Malaya Journal of Matematik, Vol. 8, No. 2, 454-458, 2020

https://doi.org/10.26637/MJM0802/0021

Coefficient bounds for a class of univalent functions involving the modified Sigmoid function

Olubunmi A. Fadipe-Joseph[1]*, O. J. Windare[2] and Esther O. Davids[3]

Abstract

New class of univalent functions which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ using the modified Salagean operator involving the modified sigmoid function was defined. Coefficient bounds, the Fekete-Szego functional and some consequences of the results obtained were established.

Keywords

Analytic function, Sigmoid function, Coefficient bounds, Sălăgean differential operator, Fekete-Szego functional.

AMS Subject Classification

30C45.

[1,2,3] *Department of Mathematics, University of Ilorin, P.M.B.1515, Ilorin, Nigeria.*

***Corresponding author**: [1] famelov@unilorin.edu.ng;famelov@gmail.com [2]windare14@gmail.com [3]yemisioduwole1@gmail.com; davids.esther@lmu.edu.ng

Article History: Received **24** March **2017**; Accepted **09** September **2017**

Contents

1. Introduction

Let A be the class of functions $f(z)$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ satisfying the conditions $f(0) = 0$ and $f'(0) = 1$. Furthermore, let S denote the family of all functions in A which are univalent in $\mathbb{U}$.

For two functions f and g analytic in $\mathbb{U}$, we say that the function $f(z)$ is subordinate to $g(z)$ in $\mathbb{U}$ and write

$$f(z) \prec g(z)$$

$(z \in \mathbb{U})$ if there exists a Schwartz function $w(z)$ analytic in $\mathbb{U}$ with $w(0) = 0$ and $|w(z)| < 1$ $(z \in \mathbb{U})$ such that

$$f(z) = g(w(z))$$

In particular, if the function g is univalent in $\mathbb{U}$, the above subordination is equivalent to $f(0) = g(0)$ and $f(\mathbb{U}) \subset g(\mathbb{U})$, [4].

Sălăgean [9] introduced the differential operator $D^n f, n \in N_0$ for functions $f(z)$ belonging to class $\mathbf{A}$ of analytic functions in the unit disk $\mathbb{U}$;

$$D^n f(z) = z + \sum_{k=2}^{\infty} k^n a_k z^k; \quad n \in N_0$$

Fadipe-Joseph et al. [2] studied the modified sigmoid function

$$G(z) = \frac{2}{1+e^{-z}}$$

and obtained a series form of the modified sigmoid function as

$$G(z) = 1 + \left(\sum_{m=1}^{\infty} \frac{(-1)^m}{2^m} \left(\frac{(-1)^n}{n!} z^n \right)^m \right)$$

The following classes of functions and many others are well known and have been studied repeatedly by many authors, namely, Salagean, Abdul Falim and Darus to mention but a few.

i $S_0 = \{f \in A : Re(f(z)/z) > 0, \ z \in \mathbb{U}\}$

ii $B(\alpha) = \{f \in A : Re(f(z)/z) > \alpha, \ 0 \le \alpha \le 1 \ z \in \mathbb{U}\}$

iii $\delta(\alpha) = \{f \in A : Re(f'(z)) > \alpha, \ 0 \le \alpha < 1 \ z \in \mathbb{U}\}$

iv $B_n(\beta) = \{f \in A : Re(D^n f(z)^\beta / z^\beta) > 0, \ n \in N_0 \ \beta > 0 z \in \mathbb{U}\}$

Khalifa Al-Shaqsi et al. [5] established the family of harmonic functions such that

$$Re\left(\frac{D^{n+1}f(z)^{\beta}}{D^{n}f(z)^{\beta}}\right) > \alpha$$

$\beta \geq 1, 0 \leq \alpha < 1$

Fadipe-Joseph et al. [3] defined the Salagean differential operator involving modified sigmoid function D_{γ}^{n} as

$$D^{n}f_{\gamma}(z) = \gamma^{n}(s)z + \sum_{k=2}^{\infty}\gamma^{n+1}(s)k^{n}a_{k}z^{k}, \tag{1.2}$$

where

$$\gamma(s) = \frac{2}{1+e^{-s}} = 1 + \frac{1}{2}s - \frac{1}{24}s^{3} + \frac{1}{240}s^{5}\ldots$$

and

$$f_{\gamma}(z) = z + \sum_{k=2}^{\infty}\gamma(s)a_{k}z^{k} \tag{1.3}$$

Remark: The function f_{γ} which is analytic and univalent in the unit disk belongs to the class A_{γ}. If $\gamma = 1$, $A_{\gamma} = A_{1} = A$.

Ramachandran and Dhanalakshmi [8] investigated coefficient estimates for a class of spirallike function in the space of sigmoid function and estimated the relevant connection to the Fekete-Szego inequality of functions belonging to the class.

More results can be found in Altinkaya and Ȯzkan [1], Mini and Keerthi [6] , Murugusundaramoorthy and Janani [7], Singh and Singh [10] and Wang and Wang [11] to mention just but a few.

Definition 1.1. *A function $f_{\gamma} \in A_{\gamma}$ is said to be in the class $S_{\gamma}(n,\lambda,\beta)$, $0 \leq \lambda \leq 1$, $\beta > 0$ and $n \in N_{0}$ if the following subordination holds*

$$\left(\frac{D^{n+1}f_{\gamma}(z)^{\beta}}{D^{n}f_{\gamma}(z)^{\beta}}\right)^{1-\lambda}\left(\frac{D^{n+2}f_{\gamma}(z)^{\beta}}{D^{n+1}f_{\gamma}(z)^{\beta}}\right)^{\lambda} \prec G(z) \tag{1.4}$$

$G(z)$ is the modified sigmoid function

2. Main Results

Theorem 2.1. *Let $f_{\gamma} \in A_{\gamma}$ be in $S_{\gamma}(n,\lambda,\beta)$, $0 \leq \lambda \leq 1$, $\beta > 0$ and $n \in N_{0}$, then*

$$|a_{\beta+1}| \leq \frac{\beta^{n+1}}{2(\lambda+\beta)(\beta+1)^{n}\gamma^{2}(s)} \tag{2.1}$$

$$|a_{\beta+2}| \leq \frac{\beta^{n}}{4(2\lambda+\beta)(\beta+2)^{n}}\left[\frac{\beta}{\gamma^{2}(s)} + \frac{\gamma(s)A_{1}}{4(\lambda+\beta)^{2}}\right] \tag{2.2}$$

$$|a_{\beta+3}| \leq \frac{\beta^{n}}{8(3\lambda+\beta)(\beta+3)^{n}}\left|\frac{4\beta}{3\gamma^{2}(s)} + \frac{A_{4}}{36\beta^{2}\gamma^{2}(s)(\lambda+\beta)^{3}(2\lambda+\beta)} + \frac{A_{3}\beta}{\gamma(s)(\lambda+\beta)(2\lambda+\beta)}\right| \tag{2.3}$$

where,

$$A_{1} = -\lambda^{2} + \lambda + 2\lambda\beta + 4\lambda\lambda^{2} + 2\beta^{3}$$

$$A_{2} = \lambda^{3} + 12\lambda^{2}\beta^{4} + 36\lambda^{2}\beta^{3} + 24\lambda^{2}\beta^{2} + 3\lambda^{2}\beta - 3\lambda^{2} + 6\lambda\beta^{4}$$
$$-18\lambda\beta^{3} - 18\lambda\beta^{2} - 3\lambda\beta + 2\lambda + 6\beta^{5}$$

$$A_{3} = 2\lambda^{2}\beta^{2} + 2\lambda^{2}\beta^{2} - 2\lambda - 11\lambda\beta^{2} - 6\lambda\beta - 3\beta^{3}$$

$$A_{4} = 4\beta^{2}\gamma^{2}(s)(\lambda+\beta)^{3}(2\lambda+\beta) + 2A_{2}(2\lambda+\beta) + 9\lambda^{4}(s)A_{1}A_{3}\beta^{2}.$$

Proof.

$$D^{n}f_{\gamma}(z)^{\beta} = \gamma^{n}(s)\beta^{n}z^{\beta} + \sum_{k=\beta+1}^{\infty}\gamma^{n+1}(s)k^{n}a_{k}z^{k}$$

$$D^{n+1}f_{\gamma}(z)^{\beta} = \gamma^{n+1}(s)\beta^{n+1}z^{\beta} + \sum_{k=\beta+1}^{\infty}\gamma^{n+2}(s)k^{n+1}a_{k}z^{k}$$

$$D^{n+2}f_{\gamma}(z)^{\beta} = \gamma^{n+2}(s)\beta^{n+2}z^{\beta} + \sum_{k=\beta+1}^{\infty}\gamma^{n+3}(s)k^{n+2}a_{k}z^{k}$$

$$\frac{D^{n+1}f_\gamma(z)^\beta}{D^n f_\gamma(z)^\beta} = \frac{\gamma^{n+1}(s)\beta^{n+1}z^\beta + \sum_{k=\beta+1}^{\infty}\gamma^{n+2}(s)k^{n+1}a_k z^k}{\gamma^n(s)\beta^n z^\beta + \sum_{k=\beta+1}^{\infty}\gamma^{n+1}(s)k^n a_k z^k}$$

$$\frac{D^{n+1}f_\gamma(z)^\beta}{D^n f_\gamma(z)^\beta} = \gamma(s)\beta\left[\begin{array}{c} 1+\frac{\gamma(s)(\beta+1)^n}{\beta^{n+1}}a_{\beta+1}z+\left(\frac{2\gamma(s)(\beta+2)^n}{\beta^{n+1}}a_{\beta+2}-\frac{\gamma^2(s)(\beta+1)^{2n}}{\beta^{2n+1}}a_{\beta+1}^2\right)z^2 \\ +\left(\frac{3\gamma(s)(\beta+3)^n}{\beta^n}a_{\beta+3}-\frac{3\gamma^2(s)(\beta+1)^n(\beta+2)^n}{\beta^{2n+1}}a_{\beta+1}a_{\beta+2}+\frac{\gamma^3(s)(\beta+1)^{3n}}{\beta^{3n+1}}a_{\beta+1}^3\right)z^3\ldots \end{array}\right]$$

Similarly,

$$\frac{D^{n+2}f_\gamma(z)^\beta}{D^{n+1}f_\gamma(z)^\beta} = \frac{\gamma^{n+2}(s)\beta^{n+2}z^\beta + \sum_{k=\beta+1}^{\infty}\gamma^{n+3}(s)k^{n+2}a_k z^k}{\gamma^{n+1}(s)\beta^{n+1}z^\beta + \sum_{k=\beta+1}^{\infty}\gamma^{n+2}(s)k^{n+1}a_k z^k} = \gamma(s)\beta\,\frac{1+\sum_{k=\beta+1}^{\infty}\gamma(s)\left(\frac{k}{\beta}\right)^{n+2}a_k z^{k-\beta}}{1+\sum_{k=\beta+1}^{\infty}\gamma(s)\left(\frac{k}{\beta}\right)^{n+1}a_k z^{k-\beta}}$$

$$\frac{D^{n+2}f_\gamma(z)^\beta}{D^{n+1}f_\gamma(z)^\beta} = \gamma(s)\beta\left[\begin{array}{c} 1+\frac{\gamma(s)(\beta+1)^{n+1}}{\beta^{n+2}}a_{\beta+1}z+\left(\frac{2\gamma(s)(\beta+2)^{n+1}}{\beta^{n+2}}a_{\beta+2}-\frac{\gamma^2(s)(\beta+1)^{2n+2}}{\beta^{2n+3}}a_{\beta+1}^2\right)z^2 \\ +\left(\frac{3\gamma(s)(\beta+3)^{n+1}}{\beta^{n+2}}a_{\beta+3}-\frac{3\gamma^2(s)(\beta+1)^{n+1}(\beta+2)^{n+1}}{\beta^{2n+2}}a_{\beta+1}a_{\beta+2}+\frac{\gamma^3(s)(\beta+1)^{3n+3}}{\beta^{3n+4}}a_{\beta+1}^3\right)z^3\ldots \end{array}\right]$$

$$\left(\frac{D^{n+1}f_\gamma(z)^\beta}{D^n f_\gamma(z)^\beta}\right)^{1-\lambda} = (\gamma(s)\beta)^{1-\lambda}\left[\begin{array}{c} 1+\frac{(1-\lambda)(\beta+1)^n\gamma(s)}{\beta^{n+1}}a_{\beta+1}z+ \\ \left(\frac{2\gamma(s)(1-\lambda)(\beta+2)^n}{\beta^{n+1}}a_{\beta+2}-\frac{\gamma^2(s)(1-\lambda)(\beta+1)^{2n}(\lambda+2\beta)}{2\beta^{2+2n}}a^2{}_{\beta+1}\right)z^2+ \\ \left(\frac{3\gamma(s)(1-\lambda)(\beta+3)^n}{\beta^{n+1}}a_{\beta+3}+\frac{\gamma^3(s)(1-\lambda)(\beta+1)^{3n}(\lambda(\lambda+1)+6\beta(\lambda+\beta))}{6\beta^{3+3n}}a^3{}_{\beta+1}\right. \\ \left.-\frac{\gamma^2(s)(1-\lambda)(\beta+1)^n(\beta+2)^n(3\beta+2\lambda)}{\beta^{2+2n}}a_{\beta+1}a_{\beta+2}\right)z^3\ldots \end{array}\right]$$

and

$$\left(\frac{D^{n+2}f_\gamma(z)^\beta}{D^{n+1}f_\gamma(z)^\beta}\right)^{\lambda} = (\gamma(s)\beta)^{\lambda}\left[\begin{array}{c} 1+\frac{\lambda(\beta+1)^{n+1}\gamma(s)a_{\beta+1}z}{\beta^{n+2}}+ \\ \left(\frac{2\gamma(s)\lambda(\beta+2)^{n+1}a_{\beta+2}}{\beta^{n+2}}-\frac{\gamma^2(s)\lambda(\beta+1)^{2n+2}(-\lambda+2\beta+1)a_{\beta+1}^2}{2\beta^{2+2n}}\right)z^2+ \\ \left(\frac{3\gamma(s)(1-\lambda)(\beta+3)^n a_{\beta+3}}{\beta^{n+1}}+\frac{(\gamma(s))^3(1-\lambda)(\beta+1)^{3n}(\lambda(\lambda+1)+6\beta(\lambda+\beta))a_{\beta+1}^3}{6\beta^{3+3n}}\right. \\ \left.-\frac{(\gamma(s))^2(1-\lambda)(\beta+1)^n(\beta+2)^n(3\beta+2\lambda)a_{\beta+1}a_{\beta+2}}{\beta^{2+2n}}\right)z^3 \end{array}\right]$$

$$\left(\frac{D^{n+2}f_\gamma(z)^\beta}{D^{n+1}f_\gamma(z)^\beta}\right)^{\lambda}\left(\frac{D^{n+1}f_\gamma(z)^\beta}{D^n f_\gamma(z)^\beta}\right)^{1-\lambda} = \gamma(s)\beta\left[\begin{array}{c} 1+\frac{(\lambda+\beta)(\beta+1)^n\gamma(s)}{\beta^{2+n}}a_{\beta+1}z+ \\ \left(\frac{2\gamma(s)(\beta+2)^n(\beta+2\lambda)}{\beta^{2+n}}a_{\beta+2}-\frac{(\beta+1)^{2n}\gamma^2(s)A_1}{2\beta^{4+2n}}a_{\beta+1}^2\right)z^2 \\ \left(\frac{3\gamma(s)(\beta+3\lambda)(\beta+3)}{\beta^{2+n}}a_{\beta+3}+\right. \\ \left(\frac{\gamma^2(s)(\beta+1)^n(\beta+2)^n(-\beta^2)(1-\lambda)(3\beta+2\lambda)-\lambda(\beta+1)(\beta+2)(3\beta+2-2\lambda)}{\beta^{2+2n}}\right. \\ \left.+\frac{2\beta\lambda(1-\lambda)(\beta+2)+2(1-\lambda)\lambda(\beta+1))}{\beta^{2+2n}}\right)a_{\beta+1}a_{\beta+2}+ \\ \left(\frac{\gamma^3(s)(\beta+1)^{3n}[\beta^3(1-\lambda)(6\beta(\beta+\lambda)+\lambda(\lambda+1))+\lambda(\beta+1)((\beta+1)^2(6\beta^2+(\lambda-1)(6\beta+\lambda-2)))]}{6\beta^{6+3n}}-\right. \\ \left.\frac{3\beta(1-\lambda)((\beta+1)(2\beta-\lambda+1)-\beta(\lambda+2\beta))}{6\beta^{6+3n}}a_{\beta+1}^3\right)z^3\ldots \end{array}\right]$$

$$G(w(z)) = 1+\frac{1}{2}w(z)-\frac{1}{24}w^3(z)+\frac{1}{240}w^5(z)\ldots$$

$$G(w(z)) = 1+\frac{c_1}{2}z+\frac{c_2}{2}z^2+\left(\frac{c_3}{2}-\frac{c_1^3}{24}\right)z^3\ldots$$

then,

$$\gamma(s)\beta\left[\begin{array}{c} 1+\frac{(\lambda+\beta)(\beta+1)^n\gamma(s)}{\beta^{2+n}}a_{\beta+1}z+ \\ \left(\frac{2\gamma(s)(\beta+2)^n(\beta+2\lambda)}{\beta^{2+n}}a_{\beta+2}-\frac{(\beta+1)^{2n}\gamma^2(s)A_1}{2\beta^{4+2n}}a_{\beta+1}^2\right)z^2\ldots \end{array}\right]$$

$$= 1+\frac{c_1}{2}z+\frac{c_2}{2}z^2+\left(\frac{c_3}{2}-\frac{c_1^3}{24}\right)z^3$$

$$\frac{(\lambda+\beta)(\beta+1)^n\gamma^2(s)}{\beta^{1+n}}a_{\beta+1} = \frac{c_1}{2}$$

$$a_{\beta+1} = \frac{c_1\beta^{n+1}}{2(\lambda+\beta)(\beta+1)^n\gamma^2(s)}$$

$$|a_{\beta+1}| \leq \frac{\beta^{n+1}}{2(\lambda+\beta)(\beta+1)^n\gamma^2(s)}$$

$$\frac{2\gamma^2(s)(\beta+2)^n(\beta+2\lambda)}{\beta^{1+n}}a_{\beta+2} - \frac{(\beta+1)^{2n}\gamma^3(s)A_1}{2\beta^{3+2n}}a^2_{\beta+1} = \frac{c_2}{2}$$

$$a_{\beta+2} = \frac{c_2\beta^{1+n}}{4\gamma^2(s)(\beta+2)^n(\beta+2\lambda)} + \frac{c_1^2\beta^n\gamma(s)A_1}{16(\lambda+\beta)(2\lambda+\beta)(\beta+2)^n}$$

$$|a_{\beta+2}| \leq \frac{\beta^n}{4(2\lambda+\beta)(\beta+2)^n}\left[\frac{\beta}{\gamma^2(s)} + \frac{\gamma(s)A_1}{4(\lambda+\beta)^2}\right]$$

and

$$|a_{\beta+3}| \leq \frac{\beta^n}{8(3\lambda+\beta)(\beta+3)^n}\left|\frac{4\beta}{3\gamma^2(s)} + \frac{A_4}{36\beta^2\gamma^2(s)(\lambda+\beta)^3(2\lambda+\beta)} + \frac{A_3\beta}{\gamma(s)(\lambda+\beta)(2\lambda+\beta)}\right|$$

□

Corollary 2.2. *Let $f_\gamma \in A_\gamma$ be in $S_\gamma(n,\lambda,1)$, $0 \leq \lambda \leq 1$, $\beta > 0$ and $n \in N_0$, then*

$$|a_2| \leq \frac{1}{2^{n+1}(\lambda+1)\gamma^2(s)}$$

$$|a_3| \leq \frac{1}{4(2\lambda+1)3^n}\left[\frac{1}{\gamma^2(s)} + \frac{\gamma(s)A_1'}{4(\lambda+1)^2}\right]$$

$$|a_4| \leq$$

$$\frac{1}{8(3\lambda+1)4^n}\left|\frac{4}{3\gamma^2(s)} + \frac{A_4'}{36\gamma^2(s)(\lambda+1)^3(2\lambda+1)} + \frac{A_3'}{\gamma(s)(\lambda+1)(2\lambda+1)}\right|$$

where,

$$A_1' = 2+7\lambda-\lambda^2$$

$$A_2' = \lambda^3+72\lambda^2-31\lambda+6$$

$$A_3' = 4\lambda^2-19\lambda-3$$

$$A'_4 = 4\gamma^2(s)(\lambda+1)^3(2\lambda+1)+2A_2'(2\lambda+1)+9\gamma^4(s)A_1'A_3'$$

Corollary 2.3. *Let $f_\gamma \in A_\gamma$ be in $S_1(n,\lambda,1)$, $0 \leq \lambda \leq 1$ and $n \in N_0$, then*

$$|a_2| \leq \frac{1}{2^{n+1}(\lambda+1)}$$

$$|a_3| \leq \frac{1}{4(2\lambda+1)3^n}\left[1 + \frac{A_1'}{4(\lambda+1)^2}\right]$$

$$|a_4| \leq \frac{1}{8(3\lambda+1)4^n}\left|\frac{4}{3} + \frac{A_4''}{36(\lambda+1)^3(2\lambda+1)} + \frac{A_3'}{(\lambda+1)(2\lambda+1)}\right|$$

where,

$$A_1' = 2+7\lambda-\lambda^2$$

$$A_2' = \lambda^3+72\lambda^2-31\lambda+6$$

$$A_3' = 4\lambda^2-19\lambda-3$$

$$A''_4 = 4(\lambda+1)^3(2\lambda+1)+2A_2'(2\lambda+1)+9A_1'A_3'$$

Corollary 2.4. *Let $f_\gamma \in A_\gamma$ be in $S_1(0,\lambda,1)$, $0 \leq \lambda \leq 1$, then*

$$|a_2| \leq \frac{1}{2(\lambda+1)}$$

$$|a_3| \leq \frac{1}{4(2\lambda+1)}\left[1 + \frac{A_1'}{4(\lambda+1)^2}\right]$$

$$|a_4| \leq \frac{1}{8(3\lambda+1)}\left|\frac{4}{3} + \frac{A_4''}{36(\lambda+1)^3(2\lambda+1)} + \frac{A_3'}{(\lambda+1)(2\lambda+1)}\right|$$

where,

$$A_1' = 2+7\lambda-\lambda^2$$

$$A_2' = \lambda^3+72\lambda^2-31\lambda+6$$

$$A_3' = 4\lambda^2-19\lambda-3$$

$$A''_4 = 4(\lambda+1)^3(2\lambda+1)+2A_2'(2\lambda+1)+9A_1'A_3'$$

Corollary 2.5. *Let $f_\gamma \in A_\gamma$ be in $S_1(0,0,1)$, $0 \leq \lambda \leq 1$, then*

$$|a_2| \leq \frac{1}{2}$$

$$|a_3| \leq \frac{3}{8}$$

Next, we define the Fekete-Szego Inequality for the class $S_\gamma(n,\lambda,\beta)$.

Theorem 2.6. *Let $f_\gamma \in A_\gamma$ be in $S_\gamma(n,\lambda,\beta)$, $0 \le \lambda \le 1$, $\beta > 0$ and $n \in N_0$, then*

$$|a_{\beta+2} - \mu a_{\beta+1}^2| \le$$

$$max\left|1, \begin{array}{c} \frac{\beta^{1+n}}{4\gamma^2(s)(\beta+2)^n(\beta+2\lambda)} + \frac{\beta^n\gamma(s)A_1}{16(\lambda+\beta)(2\lambda+\beta)(\beta+2)^n} \\ -\mu\frac{\beta^{2n+2}}{4(\lambda+\beta)^2(\beta+1)^{2n}\gamma^4(s)} \end{array}\right|$$

Proof.

$$a_{\beta+1} = \frac{c_1\beta^{n+1}}{2(\lambda+\beta)(\beta+1)^n\gamma^2(s)}$$

and

$$a_{\beta+2} = \frac{c_2\beta^{1+n}}{4\gamma^2(s)(\beta+2)^n(\beta+2\lambda)} + \frac{c_1^2\beta^n\gamma(s)A_1}{16(\lambda+\beta)(2\lambda+\beta)(\beta+2)^n}$$

then,

$$a_{\beta+2} - \mu a_{\beta+1}^2 =$$

$$\frac{c_2\beta^{1+n}}{4\gamma^2(s)(\beta+2)^n(\beta+2\lambda)} + \frac{c_1^2\beta^n\gamma(s)A_1}{16(\lambda+\beta)(2\lambda+\beta)(\beta+2)^n} -$$

$$\mu\frac{c_1^2\beta^{2n+2}}{4(\lambda+\beta)^2(\beta+1)^{2n}\gamma^4(s)}$$

$$|a_{\beta+2} - \mu a_{\beta+1}^2| \le$$

$$max\left|1, \begin{array}{c} \frac{\beta^{1+n}}{4\gamma^2(s)(\beta+2)^n(\beta+2\lambda)} + \frac{\beta^n\gamma(s)A_1}{16(\lambda+\beta)(2\lambda+\beta)(\beta+2)^n} \\ -\mu\frac{\beta^{2n+2}}{4(\lambda+\beta)^2(\beta+1)^{2n}\gamma^4(s)} \end{array}\right|$$

□

Corollary 2.7. *Let $f_\gamma \in A_\gamma$ be in the class $S_\gamma(n,\lambda,1)$, $0 \le \lambda \le 1$, then*

$$|a_3 - \mu a_2^2| \le$$

$$max\left|1, \frac{1}{4\gamma^2(s)3^n(1+2\lambda)} + \frac{\gamma(s)A_1'}{16(\lambda+1)(2\lambda+1)3^n} - \mu\frac{1}{2^{2n+2}(\lambda+1)^2\gamma^4(s)}\right|.$$

Acknowledgment

The authors appreciate the reviewers for their useful comments.

References

[1] S. Altinkaya , Y. S. Ȯzkan , On Salagean type pseudo-starlike functions, *Acta Et Commentationes Universitatis Tartuensis De Mathematica* , 21(2)(2017), 275–285.

[2] O.A. Fadipe-Joseph, A.T. Oladipo and A.U. Ezeafulukwe, Modified Sigmoid Function in Univalent Theory. *International Journal of Mathematics Science and Engineering Applications (IJMSEA)*. 7(v)(2013), 313–317.

[3] O.A. Fadipe-Joseph, B.O. Moses and M.O Olayemi, Certain New Classes of Analytic Functions defined by using Sigmoid Function. *Advances in Mathematics: Scientific Journal* 5(1)(2016), 83–89.

[4] A.W. Goodman, Univalent Functions. *Vols. 1-2, Mariner, Tampa* Florida. (1983).

[5] Khalifa Al-Shaqsi, Maslina Darus and O.A. Fadipe-Joseph, A New Subclass of Salagean-Type Harmonic Univalent Functions, *Abstract and Applied Analysis* (2010), 2010.

[6] P. Mini and S. Keerthi , Coefficient Inequality for New Subclass of Sakaguchi Type Function Related to Sigmoid Functions. *International Journal of Recent Technology and Engineering*, 7(5S)(2019), 467–471.

[7] G. Murugusundaramoorthy and T. Janani, Sigmoid Function in the Space of Univalent $\lambda - Pseudo$ Starlike Functions. *International Journal of Pure and Applied Mathematics*, 101(1)(2015), 33–41.

[8] C. Ramachandran and K. Dhanalakshmi The Fekete-Szego problem for a subclass of analytic functions related to Sigmoid function. *International Journal of Pure and Applied Mathematics,* 113(3)(2017), 389–398.

[9] G.S. Sălăgean, Subclasses of Univalent Functions. *Lecture Note in Math.* 1013, Springer-Verlag, Berlin, Heidelberg and New York, (1983), 362–372.

[10] G. Singh and G. Singh , A Generalized Subclass of Multivalent Functions Related to Sigmoid function. *Journal of Ultra Scientist of Physical Sciences*, 30(3)(2018), 164–171.

[11] X. Wang and Z. Wang, Coefficient inequality for a new subclass of analytic and univalent functions related to sigmoid function, *International Journal of Modern Mathematical Sciences*, 16(1)(2018), 51–57.

★★★★★★★★★
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
★★★★★★★★★