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Extended energy of some standard graphs

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Abstract

This paper finds the extended energy of some special class of graphs and their complement graphs. Let $G=(V,E)$ be a simple graph with vertex set $V(G)=\{v_1,v_2,...v_n\}$ and edge set $E(G)$.The Extended energy $E_{ext}(G)$ is defined to be the sum of the absolute eigen values of its extended adjacency matrix $A_{ext}(G)$.

Keywords

Extended adjacency matrix, extended energy, spectral radius, spectrum of graphs.

AMS Subject Classification

05C50.

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1. Introduction

The energy of a graph was first coined by Ivan Gutman [1]. However, the motivation for his work appeared in 1930's. In 1930, German scholar Erich Huckel put forward a method for finding approximate solution of the Schrodinger equation of a class of organic molecules called unsaturated conjugated hydrocarbons. This approach is referred to as Huckel molecular orbital theory(HMO). This is in accordance with the π electron energy of the molecule [2]. Earlier some chemical problems were converted to graph and were then solved using spectral graph theory. For example, upper and lower bounds for energy of various classes of graphs were calculated and this can be further used to calculate the total $\pi-$ electron energy of molecular graphs. One can find various results about the eigen values, energy of graphs, Laplacian energy, color energy and color Laplacian energy can be inferred from [3–13, 15]. Let d_i the degree of the vertex $v_i, 1 \leq i \leq n$. The concept of extended adjacency matrix $A_{ext}(G)$ was first explored by Yang Xu et al., [14] and defined as

$$A_{ext}(G) = \begin{cases} \frac{1}{2}\left(\frac{d_i}{d_j}+\frac{d_j}{d_i}\right), & \text{if } v_iv_j \in E(G) \\ 0 & \text{elsewhere.} \end{cases}$$

Extended adjacency matrix can account for heterotoms and multiple bonds which possess high discriminating power and correlate a number of physico-chemical properties and biological index of organic compounds. Also it is used for removing the degeneracy of the entries of the adjacency matrices. In this paper, the extended energy of complete graph, complete bipartite graph, star graph, bistar, crown graph, cocktail party graph, friendship graph and their complement graph are calculated.

2. Preliminaries

In this section, we give some necessary existing results which are required for the development of our main results.

Lemma 2.1. *[16] Let M,N,P,Q be matrices and M be invertible. If $S=\begin{pmatrix} M & N \\ P & Q \end{pmatrix}$ then $\det(S)=\det(M)\det(Q-PM^{-1}N)$. Also, if M and P commute then, $\det S=\det(MQ-PN)$.*

Lemma 2.2. *[16] Let $A=\begin{pmatrix} A_0 & A_1 \\ A_1 & A_0 \end{pmatrix}$ be a symmetric matrix partitioned into blocks. Then the eigen values of A are the eigen values of the matrices A_0+A_1 and A_0-A_1.*

3. Main Results

This section includes some important results such as extended energy calculation of complete bipartite graph, star graph, cocktail party graph etc.,. Also, finds the extended energy of complement of some of their graphs.

Theorem 3.1. *If K_n is a complete graph of order n, then*

$$E_{ext}(K_n) = 2n-2.$$

Proof. Let K_n be the complete graph with vertex set

$$V = \{v_1, v_2, ...v_n\}.$$

$$A_{ext}(K_n) = \begin{bmatrix} 0 & 1 & 1 & \dots & 1 & 1 \\ 1 & 0 & 1 & \dots & 1 & 1 \\ 1 & 1 & 0 & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 0 & 1 \\ 1 & 1 & 1 & \dots & 1 & 0 \end{bmatrix}$$

Then the characteristic polynomial

$$P(K_n, \lambda) = (\lambda+1)^{n-1}(\lambda-(n-1)).$$

Spectrum of

$$K_n = \begin{pmatrix} -1 & n-1 \\ n-1 & 1 \end{pmatrix}$$

$$E_{ext}(K_n) = \sum_{i=1}^{n} |\eta_i| = 2n-2.$$

□

Remark 3.2. *The Extended energy of a complete graph is same as the energy of the complete graph.*

Theorem 3.3. *Let $K_{m,n}$ be the complete bipartite graph on $m+n$ vertices, then*

$$E_{ext}(K_{m,n}) = \sqrt{mn}\left(\frac{m}{n}+\frac{n}{m}\right).$$

Proof. Let $K_{m,n}$ be the complete bipartite graph of order $m+n$, then

$$A_{ext}(K_{m,n}) = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \cdots & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) \\ 0 & 0 & \cdots & 0 & 0 & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \cdots & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \cdots & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) \\ 0 & 0 & \cdots & 0 & 0 & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \cdots & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) \\ \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \cdots & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & 0 & 0 & \cdots & 0 & 0 \\ \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \cdots & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \cdots & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & 0 & 0 & \cdots & 0 & 0 \\ \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \cdots & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & \frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right) & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}_{(m+n)\times(m+n)}$$

Consider, $\det(\lambda I - A_{ext}(K_{m,n}))$.

Steps:

1. $C_k \to C_k - C_{m+1}$ for $k = m+2, m+3, ..., m+n$. Then we get $\lambda^{m+n-2}\det B$.

2. In $\det B, R_k \to R_k - R_{m+1}$ for $k = m+2, m+3, ..., m+n$ and simplifying we get,

$$\det B = \lambda^2 - mn\left[\frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right)\right]^2$$

The characteristic polynomial of $K_{m,n}$is

$$P(K_{m,n},\lambda) = \lambda^{m+n-2}\left(\lambda^2 - mn\left[\frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right)\right]^2\right).$$

Spectrum of $K_{m,n}$ is

$$\begin{pmatrix} 0 & \sqrt{mn}\left[\frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right)\right] & -\sqrt{mn}\left[\frac{1}{2}\left(\frac{m}{n}+\frac{n}{m}\right)\right] \\ m+n-2 & 1 & 1 \end{pmatrix}$$

$$E_{ext}(K_{m,n}) = \sum_{i=1}^{n}|\eta_i| = \sqrt{mn}\left(\frac{m}{n}+\frac{n}{m}\right).$$

□

Definition 3.4. *A star graph S_n, is a connected graph on n vertices where one vertex has degree $n-1$ and the other $n-1$ vertices have degree 1. A star graph is a special case of a complete bipartite graph in which one set has 1 vertex and the other set has $n-1$ vertices. It is also denoted by $K_{1,n-1}$.*

Theorem 3.5. *If $K_{1,n-1}$ is a star graph on vertices, then*

$$E_{ext}(K_{1,n-1}) = \frac{n^2-2n+2}{\sqrt{n-1}}.$$

Proof. Let $K_{1,n-1}$ be a star graph on n vertices, then

$$A_{ext}(K_{m,n}) = \begin{bmatrix} 0 & \frac{1}{2}\left(n-1+\frac{1}{n-1}\right) & \frac{1}{2}\left(n-1+\frac{1}{n-1}\right) & \cdots & \frac{1}{2}\left(n-1+\frac{1}{n-1}\right) & \frac{1}{2}\left(n-1+\frac{1}{n-1}\right) \\ \frac{1}{2}\left(n-1+\frac{1}{n-1}\right) & 0 & 0 & \cdots & 0 & 0 \\ \frac{1}{2}\left(n-1+\frac{1}{n-1}\right) & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{2}\left(n-1+\frac{1}{n-1}\right) & 0 & 0 & \cdots & 0 & 0 \\ \frac{1}{2}\left(n-1+\frac{1}{n-1}\right) & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

Consider, $\det(\lambda I - A_{ext}(K_{1,n-1}))$.

Steps:

1. $R_k \to R_k - R_n$ for $k = 2, 3 ... n-1$. Then we get,

$$\det(\lambda I - A_{ext}(K_{1,n-1})) = \lambda^{n-2}\det(B).$$

2. In $\det(B), C_k \to C_k - C_2$ for $k = 3, 4 ... n$ and expanding $\det(B)$ we get,

$$\det(B) = \lambda^2 - \frac{n-1}{4}\left(\frac{1}{n-1}+n-1\right)^2.$$

Thus the characteristic polynomial of $K_{1,n-1}$ is

$$P(K_{1,n-1},\lambda) = \lambda^{n-2}\left(\lambda^2 - \frac{n-1}{4}\left(\frac{1}{n-1}+n-1\right)^2\right).$$

Therefore, spectrum of $K_{1,n-1}$ is

$$\begin{pmatrix} 0 & \frac{\sqrt{n-1}}{2}\left(\frac{1}{n-1}+n-1\right) & -\frac{\sqrt{n-1}}{2}\left(\frac{1}{n-1}+n-1\right) \\ n-2 & 1 & 1 \end{pmatrix}$$

Hence,

$$\begin{aligned} E_{ext}(K_{1,n-1}) &= \sum_{i=1}^{n} |\eta_i| \\ &= \sqrt{n-1}\left(\frac{1}{n-1}+n-1\right) \\ &= \frac{n^2-2n+2}{\sqrt{n-1}}. \end{aligned}$$

□

Definition 3.6. *Bistar $B_{n,n}$ is the graph obtained by joining the apex vertices of two copies of star $K_{1,n-1}$.*

Theorem 3.7. *If $B_{n,n}$ is a bistar on $2n$ vertices then*

$$E_{ext}(B_{n,n}) = 2\sqrt{1+\left((n-1)\left(\frac{1}{n}+n\right)^2\right)}.$$

Proof. Let $B_{n,n}$ is a bistar on $2n$ vertices then

$$A_{ext}(B_{n,n}) = \begin{bmatrix}
0 & \frac{1}{2}\left(n+\frac{1}{2}\right) & \vdots & \frac{1}{2}\left(n+\frac{1}{2}\right) & \frac{1}{2}\left(n+\frac{1}{2}\right) & 1 & 0 & \vdots & 0 & 0 \\
\frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 0 \\
\cdots & \cdots & \ddots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots \\
\frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 0 \\
\frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & \vdots & 0 & 0 & 0 & \frac{1}{2}\left(n+\frac{1}{2}\right) & \vdots & \frac{1}{2}\left(n+\frac{1}{2}\right) & \frac{1}{2}\left(n+\frac{1}{2}\right) \\
1 & 0 & \vdots & 0 & 0 & \frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & \vdots & 0 & 0 \\
0 & 0 & \vdots & 0 & 0 & \frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & \vdots & 0 & 0 \\
\cdots & \cdots & \ddots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots \\
0 & 0 & \vdots & 0 & 0 & \frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & \vdots & 0 & 0 \\
0 & 0 & \vdots & 0 & 0 & \frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & \vdots & 0 & 0
\end{bmatrix}$$

Using lemma 2.2, the above matrix is of the form

$$A_{ext}(B_{n,n}) = \begin{bmatrix} A_0 & A_1 \\ A_1 & A_0 \end{bmatrix}$$

$$A_0 + A_1 = \begin{bmatrix} 1 & \frac{1}{2}\left(n+\frac{1}{2}\right) & \frac{1}{2}\left(n+\frac{1}{2}\right) & \cdots & \frac{1}{2}\left(n+\frac{1}{2}\right) & \frac{1}{2}\left(n+\frac{1}{2}\right) \\ \frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & 0 & \cdots & 0 & 0 \\ \frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & 0 & \cdots & 0 & 0 \\ \frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

Spectrum of $A_0 + A_1$ can be found using the same elementary steps as in theorem 3.5. Similarly, we get the spectrum of $A_0 - A_1$. Thus the characteristic polynomial of $B_{n,n}$ is

$$P(B_{n,n}, \lambda) = \lambda^{2n-4}\left(\lambda^2 - \lambda - \frac{(n-1)}{4}\left(\frac{1}{n+n}\right)^2\right)\left(\lambda^2 + \lambda - \frac{(n-1)}{4}\left(\frac{1}{n}+n\right)^2\right)$$

Spectrum of $B_{n,n}$ is

$$\begin{pmatrix} 0 & 1 \pm \frac{\sqrt{1+\left((n-1)\left(\frac{1}{n}+n\right)^2\right)}}{2} & -1 \pm \frac{\sqrt{1+\left((n-1)\left(\frac{1}{n}+n\right)^2\right)}}{2} \\ 2n-4 & 1 & 1 \end{pmatrix}$$

Hence

$$E_{ext}(B_{n,n}) = 2\sqrt{1 + \left((n-1)\left(\frac{1}{n}+n\right)^2\right)}.$$

□

Definition 3.8. *The crown graph $H_{n,n}$ is the graph obtained from the complete bipartite graph $K_{n,n}$ by removing a perfect matching.*

Theorem 3.9. *If $H_{n,n}$ is a crown graph of order $2n$ with $n \geq 3$, then $E_{ext}(H_{n,n}) = 4(n-1)$.*

Proof. Let $H_{n,n}$ is a crown graph of order $2n$, then

$$A_{ext}(H_{n,n}) = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 & 1 & \cdots & 1 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 1 & 1 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 1 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

Using lemma 2.2, the above matrix is of the form

$$A_{ext}(H_{n,n}) = \begin{bmatrix} A_0 & A_1 \\ A_1 & A_0 \end{bmatrix}$$

$$A_0 + A_1 = \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{bmatrix}$$

Consider, $\det(\lambda I - (A_0 + A_1))$

Steps:

1. $R_1 \to R_1 + R_2 + ... + R_n$ Then we get

$$(\lambda - (n-1)) \det B.$$

2. In $\det B, C_k \to C_k - C_1$ for $k = 2, 3, ..., n$, then

$$\det B = (\lambda + 1)^{n-1}.$$

Thus characteristic polynomial of $A_0 + A_1$ is

$$(\lambda - (n-1))(\lambda + 1)^{n-1}.$$

Similarly, we can find the characteristic polynomial of $A_0 - A_1$ is

$$(\lambda + (n-1))(\lambda - 1)^{n-1}$$

By lemma 2.2, The characteristic polynomial of $(H_{n,n})$ is

$$(\lambda + 1)^{n-1}(\lambda - (n-1))(\lambda + (n-1))(\lambda - 1)^{n-1}.$$

Spectrum of

$$H_{n,n} = \begin{pmatrix} -(n-1) & n-1 & -1 & 1 \\ 1 & 1 & n-1 & n-1 \end{pmatrix}$$

Hence,

$$E_{ext}(H_{n,n}) = 4(n-1).$$

□

Theorem 3.10. *If $\overline{H_{n,n}}$ is the complement of the crown graph $H_{n,n}$ then*

$$E_{ext}(\overline{H_{n,n}}) = 4(n-1).$$

Proof.

$$A_{ext}(\overline{H_{n,n}}) = \begin{bmatrix} 0 & 1 & \cdots & 1 & 1 & 1 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 1 & 1 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 0 & 1 & 0 & 0 & \cdots & 1 & 0 \\ 1 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 & 0 & 1 & \cdots & 1 & 1 \\ 0 & 1 & \cdots & 0 & 0 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 1 & 1 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 1 & 1 & 1 & \cdots & 1 & 0 \end{bmatrix}$$

$$A_{ext}(\overline{H_{n,n}}) = \begin{bmatrix} J_n - I_n & I_n \\ I_n & J_n - I_n \end{bmatrix}$$

Using lemma 2.2, the above matrix is of the form

$$A_{ext}(\overline{H_{n,n}}) = \begin{bmatrix} A_0 & A_1 \\ A_1 & A_0 \end{bmatrix}$$

The characteristic polynomial of $(\overline{H_{n,n}})$ is

$$(\lambda - (n-2))(\lambda - n)\lambda^{n-1}(\lambda + 2)^{n-1}$$

Spectrum of

$$(\overline{H_{n,n}}) = \begin{pmatrix} n-2 & n & 0 & -2 \\ 1 & 1 & n-1 & n-1 \end{pmatrix}$$

$$E_{ext}(\overline{H_{n,n}}) = 4(n-1)$$

□

Definition 3.11. *The cocktail party graph denoted by $K_{n\times 2}$ is a graph having vertex set*

$$V = \bigcup_{i=1}^{n} \{u_i, v_i\}$$

and edge set

$$E = \{u_i u_j, u_i v_j, v_i u_j, v_i v_j : 1 \leq i < j \leq n\}.$$

Theorem 3.12. *If $K_{n\times 2}$ is a cocktail party graph of $2n$ vertices, then*

$$E_{ext}(K_{n\times 2}) = 4(n-1).$$

Proof. Let $K_{n\times 2}$ be a cocktail party graph on $2n$ vertices, then

$$A_{ext}(K_{n\times 2}) = \begin{bmatrix} 0 & 1 & \cdots & 1 & 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 0 & \cdots & 1 & 1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 0 & 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & \cdots & 1 & 0 & 1 & 1 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 1 & 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 0 & \cdots & 1 & 1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 0 & 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & \cdots & 1 & 0 & 1 & 1 & \cdots & 1 & 0 \end{bmatrix}$$

$$A_{ext}(K_{n\times 2}) = \begin{bmatrix} J_n - I_n & J_n - I_n \\ J_n - I_n & J_n - I_n \end{bmatrix}$$

From lemma 2.2,The spectrum of $(K_{n\times 2})$ is

$$\begin{pmatrix} 2(n-1) & -2 & 0 \\ 1 & n-1 & n \end{pmatrix}$$

$$E_{ext}(K_{n\times 2}) = 4(n-1).$$

□

Theorem 3.13. *If $(\overline{K_{n\times 2}})$ is the complement of the cocktail party graph $K_{n\times 2}$ of order $2n$, then*

$$E_{ext}(\overline{K_{n\times 2}}) = 2n.$$

Proof. Let $(\overline{K_{n\times 2}})$ is the complement of the cocktail party graph $K_{n\times 2}$ of order $2n$, then

$$A_{ext}(\overline{K_{n\times 2}}) = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$A_{ext}(\overline{K_{n\times 2}}) = \begin{bmatrix} 0 & I_n \\ I_n & 0 \end{bmatrix}$$

Using lemma 2.2, the above matrix is of the form

$$A_{ext}(\overline{K_{n\times 2}}) = \begin{bmatrix} A_0 & A_1 \\ A_1 & A_0 \end{bmatrix}$$

Spectrum of

$$(\overline{K_{n\times 2}}) = \begin{pmatrix} 1 & -1 \\ n & n \end{pmatrix}$$

Hence

$$E_{ext}(\overline{K_{n\times 2}}) = 2n.$$

□

Definition 3.14. *Friendship graph $F_n, n \geq 2$ is a planar undirected graph with $2n+1$ vertices and $3n$ edges.*

It is constructed by joining n copies of the cycle C_3 with a common vertex.

Theorem 3.15. *If $F_n, n \geq 2$ is a friendship graph of order $2n+1$, then*

$$E_{ext}(F_n) = 2n - 1 + \sqrt{1 + 2n\left(\frac{n+1}{n}\right)^2}.$$

Proof. let $F_n, n \geq 2$ be a friendship graph of order $2n+1$, then

$$A_{ext}(F_n) = \begin{bmatrix} 0 & 1 & \frac{1}{2}\left(n+\frac{1}{2}\right) & \cdots & 0 & 0 \\ 1 & 0 & \frac{1}{2}\left(n+\frac{1}{2}\right) & \cdots & 0 & 0 \\ \frac{1}{2}\left(n+\frac{1}{2}\right) & \frac{1}{2}\left(n+\frac{1}{2}\right) & 0 & \cdots & \frac{1}{2}\left(n+\frac{1}{2}\right) & \frac{1}{2}\left(n+\frac{1}{2}\right) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \frac{1}{2}\left(n+\frac{1}{2}\right) & \cdots & 0 & 1 \\ 0 & 0 & \frac{1}{2}\left(n+\frac{1}{2}\right) & \cdots & 1 & 0 \end{bmatrix}$$

Consider, $\det(\lambda I - A_{ext}(F_n))$

Steps:

1. $R_k \to R_k - R_4$ for $k = 5, 6, ..., 2n+1$, then we get $(\lambda + 1)^{2n-3} \det B$.

2.In $\det B$, use elementary row and column operations to simplify it. Then we get, the characteristic polynomial as

$$P(F_n, \lambda) = (\lambda+1)^n (\lambda-1)^{n-1} \left(\lambda^2 - \lambda - \frac{n}{2}\left(\frac{1}{n} + n\right)^2\right)$$

Spectrum of F_n is $\begin{pmatrix} -1 & 1 & \frac{1 \pm \sqrt{1+2n\left(\frac{n+1}{n}\right)^2}}{2} \\ n & n-1 & 1 \end{pmatrix}$

$$E_{ext}(F_n) = 2n - 1 + \sqrt{1 + 2n\left(\frac{n+1}{n}\right)^2}.$$

□

4. Conclusion

In order to explore the concept of extended energy mathematically, the extended energy of some graphs are found. Of which, it is observed that the extended energy and energy of complete graph, crown graph and cocktail party graph are same. Further, the relation between the energy, extended energy, Laplacian and signless Laplacian energies of the graphs discussed in this work may be explored and study its characteristics.

References

[1] I. Gutman, The energy of a graph, *Ber. Math. Statist. Sekt. Forsch-ungszentram. Graz*, 22(103)(1978), 1–22.

[2] I. Gutman, Topology and stability of conjugated hydrocarbons: the dependence of total $\pi-$ electron energy on molecular topology, *J. Serb. Chem. Soc*, 70(2005), 441–456.

[3] I. Gutman, X.Li, Y.Shi, *Graph Energy*, New York, Springer, 2012.

[4] A.E. Brouwer, W.H. Haemers, *Spectra of Graphs*, Springer, 2010.

[5] I. Gutman, *The Energy of Graph-Old and New Results, Algebraic Combinatorics and Applications*, Springer, Berlin, (2000), 196-211.

[6] R.B. Bapat, S. Pati, Energy of a graph is never an odd integer, *Bull. Kerala Math. Assoc.*, 1(2004), 129–132.

[7] I. Gutman, B. Zhou, Laplacian energy of a graph, *Linear Algebra and its Applications*, 414(2006), 29–37.

[8] T. Aleksic, Upper bounds for laplacian energy of graphs, *Match Communications. Math. Comput. Chem.*, 60(2008), 435–439.

[9] R. Li, Some lower bounds for laplacian energy of graphs, *Int. J. Contemp. Math. Sciences*, 4(5)(2009), 219–223.

[10] S. Pirzada, H.A.Ganie, On the laplacianeigen values of a graph and laplacian energy, *Linear Algebra and its Applications*, 486(2015), 454–468.

[11] C. Adiga, E. Sampathkumar, M.A. Sriraj and A.S. Shrikanth, Color energy of graphs, *Proc. Jangjeon Math.Soc.*, 16(2013), 335-351.

[12] P.G. Bhat, S.D'Souza, Color Laplacian energy of a graph, *Proc. Jangjeon Math.Soc.*, 18(2015), 321–330.

[13] P.G. Bhat, S.D'Souza, Color Signless Laplacian energy of a graphs, *AKCE International Journal of Graphs and Combinatorics*, (2017), 1–12.

[14] Y. Yang, L. Xu, C.Y.Hu, Extended adjacency matrix indices and their applications, *J. Chem. Inf. Comput. Sci*, 34(1994), 1140–1145.

[15] K. C. Das, I. Gutman, B. Furtula, On spectral radius and energy of extended adjacency matrix of graphs, *Applied Mathematics and Computations*, 296(2017), 116–123.

[16] G. Indulal, I. Gutman, A. Vijayakumar, On distance energy of graphs, *Match Communications. Math. Comput. Chem.*, 60(2008), 461–472.

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