Malaya Journal of Matematik, Vol. 8, No. 2, 450-453, 2020

https://doi.org/10.26637/MJM0802/0020

On some strong forms of $\psi\omega$-continuity

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Abstract

In this paper is to introduce a new class of functions called $\psi\omega$-irresolute functions, strongly $\psi\omega$-continuous functions and perfectly $\psi\omega$-continuous functions in topological spaces and study some of their properties and relations among them.

Keywords

$\psi\omega$-irresolute functions, strongly $\psi\omega$-continuous functions and perfectly $\psi\omega$-continuous functions.

AMS Subject Classification

54A05, 54A10, 54C08, 54C10.

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Article History: Received **23** November **2019**; Accepted **17** February **2020**

Contents

1. Introduction and Preliminaries

N. Levine [9] introduced generalized closed (briefly g-closed) sets and studied their basic properties. P. Sundaram and M. Sheik John [16] was introduced ω-closed sets in topological spaces. Recently I have introduced and investigated $\psi\omega$-closed sets in a topological spaces.

In this paper is to introduce a new class of functions called $\psi\omega$-irresolute functions, strongly $\psi\omega$-continuous functions and perfectly $\psi\omega$-continuous functions in topological space and study some of their properties and relations among them.

Throughout this paper (X,τ) and (Y,σ) and (Z,η) represent non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

We recollect some notations and definitions which are used in this paper.

Definition 1.1. *A subset A of a topological space (X,τ) is called a*

1. *semi open [8] if $A \subseteq cl(int(A))$.*
2. *α-open [11] if $A \subseteq int(cl(int(A)))$.*
3. *regular open [13] if $A \subseteq int(cl(A))$.*
4. *pre open [10] if $A \subseteq int(cl(A))$.*

The complements of the above mentioned open sets are called their respective closed sets.

Definition 1.2. *A subset A of a topological space (X,τ) is called a*

1. *generalized closed (briefly g-closed) set [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.*
2. *semi-generalized closed (briefly sg-closed) set [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.*
3. *generalized semi-closed (briefly gs-closed) set [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.*
4. *ω-closed set [16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.*
5. *ψ-closed set [12] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open.*
6. *ψ-generalized closed set (briefly ψg-closed) [12] if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.*
7. *a $\psi\omega$-closed set [6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ω-open.*

The complements of the above mentioned closed sets are called their respective open sets.

Definition 1.3. *A function $f : (X,\tau) \to (Y,\sigma)$ is called*

1. *semi-continuous [8] if $f^{-1}(V)$ is open in (X,τ) for every open set V in (Y,σ).*
2. *g-continuous [2] if $f^{-1}(V)$ is g-open in (X,τ) for every open set V in (Y,σ).*
3. *sg-continuous [14] if $f^{-1}(V)$ is sg-open in (X,τ) for every open set V in (Y,σ).*
4. *gs-continuous [15] if $f^{-1}(V)$ is gs-open in (X,τ) for every open set V in (Y,σ).*
5. *ω-continuous [16] if $f^{-1}(V)$ is ω-open in (X,τ) for every open set V in (Y,σ).*
6. *ψ-continuous [12] if $f^{-1}(V)$ is ψ-open in (X,τ) for every open set V in (Y,σ).*
7. *ψg-continuous [12] if $f^{-1}(V)$ is ψg-open in (X,τ) for every open set V in (Y,σ).*
8. *$\psi\omega$-continuous [7] if $f^{-1}(V)$ is $\psi\omega$-open in (X,τ) for every open set V in (Y,σ).*

2. $\psi\omega$-irresolute functions

Definition 2.1. *A function $f:X\to Y$ from a topological space (X,τ) into a topological space (Y,σ) is called $\psi\omega$-irresolute if the inverse image of every $\psi\omega$-closed set in Y is $\psi\omega$-closed set in X.*

Theorem 2.2. *A function $f:X\to Y$ is $\psi\omega$-irresolute $\iff$ the inverse image of each $\psi\omega$-open in Y is $\psi\omega$-open in X.*

Proof. Assume that f is $\psi\omega$-irresolute. Let A be any $\psi\omega$-open set in Y. Then A^c is $\psi\omega$-closed in Y. Since f is $\psi\omega$-irresolute, $f^{-1}(A^c)$is $\psi\omega$-closed in X. But $f^{-1}(A^c)=X-f^{-1}(A)$ and so $f^{-1}(A)$ is $\psi\omega$-open in X. Hence the inverse image of every $\psi\omega$-open in Y is $\psi\omega$-open in X.

Conversely, suppose that the inverse image of every $\psi\omega$-open set in Y is $\psi\omega$-open in X. Let A be any $\psi\omega$-closed in Y. Then A^c $\psi\omega$-open in Y. By assumption, $f^{-1}(A^c)$ is $\psi\omega$-open in X. But $f^{-1}(A^c)=X-f^{-1}(A)$ and so $f^{-1}(A)$ is $\psi\omega$-closed in X. Therefore f is $\psi\omega$-irresolute.

Theorem 2.3. *A function $f:X\to Y$ is $\psi\omega$-irresolute $\iff$ it is $\psi\omega$-continuous.*

Proof. Assume that f is $\psi\omega$-irresolute. Let F be any closed set in Y. By Theorem 2.2, each closed set is $\psi\omega$-closed, F is $\psi\omega$-closed in Y. Since f is $\psi\omega$-irresolute, $f^{-1}(F)$ is $\psi\omega$-closed in X. Therefore f is $\psi\omega$-continuous.

Conversely, assume that f is $\psi\omega$-continuous. Let F be any closed set in Y. By Theorem 2.2, each closed set is $\psi\omega$-closed, F is $\psi\omega$-closed in Y. Since f is $\psi\omega$-continuous, $f^{-1}(F)$ is $\psi\omega$-closed in X. Therefore f is $\psi\omega$-irresolute.

Theorem 2.4. *Let X,Y and Z be any topological spaces. For any $\psi\omega$-irresolute map $f:X\to Y$ and any $\psi\omega$-continuous map $g:Y\to Z$, the composition $g\circ f:X\to Z$ is $\psi\omega$-continuous.*

Proof. Let F be any closed set in Z. Since g is $\psi\omega$-continuous, $g^{-1}(F)$ is $\psi\omega$-closed in Y. Since f is $\psi\omega$-irresolute, $f^{-1}(g^{-1}(F))$ is $\psi\omega$-closed in X. But $f^{-1}(g^{-1}(F))=(g\circ f)^{-1}(F)$. Therefore $g\circ f$ is $\psi\omega$-continuous.

3. Strongly $\psi\omega$-continuous and Perfectly $\psi\omega$-continuous

Definition 3.1. *A function $f:X\to Y$ from a topological space (X,τ) into a topological space (Y,σ) is said to be strongly $\psi\omega$-continuous if the inverse image of every $\psi\omega$-open set in Y is open set in X.*

Theorem 3.2. *If a function $f:X\to Y$ from a topological space (X,τ) into a topological space (Y,σ) is strongly $\psi\omega$-continuous, then it is $\psi\omega$-continuous.*

Proof. Assume that f is strongly $\psi\omega$-continuous. Let G be any closed set in Y. By Theorem 2.2, each closed set is $\psi\omega$-closed in Y,G is $\psi\omega$-closed in Y. Since f is strongly $\psi\omega$-continuous, $f^{-1}(G)$ is closed in X. Therefore f is $\psi\omega$-continuous.

Remark 3.3. *The converse of Theorem 3.2, is need not be true as seen from the following Example.*

Example 3.4. *Let $X=Y=\{a,b,c\}$ with $\tau=\{\phi,X,\{c\}\}$ and $\sigma=\{\phi,X,\{c\},\{a,b\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be an identity map. Then f is $\psi\omega$-continuous. But f is not strongly $\psi\omega$-continuous, since for the $\psi\omega$-closed set $A=\{c\}$ in Y, $f^{-1}(A)=\{c\}$ is not closed in X.*

Theorem 3.5. *A function $f:X\to Y$ from a topological space (X,τ) into a topological space (Y,σ) strongly $\psi\omega$-continuous $\iff$ the inverse image of every $\psi\omega$-closed set in Y is closed in X.*

Proof. Assume that f is strongly $\psi\omega$-continuous. Let F be any $\psi\omega$-closed set in Y. Then F^c is $\psi\omega$-open in Y. Since f is strongly $\psi\omega$-continuous, $f^{-1}(F^c)$ is open in X. But $f^{-1}(F^c)=X-f^{-1}(F)$ and $f^{-1}(F)$ is closed in X.

Conversely assume that the inverse image of every $\psi\omega$-closed set in Y is closed in X. Let G be any $\psi\omega$-open set in Y. Then G^c is $\psi\omega$-closed set in Y. By assumption, $f^{-1}(G^c)$-closed in X. But $f^{-1}(G^c)=X-f^{-1}(G)$ and so $f^{-1}(G)$ is open in X. Therefore f is strongly $\psi\omega$-continuous.

Theorem 3.6. *If a function $f:X\to Y$ is strongly $\psi\omega$continuous and a map $g:Y\to Z$ is $\psi\omega$-continuous, then the composition $g\circ f:X\to Z$ is strongly $\psi\omega$-continuous.*

Proof. Let G be any closed set in Z. Since g is $\psi\omega$-continuous, $g^{-1}(G)$ is $\psi\omega$-closed in Y. Since f is strongly $\psi\omega$-continuous $f^{-1}(g^{-1}(G))$ is closed in X. But $(g\circ f)^{-1}(G)=f^{-1}(g^{-1}(G))$. Therefore $g\circ f$ is strongly $\psi\omega$-continuous.

Theorem 3.7. *If a function $f:X\to Y$ is strongly $\psi\omega$-continuous and a map $g:Y\to Z$ is $\psi\omega$-continuous, then the composition $g\circ f:X\to Z$ is $\psi\omega$-continuous.*

Proof. Let G be any closed set in Z. Since g is $\psi\omega$-continuous, $g^{-1}(G)$ is $\psi\omega$-closed in Y. Since f is strongly $\psi\omega$-continuous $f^{-1}(g^{-1}(G))$ is closed in X. By Theorem 2.2, each closed set is $\psi\omega$-closed, $f^{-1}(g^{-1}(G))$ is $\psi\omega$-closed. But $f^{-1}(g^{-1}(G)) = (g\circ f)^{-1}(G)$. Therefore $g\circ f$ is $\psi\omega$-continuous.

Theorem 3.8. *If a function $f : X \to Y$ from a topological spaces (X,τ) into a topological spaces (Y,σ) is continuous then it is strongly $\psi\omega$-continuous but not conversely.*

Proof. Let $f : X \to Y$ be continuous. Let F be a closed set in Y. Since f is continuous, $f^{-1}(F)$ is closed in X. By Theorem 2.2, each closed set is $\psi\omega$-closed, $f^{-1}(F)$ is $\psi\omega$-closed. Hence f is $\psi\omega$-closed.

Remark 3.9. *The converse of Theorem 3.8 is need not be true as seen from the following Example.*

Example 3.10. *Let $X = Y = \{a,b,c\}$ with $\tau = \{X,\phi,\{a\},\{b,c\}\}$ and $\sigma = \{X,\phi,\{a\},\{c\},\{a,c\}\}$. Let $f : (X,\tau) \to (Y,\sigma)$ be an identity map. Then f is strongly $\psi\omega$-continuous. But f is not continuous, since for the $\psi\omega$-closed set $A = \{b\}$ in $Y, f^{-1}(A) = \{b\}$ is not closed in X.*

Definition 3.11. *A function $f : X \to Y$ from a topological space (X,τ) into a topological space (Y,σ) is said to be perfectly $\psi\omega$-continuous if the inverse image of every $\psi\omega$-closed set in Y is both open and closed in X.*

Theorem 3.12. *If a function $f : X \to Y$ from a topological space (X,τ) into a topological space (Y,σ) is perfectly $\psi\omega$-continuous then it is strongly $\psi\omega$-continuous.*

Proof. Assume that f is perfectly $\psi\omega$-continuous. Let G be any $\psi\omega$-closed set in Y. Since f is perfectly $\psi\omega$-continuous, $f^{-1}(G)$ is closed in X. Therefore f is strongly $\psi\omega$-continuous.

Remark 3.13. *The converse of Theorem 3.12 is need not be true as seen from the following Example.*

Example 3.14. *Let $X = Y = \{a,b,c\}$ with $\tau = \{X,\phi,\{a\}\}$ and $\sigma = \{X,\phi,\{a\},\{b\},$
$\{a,b\}\}$. Let $f : (X,\tau) \to (Y,\sigma)$ be an identity map. Then f is strongly $\psi\omega$-continuous. But f is not perfectly $\psi\omega$-continuous, since for the $\psi\omega$-closed set $A = \{b,c\}$ in $Y, f^{-1}(A) = \{b,c\}$ is not closed in X.*

Theorem 3.15. *A function $f : X \to Y$ from a topological space (X,τ) into a topological space (Y,σ) is perfectly $\psi\omega$-continuous $\iff$ the inverse image of every $\psi\omega$-closed set in Y is both open and closed in X.*

Proof. Assume that f is perfectly $\psi\omega$-continuous. Let F be any $\psi\omega$-closed set in Y. Then F^c is $\psi\omega$-open in Y. Since f is perfectly $\psi\omega$-continuous, $f^{-1}(F^c)$ is both open and closed in X. But $f^{-1}(F^c) = X - F^{-1}(F^c)$ and so $f^{-1}(F)$ is both open and closed in X.

Conversely assume that the inverse image of every $\psi\omega$-closed set in Y is both open and closed in X. Let G be any $\psi\omega$-open in Y. By assumption $f^{-1}(G^c) = X - f^{-1}(G)$ and so $f^{-1}(G)$ is both open and closed in X. Therefore f is perfectly $\psi\omega$-continuous.

Theorem 3.16. *If a function $f : X \to Y$ from a topological space (X,τ) into a topological space (Y,σ) is strongly $\psi\omega$-continuous then it is $\psi\omega$-irresolute.*

Proof. Let $f : X \to Y$ be strongly $\psi\omega$-continuous function. Let F be a $\psi\omega$-closed in Y. Since f is strongly $\psi\omega$-continuous, $f^{-1}(F)$ is closed in X. Each closed set is $\psi\omega$-closed, $f^{-1}(F)$ is $\psi\omega$-closed in X. Hence f is $\psi\omega$-irresolute.

Remark 3.17. *The converse of the Theorem 3.16, is need not be true as seen from the following Example.*

Example 3.18. *Let $X = Y = \{a,b,c\}$ with $\tau = \{X,\phi,\{c\}\}$ and $\sigma = \{X,\phi,\{a\},\{b\},$
$\{a,b\}\}$. Let $f : (X,\tau) \to (Y,\sigma)$ be an identity map. Then f is $\psi\omega$-irresolute. But f is not strongly $\psi\omega$- continuous, since for the $\psi\omega$-closed set $A = \{c\}$ in $Y, f^{-1}(A) = \{c\}$ is not closed in X.*

Theorem 3.19. *If a function $f : X \to Y$ from a topological space (X,τ) into a topological space (Y,σ) is perfectly $\psi\omega$-continuous, then it is $\psi\omega$-irresolute.*

Proof. Let $f : X \to Y$ be perfectly $\psi\omega$-continuous function. Let F be a $\psi\omega$-closed set in Y. Since f is perfectly $\psi\omega$-continuous, $f^{-1}(F)$ is both closed set is both open and closed in X. Then $f^{-1}(F)$ is $\psi\omega$-closed in X. Hence f is $\psi\omega$-irresolute.

Remark 3.20. *The converse of Theorem 3.19, is need not be true as seen from the following Example.*

Example 3.21. *Let $X = Y = \{a,b,c\}$ with $\tau = \{X,\phi,\{a\},\{b\},$ $\{a,b\}\}$ and $\sigma = \{X,\phi,\{b\},\{a,b\},\{b,c\}\}$. Let $f : (X,\tau) \to (Y,\sigma)$ be defined by $f(a) = \{b\}, f(b) = \{c\}, f(c) = \{a\}$. Then f is $\psi\omega$-irresolute. But f is not perfectly $\psi\omega$-continuous, since for the $\psi\omega$-closed set $A = \{c\}$ in $Y, f^{-1}(A) = \{c\}$ is open in X but not closed in X.*

4. Conclusion

One must be in ”love” with Mathematics is the intrinsic nature and beauty of Mathematics. As a result, the nature of inquisitiveness in a person gets always enkindled and triggered by new theorems, axioms, even if it is mighty small in its nature or incredibly big.

General topology is applied to many fields such as Mathematics, Physics, Chemistry, Biology, Engineering and so on. This theory is definitely an eye opener for new research works. We can apply these findings into other research areas of general topology such as Fuzzy topology, intuitionistic topology, digital topology, nano topology and so on.

Acknowledgment

I owe my gratitude and thanks to the referees for their valuable suggestions for devolvement of this article.

References

[1] S. P. Arya and T. M. Nour, Characterizations of *s*-normal spaces, *Indian J. Pure. Appl. Math.*, 21(8)(1990), 717–719.

[2] K. Balachandran, P. Sundaram and H. Maki, On generalized continuous maps topological spaces, *Mem. Fac. Sci. Kochi Univ. (Math.)* 12(1991), 5–13.

[3] P. Bhattacharrya and B. K. Lahiri, Semi-generalized closed sets in topology, *Indian J. Math.*, 29(3)(1987), 375–382.

[4] J. Dontchev, On generalizing semi-preopen sets, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math.*, 16(1995), 35–48.

[5] M. Ganster, S. Jafari and G. B. Navalagi, Semi-*g*-regular and semi-*g*-normal spaces, *Demonstratio Math.*, 35(2)(2002), 415–421.

[6] P. Jeyalakshmi, $\psi\omega$-Closed sets and it's properties in topology, *International Journal of Analytical and Experimental Modal Analysis*, 11(10)(2019), 648–654.

[7] P. Jeyalakshmi, *On $\psi\omega$-continuous maps in topological spaces*, (to appear).

[8] N. Levine, Semi open sets and semi-continuity in topological spaces, *Amer. Math. Monthly.*, 70(1963), 36–41.

[9] N. Levine, Generalized closed sets in topological spaces, *Rend. Circ. Math. Palermo.*, 19(2)(1970), 89–96.

[10] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, On precontinuous and weak precontinuous mappings, *Proc. Math. Phys. Soc. Egypt*, 53 (1982), 47–53.

[11] O. Njastad, On some classes of nearly open sets, *Pacific J.Math.*, 15(1965), 961–970.

[12] T. Noiri, On ψ-continuous functions, *Journal of the Korean Mathematical Society*, 16(2)(1981), 167–176.

[13] M. H. Stone, Application of the theory of Boolean rings to general topology, *Trans. Amer. Math. Soc.*, 41(1937), 374–381.

[14] P. Sundarm, H. Maki and K. Balachandran, *sg*-closed sets and semi-$\tau\frac{1}{2}$ spaces, *Bull. Fukuoka. Univ. Ed. Part*, 3(40)(1991), 33–40.

[15] P. Sundaram, H. Maki and K. Balachandran, *gs*-closed sets and semi-$\tau\frac{1}{2}$ spaces, *Univ. Ser.A (Maths)*, 14(1993), 41–54.

[16] P. Sundaram and M. Sheik John, On ω-closed sets in topology, *Acta Ciencia Indica*, 4(2000), 389–392.

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ISSN(P):2319 − 3786

Malaya Journal of Matematik

ISSN(O):2321 − 5666

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