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Abstract

A dominating set or **dset** of $\mathscr{G}$ is called a s-path dset of $\mathscr{G}$ $(2 \leq s \leq diam\mathscr{G})$ if any path of length $s \in \mathscr{G}$ has $\subseteq$ of one vertex in this dset. We indicate a s-path dset by D_{p_s}. The s-path dominaton number or **s-path dn** of $\mathscr{G}$ indicated by $\gamma_{p_s}(\mathscr{G})$ is the minimal cardinality or **MC** taken over all s-path dsets of $\mathscr{G}$. In that paper, we determine domination number and s-path domination number for the brick product graph $B(2n, \mathscr{P}, \mathscr{Q})$ $(\mathscr{P} = 2)$ related with even cycles.

Keywords

dset, dn, edge dn, s - path dn.

AMS Subject Classification

05C69.

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Article History: Received **24** November **2019**; Accepted **19** February **2020**

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1. Introduction

For a graph $\mathscr{G} = (V,E)$ is a finite, not directed graph, loopless and non parallel edges. If $D \subseteq V$ is called a dset of $\mathscr{G}$, if every vertex $\notin D$ is adjoining to few vertex $\in D$. The dn of $\mathscr{G}$ denoted by $\gamma(\mathscr{G})$ is the MC taken over all dset of $\mathscr{G}$.

A set F of edges $\in \mathscr{G}$ is called an edge dset if any edge $e \in E - F$ is adjoining to $\geq$ one edge in F. The edge domination number $\gamma^{'}(\mathscr{G})$ of $\mathscr{G}$ is the MC of an edge dset of $\mathscr{G}$.

The open neighbourhood of $N(e)$ is the set of all edges adjoining to $e \in \mathscr{G}$. If $e = (u,v)$ is an edge in $\mathscr{G}$, the degree of e indicate by $deg(e)$ is described as $deg(e) = deg(u) + deg(v) - 2$. The maximum degree of an edge in graph is denoted by $\triangle^{'}(\mathscr{G})$.

A dset of graph is called a s-path dset of $\mathscr{G}$ $(2 \leq s \leq diam\mathscr{G})$ if any path of length s $\in \mathscr{G}$ has $\subseteq$ one vertex in this dset. We indicate a s-path dset by D_{p_s}. The s-path dn of $\mathscr{G}$ denoted by $\gamma_{p_s}(\mathscr{G})$ is the MC taken over all s-path dsets of graph.

If any s-path dset is a dset but the converse need not be accurate . Also we well known that $|D| \leq |D_{p_s}|$. Therefore $\gamma(\mathscr{G}) \leq \gamma_{p_s}(\mathscr{G})$.

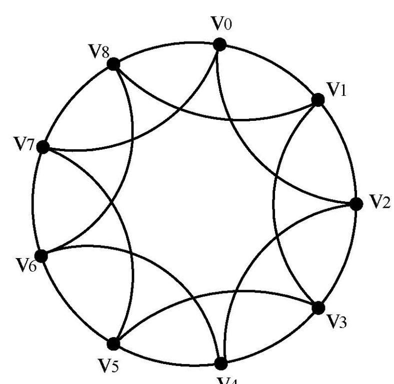

Figure 1. $\mathscr{G} = (B(9,1,2))$, $\gamma_{P_2}(G) = 4$

The graph $\mathscr{G} = B(9,1,2)$ in figure , the sets $D = \{v_0, v_4\}$, $\{v_1, v_5\}$, $\{v_2, v_7\}$, $\{v_3, v_7\}$ etc are dsets. Without loss generality let us consider the set $\{v_0, v_4\}$ as the dset but not a 2-path dset of $\mathscr{G}$ since the paths $v_3 - v_5 - v_6$, $v_3 - v_5 - v_7$,

$v_3 - v_1 - v_8$ and $v_1 - v_8 - v_6$ of length 2 does not contain either v_0 or v_4. But, the set $\{v_0, v_3, v_4, v_8\}$ is a 2-path dset, $\gamma_{P_2} = 4$. If allow that $|D| < |D_{p_s}|$

Definition 1.1. *[3]*

Let $\mathscr{P}$, n and $\mathscr{Q}$ be a positive integers.

Let $B_{2n} = a_0, a_1, a_2, ..., a_{2n-1}, a_0$ denote a cycle order 2n. The $(\mathscr{P}, \mathscr{Q})$ - brick product of B_{2n}, [2] denoted by $B(2n, \mathscr{P}, \mathscr{Q})$, is defined in two cases as follows.

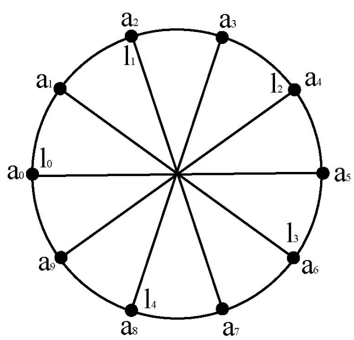

Figure 2. The brick product graph $B(10, 1, 5)$

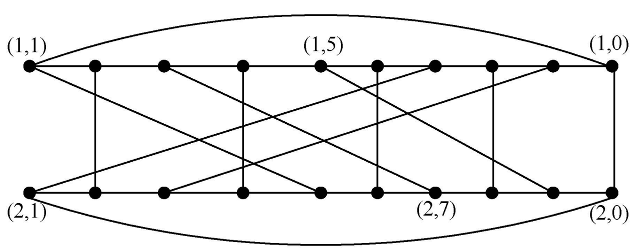

Figure 3. The brick product graph $B(10, 2, 4)$

1. *If $\mathscr{P}$ =1, we make necessary that $\mathscr{Q}$ be odd and > 1. Then, $B(2n, \mathscr{P}, \mathscr{Q})$ is attained from B_{2n} by connecting chords $a_{2k}a_{2k+\mathscr{Q}}$, k = 1,2,...,n, where the computation is performed modulo 2n.*

2. *If $\mathscr{P}$ >1, we make necessary that $\mathscr{P} + \mathscr{Q}$ be even. Then, $B(2n, \mathscr{P}, \mathscr{Q})$ is attained by first taking the disjoint union of $\mathscr{P}$ copies of B_{2n},*

   namely $B_{2n}(1), B_{2n}(2), ..., B_{2n}(\mathscr{P})$, where for each $i = 1, 2, ..., m$, $B_{2n}(i) = (i, 0)(i, 1)...(i, 2n)$. Next, for each odd $i = 1, 2, ...\mathscr{P} - 1$ and each even $k = 0, 1, 2, ...2n - 2$, an edge (called a brick edge) is drawn to join(a_i, a_k) to (a_{i+1}, a_k), whereas, for each even $i = 1, 2, ..., \mathscr{P} - 1$ and each odd $k = 1, 2, ..., 2n - 1$, an edge (also called a brick edge) is drawn to join (a_i, a_k) to (a_{i+1}, a_k). Finally, for each odd $k = 1, 2, .., 2n - 1$, an edge (called a hooking edge) is drawn to join (a_1, a_k) to $(a_{\mathscr{P}}, a_{k+\mathscr{Q}})$. An edge in $B(2n, \mathscr{P}, \mathscr{Q})$ which is not either a brick edge nor a hooking edge is called a flat edge.

2. Preliminaries

We bring the following result belonging to the dn of a graph.

Theorem 2.1. *[6] A dset D is a minimal dset ⇔ for any vertex $a \in D$, one of the following condition holds:*

1. *degree(a) = 0 of dset*

2. *$\exists$ a vertex b in $V - D$ such that $N(b) \cap D$ is equal to $\{a\}$.*

comparable to the theorem 2.1, we have the following result for a s-path dset.

Theorem 2.2. *A dset D_{p_s} is a minimal s-path dset $(s \geq 2)$ ⇔ for each vertex u in D_{p_s}, one of the following conditions holds:*

1. *degree u = 0 of D_{p_s}*

2. *$\exists$ a vertex $v \in V - D_{p_s}$ such that $N(v) \cap D_{p_s} = \{u\}$*

3. *If $\mathscr{G}$ is k- connected, $k > 1$ and $v_i, v_j \in D_{p_s}$, then $< D_{p_s} >$ is a disconnected graph and each vertex of $D_{p_s} - \{u\}$ belongs to some cycle in $\mathscr{G}$.*

Proof. Let D_{p_s} be a minimal s-path dset of $\mathscr{G}$. Then for every vertex $u \in D_{p_s}$ if the set $D_{p_s} - \{u\}$ is not a s-path dset in $\mathscr{G}$, it follows that either degree $u = 0$ of D_{p_s} or $\exists$ a vertex $v \in V - D_{p_s}$ such that$N(v) \cap D_{p_s} = \{u\}$. If $\mathscr{G}$ is k- connected and $k > 1$, then for $s \geq 2$, every vertex of $D_{p_s} - \{u\}$ belongs to some cycle in $\mathscr{G}$ and we have the following two possible cases.

Case 1 : Let $v_i, v_j \in D_{p_s}$ such that $d(v_i, v_j)$= 1. Then, $< D_{p_s} >$ is a disconnected graph with one component as K_2 and the remaining components are isolated vertices.

Case 2 : Let $v_i, v_j \in D_{p_s}$ such that $d(v_i, v_j) \geq 1$. Then $< D_{p_s} >$ is a disconnected graph in which all components are isolated vertices.

Conversely, let D_{p_s} be a s-path dset satisfying the conditions above. For the purpose of contradiction, let us assume that D_{p_s} is not minimal. Then there must exist a vertex $u \in D_{p_s}$ such that $D_{p_s} - \{u\}$ is also a s- path dset. Hence, for atleast one vertex $v \in D_{p_s} - \{u\}$, there must be a path connecting u and v in $\mathscr{G}$, so that $\{u\}$ cannot be an isolated vertex of D_{p_s} and hence condition 1 fails. Also, every vertex in $V - D_{p_s}$ lies in some path connecting atleast one vertex in $D_{p_s} - \{u\}$ so that conditions 2 also fails. For condition 3, it is easy to observe that $\{u\}$ lies in some cycle of $\mathscr{G}$ along with the vertices of $V - (D_{p_s} - \{u\})$. So condition 3 also fails. This contradicts the fact the $D_{p_s} - \{u\}$ also a minimal s-path dset.

Hence the proof. □

3. Main Results

We provide the results connected to the domination and the edge dn of some brick product graphs.

Theorem 3.1. *Let* $\mathscr{G} = B(2n,2,\mathscr{Q})$*. Then* $\gamma(\mathscr{G}) = 2\lceil \frac{n}{2} \rceil$ *for* $n \geq 3$*, where* $\mathscr{Q}$*=* $2j$*,* j*=*$1,2,3,4$*.*

Proof. We consider $V(\mathscr{G})= V_1 \cup V_2$, where $V_1= \{v_{(1,i)}\}$ and $V_2 = \{v_{(2,i)}\}$, $i = 1,2,3.....2n$, modulo $2n$ and $E(\mathscr{G}) = \mathscr{E}_1 \cup \mathscr{E}_2 \cup \mathscr{E}_3 \cup \mathscr{E}_4 \cup \mathscr{E}_5 \cup \mathscr{E}_6$, where $\mathscr{E}_1 = \{e_i/e_i = (v_{(1,i)}, v_{(1,i+1)})\}$, $\mathscr{E}_2 = \{e_i^{'}/e_i^{'} = (v_{(2,i)}, v_{(2,i+1)})\}$, $i = 1,2,...2n$, module $2n$, $\mathscr{E}_3 = \{l_p/l_p = (v_{(i,k)}, v_{(i+1,k)})\}$, for every odd $i = 1,2...m-1$ and every even $k = 0,1,2....2n-2$, $p = 1,2,...(n-1)$, $\mathscr{E}_4 = \{l_p^{'}/l_p^{'} = (v_{(1,k)}, v_{(2,k+r)})\}$, for every odd $k = 1,2,...(2n-1)$, $\mathscr{E}_5 = \{c_1/c_1 = (v_{(1,i)}, v_{(1,2n)})\}$, $\mathscr{E}_6 = \{c_1^{'}/c_1^{'} = (v_{(2,i)}, v_{(2,2n)})\}$, modulo $2n$.

Let $n \geq 3$.

Consider the set $D = D_1 \cup D_2$,
where $D_1 = \{v_{(1,4j-2)}\}$, $1 \leq j \leq \lceil \frac{n}{2} \rceil$ and $D_2 = \{v_{(2,4k)} \cup v_{(2,2n)}\}$, $1 \leq k \leq \lceil \frac{n}{2} \rceil - 1$

The above set D is a minimal dset, for any vertex $a \in D$, $D - \{a\}$ is not a dset. consequence, few vertex $b \in V - D \cup \{a\}$ is not dominated by any vertex $\in D \cup \{a\}$. If $b \in V - D$ and $b \notin$ dominated by $D - \{a\}$, but is dominated by D, then b is adjoining exclusively to vertex $a \in D$, i.e $N(a) \cap D$ is equal to $\{a\}$.

Therefore $|D| = 2\lceil \frac{n}{2} \rceil$, it follows that $\gamma(\mathscr{G}) = 2\lceil \frac{n}{2} \rceil$.

Hence the proof □

Theorem 3.2. *Let* $\mathscr{G} = B(2n,2,\mathscr{Q})$*. Then* $\gamma^{'}(\mathscr{G}) = 2\lceil \frac{2n}{3} \rceil$ *for* $n \geq 3$*, where* $\mathscr{Q}$*=* $2,8$*.*

Proof. The $V(\mathscr{G})$ and $E(\mathscr{G})$ are as theorem 3.1.

Let $n \geq 3$.

Consider set $F = F_1 \cup F_2$,

$$\text{where } F_1 = \begin{cases} \{e_{(1,p-2)}\}, & n \equiv 0(mod3) \\ \{e_{1,3q-2}\}, & n \equiv 1(mod3) \\ \{e_{1,3t-2}\} \cup \{e_{1,2n-1}\}, & n \equiv 2(mod3) \end{cases}$$

$1 \leq p \leq \dfrac{2n}{3}$, $1 \leq q \leq \dfrac{2n+1}{3}$, $1 \leq t \leq \dfrac{2n-1}{3}$ and

$$F_2 = \begin{cases} \{e_{(2,3p-1)}\}, & n \equiv 0(mod3) \\ \{e_{2,3q-1}\}, & n \equiv 1(mod3) \\ \{e_{2,3t-1}\} \cup \{e_{2,2n-1}\}, & n \equiv 2(mod3) \end{cases}$$

$1 \leq p \leq \dfrac{2n}{3}$, $1 \leq q \leq \lceil \dfrac{2n+4}{3} \rceil$, $1 \leq t \leq \dfrac{2n+2}{3}$

The above set F is a minimal edge dset, for every edge $f_i \in F$, $F - \{f_i\}$ is $\notin$ an edge dset for neighbourhood of $f_i \in \mathscr{G}$. For every set carry edges $<$ that of F cannot be a dset of graph. Also graph is a 3-regular and every edge of $\mathscr{G}$ is of degree 4 and an edge of graph be able to dominate $\leq$ *five* well defined edges of graph inclusive of itself.

Therefore $|F| = 2\lceil \frac{2n}{3} \rceil$, it follows that $\gamma^{'}(\mathscr{G}) = 2\lceil \frac{2n}{3} \rceil$.

Hence the proof □

Theorem 3.3. *Let* $\mathscr{G} = B(2n,2,\mathscr{Q})$*. Then* $\gamma^{'}(\mathscr{G}) = 2\lceil \frac{2n}{3} \rceil$ *for* $n \geq 3$*, where* $\mathscr{Q}$*=* $4,6$*.*

Proof. The $V(\mathscr{G})$ and $E(\mathscr{G})$ are as theorem 3.1.

Let $n \geq 3$.

Consider set $F = F_1 \cup F_2$,

$$\text{where } F_1 = \begin{cases} \{e_{(1,3p-1)}\}, & n \equiv 0(mod3) \\ \{e_{1,3q-1}\} \cup \{e_{1,2n-1}\}, & n \equiv 1(mod3) \\ \{e_{1,3t-1}\} \cup \{e_{1,2n-3}\} \cup \{e_{1,2n-1}\}, & n \equiv 2(mod3) \end{cases}$$

$1 \leq p \leq \dfrac{2n}{3}$, $1 \leq q \leq \lfloor \dfrac{2n}{3} \rfloor$, $1 \leq t \leq \lfloor \dfrac{2(n-1)}{3} \rfloor$ and

$$F_2 = \begin{cases} \{e_{(2,3p-2)}\}, & n \equiv 0(mod3) \\ \{e_{2,3q-2}\}, & n \equiv 1(mod3) \\ \{e_{2,3t-2}\} \cup \{e_{2,2n-1}\}, & n \equiv 2(mod3) \end{cases}$$

$1 \leq p \leq \dfrac{2n}{3}$, $1 \leq q \leq \lceil \dfrac{2n+4}{3} \rceil$, $1 \leq t \leq \lceil \dfrac{2n+2}{3} \rceil$

The above set F is a minimal edge dset, for every edge $f_i \in F$, $F - \{f_i\}$ is $\notin$ an edge dset for neighbourhood of $f_i \in \mathscr{G}$. For every set carry edges $<$ that of F cannot be a dset of graph. Also graph is a 3-regular and every edge of $\mathscr{G}$ is of degree 4 and an edge of graph be able to dominate $\leq$ *five* well defined edges of graph inclusive of itself.

Therefore $|F| = 2\lceil \frac{2n}{3} \rceil$, it follows that $\gamma^{'}(\mathscr{G}) = 2\lceil \frac{2n}{3} \rceil$.

Hence the proof □

we provide the results related to the s-path domination of some brick product graphs.

Theorem 3.4. *Let* $\mathscr{G} = B(2n,2,\mathscr{Q})$*. Then* $\gamma_{p_2}(\mathscr{G}) = 2n$ *for* $n \geq 3$*, where* $\mathscr{Q}$*=* $2j$*,* j*=*$1,2,3,4$*.*

Proof. The $V(\mathscr{G})$ and $E(\mathscr{G})$ are as theorem 3.1.

Let $n \geq 3$

Consider the set $D_{p_2} = V_1 \cup V_2$, where $V_1 = \{v_{1,2i-1}\}$ and $V_2 = \{v_{2,2k}\}$, $1 \leq i,j \leq n$

The set D_{p_2} is a minimal 2-path dominating set, for every vertex of $D_{p_2} - \{u\}$ belongs to some cycle in $\mathscr{G}$ and let $v_i, v_j \in D_{p_2}$ such that $d(v_i, v_j)$= 1. Then, $< D_{p_2} >$ is a disconnected graph with one component as K_2 and the remaining components are isolated vertices.

Therefore, $|D_{p_2}| = 2n$, it follows that $\gamma_{p_2}(\mathscr{G}) = 2n$.

Hence the proof □

Theorem 3.5. *Let* $\mathscr{G} = B(2n,2,\mathscr{Q})$*. Then* $\gamma_{p_3}(\mathscr{G}) = 2n$ *for* $n \geq 3$*, for* $n \geq 3$*, where* $\mathscr{Q}$*=* $2j$*,* j*=*$1,2,3,4$*.*

Proof. The $V(\mathscr{G})$ and $E(\mathscr{G})$ are as theorem 3.1.

Let $n \geq 3$

Consider the set $D_{p_3} = V_1 \cup V_2$, where $V_1 = \{v_{1,2i-1}\}$ and $V_2 = \{v_{2,4k-2}\}$, $1 \leq i \leq n$, $1 \leq j \leq \lceil \frac{n}{2} \rceil$

The set D_{p_3} is a minimal 3-path dset, for any $a \in D_{p_3}$, $D_{p_3} - \{a\}$ is $\notin$ a 3-path dset and also, some $b \in V - D_{p_3}$ is not dominated by any vertex in $D_{p_3} \cup \{a\}$. Hence, either a=b or $b \in V - D_{p_3}$. But, degree a of D_{p_3} if a=b, and, if $b \in V - D_{p_3}$ and b is not dominated by $D_{p_3} - \{a\}$, but is dominated by D_{p_3}, then b is adjoining exclusively to vertex $a \in D_{p_3}$, i.e. $N(a) \cap D_{p_3}$ is equal to $\{a\}$.

Therefore, $|D_{p_3}| = 2n$, it follows that $\gamma_{p_3}(\mathscr{G}) = 2n$.

Hence the proof. □

4. Conclusion

In this paper, the vertex domination number $\gamma(\mathscr{G})$ and edge domination number $\gamma^{'}(\mathscr{G})$, also s-path domination number $\gamma_{p_s}(\mathscr{G})$ associated with the brick product graphs of even cycles $B(2n,\mathscr{P},\mathscr{Q})$ $(\mathscr{P}=2)$ are determined.

Acknowledgment

The first and second authors is thankful to the Management and staff of the School of Applied Sciences (Mathematics), REVA University, Bengaluru and Guru Nanak Institutions Technical Campus (Autonomous), Hyderabad. The authors are also thankful to the Management and Research center, Department of Mathematics, Dr. AIT, Bengaluru.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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