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Analytic solution of fractional order differential equation arising in RLC electrical circuit

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Abstract

In this paper, we obtain the analytical solution of a non-integer order differential equation which is associated with a RLC electrical circuit. The order of fractional differential equation depends upon α and β, where $\alpha \in (1,2]$ and $\beta \in (0,1]$. Further, we use Elzaki transform with its different properties to obtain the solution of fractional differential equation and obtain the solution in terms of three parameter Mittag-Leffler function. In the last, we have presented an example to show effectiveness of Elzaki transform in solving electrical circuit problems.

Keywords

Model of RLC circuit, non-integer order differential equation, Elzaki transform, Mittag-Leffler function.

AMS Subject Classification

00A71, 34A08, 35A22, 33E12.

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Contents

1. Introduction

Fractional calculus is a generalization of differentiation and integration to non-integer orders. In the last few years, the number of studied on fractional differential equations have increased dramatically since it can be used in many areas of science and engineering such as electromagnetism, fluid mechanics, signal processing, electrochemistry and so on. The theory of fractional differential equations helps to translate the real world problems in a better and systematic manner. Mathematical models' involving fractional order derivatives has become a powerful and widely used tool for better modeling. Fractional model for electrical circuits such as RL, RC, RLC have already been proposed by many researchers, for details, see [1–3]. In order to stimulate more interest in subject and to show its utility, this paper is devoted to new and recent application of fractional calculus.

2. Mathematics Prerequisites

Gosta Mittag-Leffler introduced a function in 1903, is called Mittag-Leffler function $E_\beta(z)$ [4], defined as

$$E_\delta(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\delta k+1)}, \quad z \in C; R(\varepsilon) > 0, \tag{2.1}$$

In [5], Wiman gave the generalization of $E_\delta(z)$

$$E_{\delta,\rho}(z) =) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\delta k+\rho)}, \tag{2.2}$$
$$\delta, \rho \in C, R(\rho) > 0, R(\delta) > 0.$$

Prabhakar [6] investigated the three-parameter Mittag-Leffler function $E^\gamma_{\beta,\rho}(z)$ as

$$E^\gamma_{\delta,\rho}(z) = \sum_{k=0}^{\infty} \frac{((\gamma)_k)}{\Gamma(\delta k+\rho)} \frac{z^k}{k!}, \tag{2.3}$$
$$\delta, \rho, \gamma \in C, R(\rho) > 0, R(\delta) > 0, R(\gamma) > 0.$$

In Eq. (2.3), $(\gamma)_k$ is called Pochhammer symbol, introduced by Leo August Pochhammer, written in the form

$$(\gamma)_k = (\gamma)(\gamma+1)...(\gamma+(k-1)).$$

Here, we are giving some basic definitions of fractional calculus as follows [7–9]:

Definition 2.1. *A real function $(t), t > 0$ is said to be in the space C_μ if $\mu \in R$, there exists a real number $p > \mu$ and the function $f_1(t) \in C[0,\infty)$ such that $f(t) = t^p f_1(t)$. Moreover, if $f^{(n)} \in C_\mu$, then $f(t)$ is said to be in the space $C_\mu^n, n \in N$.*

Definition 2.2. *The Riemann-Liouville fractional integral of order $\sigma \geq 0$ for a function $f(t)$ is defined as*

$$I^\sigma f(t) = \begin{cases} \frac{1}{\Gamma(\sigma)} \int_0^t (t-\tau)^{\sigma-1} f(\tau) d\tau, \sigma > 0 \\ f(t), \qquad \sigma = 0 \end{cases} \tag{2.4}$$

Where $\Gamma(\cdot)$ denotes the Gamma function.

Definition 2.3. *The Riemann-Liouville fractional derivative of order $\sigma > 0$ for a function $f(t)$ is defined as*

$$D^\sigma f(t) = \frac{d^n}{dt^n} I^{n-\sigma} f(t), n \in N, n-1 < \sigma \leq n \tag{2.5}$$

Definition 2.4. *The Caputo fractional derivative of order $\sigma > 0$ is defined as*

$$D^\sigma f(t) = \begin{cases} \frac{d^n f(t)}{dt^n}, & \sigma = n,\ n \in N \\ \frac{1}{\Gamma(n-\sigma)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\sigma-n+1}} d\tau, & 0 < n-1 < \sigma \leq n, \end{cases} \tag{2.6}$$

Where n is an integer, $t > 0$ and $f(t) \in C_1^n$.

Definition 2.5. *A new integral transform called Elzaki transform [10] is defined on the set of functions*

$$A = \{f(t); \exists\, M,\ k_j > 0,\ j = 1,2,$$
$$|f(t)| < M\, e^{\frac{|t|}{k_j}},\ if\ t \in (-1)^j \times [0,\infty)\}$$

For a function $f(t)$ in the set A, the constant M must be finite number, k_1, k_2 may be finite or infinite, then Elzaki transform of $f(t)$ is defined as

$$\begin{aligned} E[f(t)] &= E[f(t), v] = T(v) \\ &= v \int_0^\infty f(t) e^{\frac{-t}{v}} dt, t \geq 0, k_1 < v < k_2, 0 \leq t < \infty. \end{aligned} \tag{2.7}$$

Equivalently form of Eq. (2.7), is

$$E[f(t)] = E[f(t), v] \tag{2.8}$$

$$= T(v) = v^2 \int_0^\infty f(vt) e^{-t} dt. \tag{2.9}$$

Using duality of Laplace [11], Elzaki transform of the Caputo fractional derivative (2.6) of order $\sigma > 0$, can be obtained and get as

$$E[D^\sigma f(t), v] = \frac{T(v)}{v^\sigma} - \sum_{k=0}^{n-1} v^{k-\sigma+2} f^{(k)}(0), n-1 < \sigma \leq n, \tag{2.10}$$

In Eq. (2.10), $T(v)$ represents the Elzaki transform of the function $f(t)$.

Following result will be used in our main findings

$$E^{-1}[v^{\rho+1}(1-\omega u^\delta)^{-\gamma}] = t^{\rho-1} E_{\delta,\rho}^{\gamma}(\omega t^\delta) \tag{2.11}$$

Definition 2.6. *(Convolution theorem) [12], Let $f(t)$ and $g(t)$ be two functions, defined in set A, have Elzaki transform is $M(v)$ and $N(v)$, then the Elzaki transform of the convolution of $f(t)$ and $g(t)$ is*

$$E[f(t) \times g(t)] = \frac{1}{v} M(v) N(v),$$

where

$$f(t) \times g(t) = \int_0^t f(t-\tau) g(\tau) d\tau, \tag{2.12}$$

whenever the integral is defined.

3. Mathematical model for RLC Electrical Circuit

The integrated process of translating real world problem into mathematical problem is termed as mathematical modeling. It includes mathematical concepts such as function, variables, constants, inequality etc. taken from different branches of mathematics. Here, we formulate the model for electrical circuit which is very useful in physics and engineering. When we connected resistor(R), inductor (L) and capacitor (C) with voltage (E) then we get an electrical circuit which is known as RLC electrical circuit. There are a lot of connecting of these three elements across voltage supply. In this proposed model, we consider an electrical circuit in which these three elements are connected in series with voltage as shown in figure 1. Here, in this model we have considered these three components i.e. capacitance (C), inductance (L) and resistor(R) are positive constants.

Here, $E(t)$ represents for power source's voltage which is measured in volts (V), $I(t)$ represents the current in the circuit at time t which is measured in amperes (A), $Q(t)$ represents the charge on the capacitor (or charge flow to capacitor plates) at time t (measured in Coulombs). As we know there is a relation between current and charge i.e. $I(t) = \frac{d}{dt} Q(t)$. Here R is denoted for the resistance of the resistor which is measured in ohms (V/A), L is denoted for the inductance of the inductor which is measured in henry (H) and C is denoted for the capacitance of the capacitor which is measured in farads ($F = C/V$).

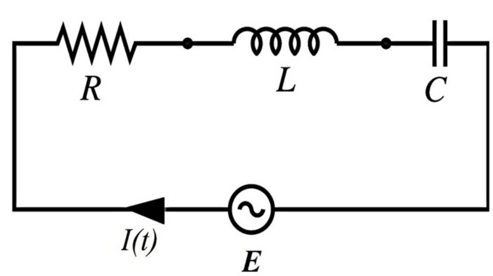

Figure 1: RLC Circuit

4. Formulation of fractional differential equation and its solution

The equations associated with resistor, inductor and capacitor in RLC circuit are:

The voltage drop across resistor, i.e.,

$$V_R(t) = RI(t) = R\frac{d}{dt}Q(t).$$

The voltage drop across inductor, i.e.,

$$V_L(t) = L\frac{d}{dt}I(t) = L\frac{d^2}{dt^2}Q(t).$$

The voltage drop across capacitor, i.e.,

$$V_C(t) = \frac{1}{C}\int_0^t I(\xi)d\xi = \frac{Q(t)}{C}.$$

On using Kirchhoff's voltage law, around any loop in a circuit, the voltage rises must equal to the voltage drops. Therefore we will get the following equation for RLC electric circuit represented in figure 1

$$L\frac{d^2}{dt^2}Q(t) + R\frac{d}{dt}Q(t) + \frac{Q(t)}{C} = E(t), \tag{4.1}$$

In the form of current the equation of the RLC electric circuit represented in figure 1 is given in [13] as

$$L\frac{d^2}{dt^2}I(t) + R\frac{d}{dt}I(t) + \frac{I(t)}{C} = E(t), \tag{4.2}$$

In this paper, we will find the analytic solution of a fractional differential equation associated with RLC electrical circuit. So convert Eq. (4.2) into fractional differential equation, as

$$LD^{\alpha}I(t) + RD^{\beta}I(t) + \frac{1}{C}I(t) = E(t), \tag{4.3}$$

where

$$D^{\alpha}I(t) = \frac{d^{\alpha}I}{dt^{\alpha}}$$

and

$$D^{\beta}I(t) = \frac{d^{\beta}I}{dt^{\beta}}, 1 < \alpha \le 2, 0 < \beta \le 1,$$

when

$$\lim_{\alpha\to 2} d\alpha \frac{d^{\alpha}I}{dt^{\alpha}} I dt \alpha = d\frac{d^2 I}{dt^2}$$

and

$$\lim_{\beta\to 1} d\alpha \frac{d^{\beta}I}{dt^{\beta}} I dt \alpha = d\frac{dI}{dt}$$

Applying Elzaki transform on Eq. (4.3) by assuming the initial condition $I(0) = A$ and $I'(0) = B$ and further using (2.10), we get

$$\begin{aligned} &E\{LD^{\alpha}I(t)\} + E\{RD^{\beta}I(t)\} + E\{\frac{1}{C}I(t)\} = E\{E(t)\}, \\ &\Rightarrow L\{v^{-\alpha}I(v) - v^{2-\alpha}I(0) - v^{3-\alpha}I'(0)\} \\ &\quad + R\{v^{-\beta}I(v) - v^{2-\beta}I(0)\} + \frac{1}{C}\{I(v)\} = E(v) \end{aligned} \tag{4.4}$$

Putting $I(0) = A$ and $I'(0) = B$, (4.4) reduces to

$$\begin{aligned} &\Rightarrow L\{v^{-\alpha}I(v) - v^{2-\alpha}A - v^{3-\alpha}B\} \\ &\quad + R\{v^{-\beta}I(v) - v^{2-\beta}A\} + \frac{1}{C}\{I(v)\} = E(v) \end{aligned}$$

$$\begin{aligned} &\Rightarrow \{Lv^{-\alpha} + Rv^{-\beta} + \frac{1}{C}\}I(v) \\ &\quad = E(v) + LAv^{2-\alpha} + LBv^{3-\alpha} + RAv^{2-\beta} \end{aligned}$$

$$\begin{aligned} \Rightarrow I(v) = {} & C\frac{E(v)}{\{LCv^{-\alpha} + RCv^{-\beta} + 1\}} \\ & + LAC\frac{v^{2-\alpha}}{\{LCv^{-\alpha} + RCv^{-\beta} + 1\}} \\ & + LBC\frac{v^{3-\alpha}}{\{LCv^{-\alpha} + RCv^{-\beta} + 1\}} \\ & + RAC\frac{v^{2-\beta}}{\{LCv^{-\alpha} + RCv^{-\beta} + 1\}} \end{aligned} \tag{4.5}$$

Some simplification will be done in equation (4.5) and then we will take the Inverse Elzaki transform of it, also Eq. (2.11) and (2.12) will get use, after that we will get following result

$$\begin{aligned} I(t) = {} & C\int_0^t E(t-\tau)\sum_{r=0}^{\infty}(-RC)^r(LC)^{-r-1}\tau^{(\alpha-\beta)r+\alpha-1} \\ & E^{r+1}_{\alpha,(\alpha-\beta)r+\alpha}(-\frac{1}{LC}\tau^{\alpha})d\tau \\ & + LAC\sum_{r=0}^{\infty}(-RC)^r(LC)^{-r-1}\tau^{(\alpha-\beta)r} \\ & E^{r+1}_{\alpha,(\alpha-\beta)r+1}(-\frac{1}{LC}\tau^{\alpha}) \\ & + LBC\sum_{r=0}^{\infty}(-RC)^r(LC)^{-r-1}\tau^{(\alpha-\beta)r+1} \\ & E^{r+1}_{\alpha,(\alpha-\beta)r+2}(-\frac{1}{LC}\tau^{\alpha}) \\ & + RAC\sum_{r=0}^{\infty}(-RC)^r(LC)^{-r-1}\tau^{(\alpha-\beta)(r+1)} \\ & E^{r+1}_{\alpha,(\alpha-\beta)(r+1)+1}(-\frac{1}{LC}\tau^{\alpha}) \end{aligned} \tag{4.6}$$

where $1 < \alpha \le 2$ and $0 < \beta \le 1$ also $\alpha - \beta > 0$. Eq. (4.6) is the required analytic solution of Eq. (4.3).

Special cases

Case I. When constant electromotive force is applied, i.e., $E(t) = E_0$, then (4.3) reduces into

$$L\frac{d^{\alpha}I}{dt^{\alpha}} + R\frac{d^{\beta}I}{dt^{\beta}} + \frac{1}{C}I(t) = E_0, \tag{4.7}$$

The technique we have applied to find out the analytic solution of Eq. (4.3), same technique we will apply to find out the solution of Eq. (4.7) and the analytic solution of (4.7) will be

$$I(t) = CE_0 \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)r+\alpha-1} E^{r+1}_{\alpha,(\alpha-\beta)r+\alpha+1}(-\frac{1}{LC}\tau^\alpha) + LAC \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)r} E^{r+1}_{\alpha,(\alpha-\beta)r+1}(-\frac{1}{LC}\tau^\alpha) + LBC \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)r+1} E^{r+1}_{\alpha,(\alpha-\beta)r+2}(-\frac{1}{LC}\tau^\alpha) + RAC \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)(r+1)} E^{r+1}_{\alpha,(\alpha-\beta)(r+1)+1}(-\frac{1}{LC}\tau^\alpha)$$

Case II. When periodic electromotive force is applied, i.e., $E(t) = E_0 cos\omega t$, where E_0 and ω are constants, then (4.3) yields,

$$L\frac{d^\alpha I}{dt^\alpha} + R\frac{d^\beta I}{dt^\beta} + \frac{1}{C}I(t) = E_0 cos\omega t, \tag{4.8}$$

The technique with which we have solved Eq. (4.3), same technique we will use to find out the solution of Eq. (4.8) and the analytic solution of (4.8) will be

$$I(t) = CE_0 \int_0^t cos\omega(t-\tau) \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)r+\alpha-1} E^{r+1}_{\alpha,(\alpha-\beta)r+\alpha}(-\frac{1}{LC}\tau^\alpha)d\tau + LAC \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)r} E^{r+1}_{\alpha,(\alpha-\beta)r+1}(-\frac{1}{LC}\tau^\alpha) + LBC \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)r+1} E^{r+1}_{\alpha,(\alpha-\beta)r+2}(-\frac{1}{LC}\tau^\alpha) + RAC \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)(r+1)} E^{r+1}_{\alpha,(\alpha-\beta)(r+1)+1}(-\frac{1}{LC}\tau^\alpha)$$

Case III. When periodic electromotive force is applied, i.e., $E(t) = E_0 sin\omega t$, where E_0 and ω are constants, then (4.3) yields,

$$L\frac{d^\alpha I}{dt^\alpha} + R\frac{d^\beta I}{dt^\beta} + \frac{1}{C}I(t) = E_0 sin\omega t, \tag{4.9}$$

On the same line as we got the analytic solution of Eq. (4.3), we will solve the Eq. (4.8) and the analytic solution of (4.9) will be

$$I(t) = CE_0 \int_0^t sin\omega(t-\tau) \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)r+\alpha-1} E^{r+1}_{\alpha,(\alpha-\beta)r+\alpha}(-\frac{1}{LC}\tau^\alpha)d\tau + LAC \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)r} E^{r+1}_{\alpha,(\alpha-\beta)r+1}(-\frac{1}{LC}\tau^\alpha) + LBC \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)r+1} E^{r+1}_{\alpha,(\alpha-\beta)r+2}(-\frac{1}{LC}\tau^\alpha) + RAC \sum_{r=0}^{\infty} (-RC)^r (LC)^{-r-1} \tau^{(\alpha-\beta)(r+1)} E^{r+1}_{\alpha,(\alpha-\beta)(r+1)+1}(-\frac{1}{LC}\tau^\alpha)$$

Case IV. When we take $\beta = 1$ in (4.3), it reduces to the form,

$$L\frac{d^\alpha I}{dt^\alpha} + R\frac{dI}{dt} + \frac{1}{C}I(t) = E_0, \quad 1 < \alpha \leq 2 \tag{4.10}$$

We will get the solution of Eq. (4.10), just as we got the solution of Eq. (4.3).

Case V. When we take $\alpha = 2$ in (4.3), it reduces to the form,

$$L\frac{d^2 I}{dt^2} + R\frac{d^\beta I}{dt^\beta} + \frac{1}{C}I(t) = E_0, \quad 0 < \beta \leq 1 \tag{4.11}$$

Solution of Eq. (4.11), can also be obtained as we got the solution of Eq. (4.3).

Case VI. When we take $\alpha = 2, \beta = 1$ and $E(t) = 0$ in Eq. (4.3), this reduces to second order homogeneous linear ordinary differential equation, as

$$L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I(t) = 0, \tag{4.12}$$

Solution of Eq. (4.12) can also be obtained by similar manners as we got the solution of Eq. (4.3).

Now we are presenting an example, to demonstrate how applicable the Elzaki transform in solving electrical circuit problems.

Example: An inductor of 2 henrys, a resistor of 16 ohms and a capacitor of 0.02 farads are connected in series with an e.m.f. of E volts. At $t = 0$ the charge on the capacitor and current in the circuit are zero. Find the charge and current at any time $t > 0$ If $(a) E = 300$(Volts) $(b) E = 100\sin 3t$ (Volts).

Solution: Let Q and I be the instantaneous charge and current respectively at time t. By Kirchhoff's laws, i.e. by Eq. (2.2), we have

$$L\frac{d^2}{dt^2}Q(t) + R\frac{d}{dt}Q(t) + \frac{Q(t)}{C} = E(t), \tag{4.13}$$

On putting the value of $L=2, R=16, C=0.02$ in (4.13)

$$2\frac{d^2}{dt^2}Q(t)+16\frac{d}{dt}Q(t)+50Q(t)=E(t), \tag{4.14}$$

The given initial conditions are $Q(0)=0, I(0)=Q'(0)=0$.

(a) If $E=300$(Volts), then (4.14) becomes

$$\frac{d^2}{dt^2}Q(t)+8\frac{d}{dt}Q(t)+25Q(t)=150, \tag{4.15}$$

Applying Elzaki transform on Eq. (4.15) and on using the various results mentioned in [10, 11, 14, 15], we get

$$\frac{E[Q(t)]}{v^2}-Q(0)-vQ'(0) \\ +8\Big[\frac{E[Q(t)]}{v}-vQ(0)\Big]+25E[Q(t)]=150v^2,$$

Taking $E[Q(t)]=T(v)$ and using given initial conditions, we get

$$\Rightarrow\frac{T(v)}{v^2}+8\frac{T(v)}{v}+25T(v)=150v^2,$$
$$\Rightarrow\Big(\frac{1}{v^2}+8\frac{1}{v}+25\Big)T(v)=150v^2,$$
$$\Rightarrow T(v)=\frac{150v^4}{25v^2+8v+1},$$
$$\Rightarrow T(v)=6v^2-\frac{48v^3}{25v^2+8v+1}-\frac{6v^2}{25v^2+8v+1},$$
$$\Rightarrow T(v)=6v^2-\frac{6[4v^3+4v^3+v^2]}{25v^2+8v+1}$$
$$\Rightarrow T(v)=6v^2-\frac{6[v^2(1+4v)+4v^3]}{25v^2+8v+1}$$
$$\Rightarrow T(v)=6v^2-6\frac{(1+4v)v^2}{(1+4v)^2+3^2v^2}-8\frac{3v^3}{(1+4v)^2+3^2v^2} \tag{4.16}$$

Taking inverse Elzaki transform on both sides of Eq.(4.16), get

$$Q(t)=6-6e^{-4t}cos3t-8e^{-4t}sin3t. \tag{4.17}$$

Since $I(t)=\frac{d}{dt}Q(t)$, so on differentiate (4.17) w.r.t. t, we get

$$I(t)=50e^{-4t}sin3t.$$

(b) If $E=100sin3t$ (Volts), then (4.14) becomes

$$\frac{d^2}{dt^2}Q(t)+8\frac{d}{dt}Q(t)+25Q(t)=50\sin 3t, \tag{4.18}$$

The technique with which we have solved Eq. (4.15), same technique we will used to find out the solution of Eq. (4.18), and we get

$$Q(t)=\frac{25}{26}sin3t-\frac{75}{52}cos3t+\frac{25}{26}e^{-4t}sin3t+\frac{75}{52}e^{-4t}cos3t \\ =\frac{25}{52}(2sin3t-3cos3t)+\frac{25}{52}e^{-4t}(2sin3t+3e^{-4t}cos3t). \tag{4.19}$$

On differentiating (4.19) w.r.t. t, we get

$$I(t)=\frac{d}{dt}Q(t)=\frac{75}{52}(2cos3t+3sin3t) \\ -\frac{25}{52}e^{-4t}(17sin3t+6cos3t).$$

For large t, those terms of $Q(t)$ or $I(t)$ which involve e^{-4t} are negligible.

5. Conclusion

In the present paper, we have obtained the analytical solution of a non-integer order differential equation which is associated with a RLC electrical circuit via Elzaki transform method. The analytic solution obtained in terms of Mittag-Leffler function which is useful for computational study of current. We can apply this same methodology in the analysis of electromagnetic transients' problems in electrical systems, machine windings and in power electronics.

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