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Rough fuzzy bi-interiorideal(biquasi-ideal) of semigroup

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Abstract

Aim of this paper is to characterize semigroups by interval valued bi-interior(bi- quasi)ideal,rough bi-interior(bi-quasi)ideal and interval valued rough fuzzy bi-interior(bi- quasi)ideal also discuss some properties of these structures.

Keywords

Interval valued bi-interior-ideal, rough bi-interior-ideal, interval valued rough fuzzy bi-interior-ideal, interval valued bi-quasi-ideal, rough bi-quasi-ideal, interval valued rough fuzzy bi-quasi-ideal.

AMS Subject Classification

20M05, 20M12, 08A72.

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Contents

1. Introduction

The famous fuzzy set theory was studied by Zedah[10], which proved a very useful tool to describe situation in which the data are imprecise or vague. Similar to fuzzy set theory, interval valued fuzzy set theory gradually developed on different algebraic structures. Rough set theory was proposed by Z.Pawlak [7] in 1982. Dubois and Prade [2]combined the rough sets and fuzzy sets together. This combination gains the great interest of researchers and becomes a useful tool in exploring the feature selection, the clustering, the control problem etc. In [8] they introduced the concept of interval-valued rough fuzzy sets in semigroups. Bi-interior-ideal,fuzzy bi-interior-ideal and fuzzy bi-quasi-ideal of semigroups are studied by M.Muralikrishna rao[4–6].

2. Preliminaries

For basic concepts used in this work see [1],[2],[3],[4],[5],[6],[8] and [9].
The notations used in this work:
$\mathcal{RB}I_nI$: Rough bi-interior-ideal
$\mathcal{R}\mathcal{B}_iQI$: Rough bi-quasi-ideal
$I_{\mathcal{V}}\mathcal{RFB}I_nI$: Interval valued rough fuzzy bi-interior-ideal.
$I_{\mathcal{V}}\mathcal{RF}\mathcal{B}_iQI$: Interval valued rough fuzzy bi-quasi-ideal.

3. $I_{\mathcal{V}}\mathcal{FBI}_nI$ of a semigroup

This section deals with Interval valued fuzzy bi-interior-ideal ($I_{\mathcal{V}}\mathcal{FBI}_nI$) which is an extension of an fuzzy bi-quasi-ideal
($\mathcal{FBI}_nI$).

Definition 3.1. *An $I_{\mathcal{V}}\mathcal{FBI}_nI$ is defined as $\tilde{\mathcal{S}}\tilde{\tau}\tilde{\mathcal{S}}\cap\tilde{\tau}\tilde{\mathcal{S}}\tilde{\tau}\subseteq\tilde{\tau}$ for an $I_{\mathcal{V}}\mathcal{F}$ subset $\tilde{\tau}$ of $\mathcal{S}$.*

Theorem 3.2. *A non-empty sub-set B of $\mathcal{S}$ is a $\mathcal{B}I_nI \iff$ the characteristic function (cf) of B is $I_{\mathcal{V}}\mathcal{FBI}_nI$ of $\mathcal{S}$.*

Proof : If B is a $\mathcal{B}I_nI$ of $\mathcal{S}$. Therefore cf_B is an $I_{\mathcal{V}}\mathcal{F}$ subsemigroup of $\mathcal{S}$. By hypothesis we've $\mathcal{S}B\mathcal{S} \cap B\mathcal{S}B \subseteq B$. Then,

$$\tilde{\mathcal{S}}(\tilde{cf})_B\tilde{\mathcal{S}} \cap (\tilde{cf})_B\tilde{\mathcal{S}}(\tilde{cf})_B = (\tilde{cf})_{\mathcal{S}B\mathcal{S}} \cap (\tilde{cf})_{B\mathcal{S}B} = (\tilde{cf})_{\mathcal{S}B\mathcal{S}\cap B\mathcal{S}B} \subseteq (\tilde{cf})_B$$

Hence $(\tilde{cf})_B$ is a $\mathcal{FB}I_nI$of $\mathcal{S}$.

Conversely, let us assume that $(\tilde{cf})_B$ is an $I_{\mathcal{V}}\mathcal{FB}I_nI$ of $\mathcal{S}$. Then B is a subsemigroup of S. We have

$$\tilde{\mathcal{S}}(\tilde{cf})_B\tilde{\mathcal{S}} \cap (\tilde{cf})_B\tilde{\mathcal{S}}(\tilde{cf})_B \subseteq (\tilde{cf})_B$$
$$(\tilde{cf})_{\tilde{\mathcal{S}}B\tilde{\mathcal{S}}} \cap (\tilde{cf})_{B\tilde{\mathcal{S}}B} \subseteq (\tilde{cf})_B$$
$$(\tilde{cf})_{\tilde{\mathcal{S}}B\tilde{\mathcal{S}}\cap B\tilde{\mathcal{S}}B} \subseteq (\tilde{cf})_B$$
$$\tilde{\mathcal{S}}B\tilde{\mathcal{S}} \cap B\tilde{\mathcal{S}}B \subseteq B$$

Thence B is a $\mathcal{B}I_nI$ of $\tilde{\mathcal{S}}$.

Theorem 3.3. *If $\tilde{\tau}(\neq \emptyset)$ be $I_{\mathcal{V}}\mathcal{F}$ sub-set of $\mathcal{S}$. Then $\tilde{\tau}$ is an $I_{\mathcal{V}}\mathcal{FB}I_nI$ of $\mathcal{S} \iff$ the $\langle[\rho_1,\rho_2]\rangle$-cut of $\tilde{\tau}$ is a $\mathcal{B}I_nI$ of $\mathcal{S}$ for every $[\rho_1,\rho_2] \in \mathscr{D}[0,1]$.*

Proof: Take $\tilde{\tau}$ is an $I_{\mathcal{V}}\mathcal{FB}I_nI$ of $\mathcal{S}$.
Let $x \in \tilde{\mathcal{S}}(\tilde{\tau},[\rho_1,\rho_2])\tilde{\mathcal{S}} \cap (\tilde{\tau},[\rho_1,\rho_2])\tilde{\mathcal{S}}(\tilde{\tau},[\rho_1,\rho_2])$. Then $x = abc = def$, $a,c,e \in \tilde{\mathcal{S}}$ and $b,d,f \in (\tilde{\tau},[\rho_1,\rho_2])$ implies $\tilde{\tau}(b) \geq [\rho_1,\rho_2]$, $\tilde{\tau}(d) \geq [\rho_1,\rho_2]$ and $\tilde{\tau}(f) \geq [\rho_1,\rho_2]$

Now, $\tilde{\mathcal{S}}\tilde{\tau}\,(x_1) = \sup_{x_1=a_1b_1} \{min\{\tilde{\mathcal{S}}(a_1),\tilde{\tau}(b_1)\}\} = \tilde{\tau}(b_1) \geq [\rho_1,\rho_2]$

$$\text{Consider, } (\tilde{\mathcal{S}}\tilde{\tau}\tilde{\mathcal{S}})\,(x_1) = \sup_{x_1=a_1b_1c_1} \left\{min\left\{\tilde{\mathcal{S}}\tilde{\tau}(a_1b_1),\tilde{\mathcal{S}}(c_1)\right\}\right\} = \sup_{x_1=a_1b_1c_1} \{min\{[\rho_1,\rho_2],1\}\} \geq [\rho_1,\rho_2]$$

Similarly, we prove $(\tilde{\tau}\tilde{\mathcal{S}}\tilde{\tau})\,(x_1) \geq [\rho_1,\rho_2]$. Therefore $\tilde{\tau}(x) \geq [\rho_1,\rho_2]$.Implies $x \in (\tilde{\tau},[\rho_1,\rho_2])$. Hence $(\tilde{\tau},[\rho_1,\rho_2])$ is a $\mathcal{B}I_nI$ of $\mathcal{S}$.

For the converse part we take $(\tilde{\tau},[\rho_1,\rho_2])$ is a $\mathcal{B}I_nI$ of $\mathcal{S}$ for all $[\rho_1,\rho_2],[\mu_1,\mu_1] \in \mathscr{D}[0,1]$. Let $x,y \in \mathcal{S}$, $\tilde{\tau}(x) = [\rho_1,\rho_2]$ and $\tilde{\tau}(y) = [\mu_1,\mu_2]$ where $[\rho_1,\rho_2] \geq [\mu_1,\mu_2]$ implies $x,y \in (\tilde{\tau},[\mu_1,\mu_2])$. We've $\tilde{\mathcal{S}}(\tilde{\tau},[\varsigma_1,\varsigma_2])\tilde{\mathcal{S}} \cap (\tilde{\tau},[\varsigma_1,\varsigma_2])\tilde{\mathcal{S}}(\tilde{\tau},[\varsigma_1,\varsigma_2]) \subseteq (\tilde{\tau},[\iota_1,\iota_2])$ for all $[\varsigma_1,\varsigma_2] \in \mathscr{D}[0,1]$. Suppose $[\varsigma_1,\varsigma_2] = min\{\mathscr{D}[0,1]\}$. Then, $\tilde{\mathcal{S}}(\tilde{\tau},[\iota_1,\iota_2])\tilde{\mathcal{S}} \cap (\tilde{\tau},[\iota_1,\iota_2])\tilde{\mathcal{S}}(\tilde{\tau},[\iota_1,\iota_2]) \subseteq (\tilde{\tau},[\iota_1,\iota_2])$. Hence $\tilde{\mathcal{S}}\tilde{\tau}\tilde{\mathcal{S}} \cap \tilde{\tau}\tilde{\mathcal{S}}\tilde{\tau} \subseteq \tilde{\tau}$.

Theorem 3.4. *Let $\tilde{\tau}(\neq \emptyset)$ be $I_{\mathcal{V}}\mathcal{F}$ sub-set of $\mathcal{S}$. Then $\tilde{\tau}$ is an $I_{\mathcal{V}}\mathcal{FB}I_nI$ of $\mathcal{S} \iff$ the $\langle(\rho_1,\rho_2)\rangle$-cut$(\neq \emptyset)$ of $\tilde{\tau}$ is a $\mathcal{B}I_nI$ of $\mathcal{S}$ for every $[\rho_1,\rho_2] \in \mathscr{D}[0,1]$.*

Proof: Similar to 3.3

Theorem 3.5. *Every $I_{\mathcal{V}}\mathcal{F}lI$ of $\mathcal{S}$ is an $I_{\mathcal{V}}\mathcal{FB}I_nI$ of $\mathcal{S}$.*

Proof: Assume that $\tilde{\tau}_l$ be an $I_{\mathcal{V}}\mathcal{F}lI$ of $\mathcal{S}$.
Take $i_1,r_1,t_1 \in \mathcal{S}$.

$$\tilde{\mathcal{S}}\tilde{\tau}_l\,(i_1) = \sup_{i_1=r_1t_1} \{min\{\tilde{\mathcal{S}}(r_1),\tilde{\tau}_l(t_1)\}\} = \sup_{i_1=r_1t_1} \left\{min\left\{\tilde{1},\tilde{\tau}_l(t_1)\right\}\right\} = \sup_{i_1=r_1t_1} \{\tilde{\tau}_l(t_1)\} \leq \sup_{i_1=r_1t_1} \{\tilde{\tau}_l(r_1t_1)\} = \tilde{\tau}_l(i_1)$$

$$\text{Consider, } \tilde{\tau}_l\tilde{\mathcal{S}}\tilde{\tau}_l(i_1) = \sup_{i_1=m_1n_1v_1} \{min\{\tilde{\tau}_l(m_1),(\tilde{\mathcal{S}}\tilde{\tau}_l)(n_1v_1)\}\} \leq \sup_{i_1=m_1n_1v_1} \{min\{\tilde{\tau}_l(m_1),\tilde{\tau}_l(n_1v_1)\}\} = \tilde{\tau}_l(i_1)$$

$$\text{Now, } \tilde{\mathcal{S}}\tilde{\tau}_l\tilde{\mathcal{S}} \cap \tilde{\tau}_l\tilde{\mathcal{S}}\tilde{\tau}_l = min\{\tilde{\mathcal{S}}\tilde{\tau}_l\tilde{\mathcal{S}}(i_1),\tilde{\tau}_l\tilde{\mathcal{S}}\tilde{\tau}_l(i_1)\} \leq min\{\tilde{\mathcal{S}}\tilde{\tau}_l\tilde{\mathcal{S}}(i_1),\tilde{\tau}_l(i_1)\} \leq \tilde{\tau}_l(i_1)$$

Hence $\tilde{\tau}_l$ is an $I_{\mathcal{V}}\mathcal{B}I_nI$ of $\mathcal{S}$.

Theorem 3.6. *Every $I_{\mathcal{V}}\mathcal{F}rI$ of $\mathcal{S}$ is an $I_{\mathcal{V}}\mathcal{FB}I_nI$ of $\mathcal{S}$.*

Proof: Take $\tilde{\tau}_r$ be an $I_{\mathcal{V}}\mathcal{F}rI$ of $\mathcal{S}$ and $i_1,a_1,b_1 \in \mathcal{S}$.

$$\text{Consider, } \tilde{\tau}_r\tilde{\mathcal{S}}\,(i_1) = \sup_{i_1=a_1b_1} \left\{min\left\{\tilde{\tau}_r(a_1),\tilde{\mathcal{S}}(b_1)\right\}\right\} = \sup_{i_1=a_1b_1} \{\tilde{\tau}_r(a_1)\} \leq \sup_{i_1=a_1b_1} \{\tilde{\tau}_r(a_1b_1)\} = \tilde{\tau}_r(i_1)$$

Also,

$$\tilde{\tau}_r\tilde{\mathcal{S}}\tilde{\tau}_r(i_1) = \sup_{i_1=u_1v_1r_1} \left\{min\left\{\tilde{\tau}_r\tilde{\mathcal{S}}(u_1v_1),\tilde{\tau}_r(r_1)\right\}\right\} \leq \sup_{i_1=u_1v_1r_1} \{min\{\tilde{\tau}_r(u_1v_1),\tilde{\tau}_r(r_1)\}\} = \tilde{\tau}_r(i_1)$$

Now,

$$\tilde{\mathcal{S}}\tilde{\tau}_r\tilde{\mathcal{S}} \cap \tilde{\tau}_r\tilde{\mathcal{S}}\tilde{\tau}_r = min\left\{\tilde{\mathcal{S}}\tilde{\tau}_r\tilde{\mathcal{S}}(i_1),\tilde{\tau}_r\tilde{\mathcal{S}}\tilde{\tau}_r(i_1)\right\} \leq min\left\{\tilde{\mathcal{S}}\tilde{\tau}_r\tilde{\mathcal{S}}(i_1),\tilde{\tau}_r(i_1)\right\} \leq \tilde{\tau}_r(i_1)$$

Hence the theorem.

Corollary 3.7. *Every $I_{\mathcal{V}}\mathcal{F}I$ of $\mathcal{S}$ is an $I_{\mathcal{V}}\mathcal{FB}I_nI$ of $\mathcal{S}$.*

Proof: By applying Theorem 3.5 and Theorem 3.6 proof is straight forward.

Theorem 3.8. *Intersection of $I_{\mathcal{V}}\mathcal{F}r$ and $I_{\mathcal{V}}\mathcal{F}lI$s of $\mathcal{S}$ is an $I_{\mathcal{V}}\mathcal{FB}I_nI$ of $\mathcal{S}$.*

4. $I_{\mathcal{V}}\mathcal{F}\,\mathcal{B}_iQI$ of a semigroup

We characterize interval valued fuzzy left bi-quasi-ideal ($I_{\mathcal{V}}\mathcal{F}l\mathcal{B}_iQI$), interval valued fuzzy right bi-quasi-ideal ($I_{\mathcal{V}}\mathcal{F}r\mathcal{B}_iQI$) of semigroup and interval valued fuzzy bi-quasi-ideal ($I_{\mathcal{V}}\mathcal{F}\,\mathcal{B}_iQI$) of a semigroup. Discuss some properties of $I_{\mathcal{V}}\mathcal{F}\,\mathcal{B}_iQI$ of semigroup.

Definition 4.1. *A non-empty sub-semigroup $\mathcal{A}$ of $\mathcal{S}$ is said to be $l\mathcal{B}_iQI(r\mathcal{B}_iQI)$ of $\mathcal{S}$ if $\mathcal{S}\mathcal{A} \cap \mathcal{A}\mathcal{S}\mathcal{A}(\mathcal{A}\mathcal{S} \cap \mathcal{A}\mathcal{S}\mathcal{A})$.*

Definition 4.2. *$\mathcal{A}$ is said to be $\mathcal{B}_iQI$ if it is both a $l\mathcal{B}_iQI$ and $r\mathcal{B}_iQI$ of $\mathcal{S}$.*

Definition 4.3. *A fuzzy sub-set ζ of semigroup is called a $\mathcal{F}l\mathcal{B}_iQI$($\mathcal{F}r\mathcal{B}_iQI$) of $\mathcal{S}$ if $\mathcal{S}\zeta \cap \zeta\mathcal{S}\zeta \subseteq \zeta(\zeta\mathcal{S} \cap \zeta\mathcal{S}\zeta \subseteq \zeta)$. ζ is said to be $\mathcal{F}\,\mathcal{B}_iQI$ of $\mathcal{S}$ if it is both $\mathcal{F}l\mathcal{B}_iQI$ and $\mathcal{F}r\mathcal{B}_iQI$*

Definition 4.4. *An $I_{\mathcal{V}}\mathcal{F}$ subset $\tilde{\zeta}$ of $\mathcal{S}$ is called a $I_{\mathcal{V}}\mathcal{F}l\mathcal{B}_iQI$ ($I_{\mathcal{V}}\mathcal{F}r\mathcal{B}_iQI$) of $\mathcal{S}$ if $\tilde{\mathcal{S}}\tilde{\zeta} \cap \tilde{\zeta}\tilde{\mathcal{S}}\tilde{\zeta} \subseteq \tilde{\zeta}(\tilde{\zeta}\tilde{\mathcal{S}} \cap \tilde{\zeta}\tilde{\mathcal{S}}\tilde{\zeta} \subseteq \tilde{\zeta})$. $\tilde{\zeta}$ is said to be $I_{\mathcal{V}}\mathcal{F}\,\mathcal{B}_iQI$ of $\mathcal{S}$ if it is both $I_{\mathcal{V}}\mathcal{F}l\mathcal{B}_iQI$ and $I_{\mathcal{V}}\mathcal{F}r\mathcal{B}_iQI$.*

Theorem 4.5. *If $\mathscr{D}$ is a non-empty sub-set of $\mathcal{S}$. Then $\mathscr{D}$ is $\mathcal{B}_i QI$ of $\mathcal{S}$ $\Longleftrightarrow$ characteristic function (cf) of $\mathscr{D}$ is $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$ of $\mathcal{S}$.*

Proof : Let us take $\mathscr{D}$ as $\mathcal{B}_i QI$ of $\mathcal{S}$ implies $(\tilde{cf})_{\mathscr{D}}$ is an $I_{\mathcal{V}}\mathcal{F}$ sub-semigroup of $\mathcal{S}$. By hypothesis we've $\tilde{\mathcal{S}}(\tilde{cf})_{\mathscr{D}} \cap (\tilde{cf})_{\mathscr{D}}\tilde{\mathcal{S}}(\tilde{cf})_{\mathscr{D}} = (\tilde{cf})_{\mathcal{S}\mathscr{D}} \cap (\tilde{cf})_{\mathscr{D}\mathcal{S}\mathscr{D}} \subseteq (\tilde{cf})_{\mathscr{D}}$. Hence $(\tilde{cf})_{\mathscr{D}}$ is a $I_{\mathcal{V}}\mathcal{F}\, l\mathcal{B}_i QI$ of $\mathcal{S}$. Similarly, we prove for $I_{\mathcal{V}}\mathcal{F}\, r\mathcal{B}_i QI$ of $\mathcal{S}$. Conversely, let us assume that $(\tilde{cf})_{\mathscr{D}}$ is an $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$ of $\mathcal{S}$. Then $\mathscr{D}$ is a subsemigroup of $\mathcal{S}$. We have

$$\tilde{\mathcal{S}}(\tilde{cf})_{\mathscr{D}} \cap (\tilde{cf})_{\mathscr{D}}\tilde{\mathcal{S}}(\tilde{cf})_{\mathscr{D}} \subseteq (\tilde{cf})_{\mathscr{D}}$$
$$(\tilde{cf})_{\mathcal{S}\mathscr{D}} \cap \tilde{\chi}_{\mathscr{D}\mathcal{S}\mathscr{D}} \subseteq (\tilde{cf})_{\mathscr{D}}$$
$$(\tilde{cf})_{\mathcal{S}\mathscr{D}\cap\mathscr{D}\mathcal{S}\mathscr{D}} \subseteq (\tilde{cf})_{\mathscr{D}}$$
$$\mathcal{S}\mathscr{D} \cap \mathscr{D}\mathcal{S}\mathscr{D} \subseteq \mathscr{D}$$

Thence, $\mathscr{D}$ is a $l\mathcal{B}_i QI$ of $\mathcal{S}$. Consequently, we verify that $r\mathcal{B}_i QI$.

Theorem 4.6. *A non-empty $I_{\mathcal{V}}\mathcal{F}$ sub-set $\tilde{\zeta}$ of $\mathcal{S}$ an $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$ of $\mathcal{S}$ $\Longleftrightarrow$ the $\langle[\rho_1,\rho_2]\rangle$-cut of $\tilde{\zeta}$ is a $\mathcal{B}_i QI$ of $\mathcal{S}$ $\forall$ $[\rho_1,\rho_2] \in \mathscr{D}[0,1]$.*

Proof: Conclude that $\tilde{\zeta}$ is an $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$ of $\mathcal{S}$.
Let $y \in \tilde{\mathcal{S}}(\tilde{\zeta},[\rho_1,\rho_2]) \cap (\tilde{\zeta},[\rho_1,\rho_2])\tilde{\mathcal{S}}(\tilde{\zeta},[\rho_1,\rho_2])$.
Then $x = gh = klm$, $g,l \in \tilde{\mathcal{S}}$ and $h,k,m \in (\tilde{\zeta},[\rho_1,\rho_2])$ implies $\tilde{\zeta}(h) \geq [\rho_1,\rho_2]$, $\tilde{\zeta}(k) \geq [\rho_1,\rho_2]$ and $\tilde{\zeta}(m) \geq [\rho_1,\rho_2]$
Now, $\tilde{\mathcal{S}}\tilde{\zeta}(y) = \sup_{y=be}\left\{min\left\{\tilde{\mathcal{S}}(b),\tilde{\zeta}(e)\right\}\right\} = \tilde{\zeta}(e) \geq [\rho_1,\rho_2]$
Consider,

$$\left(\tilde{\zeta}\tilde{\mathcal{S}}\tilde{\zeta}\right)(y) = \sup_{y=bec}\left\{min\left\{\tilde{\zeta}\tilde{\mathcal{S}}(be),\tilde{\zeta}(c)\right\}\right\}$$
$$= \sup_{y=bec}\left\{min\left\{\sup_{be=pq}\left\{min\left\{\tilde{\zeta}(p)\tilde{\mathcal{S}}(q)\right\},\tilde{\zeta}(c)\right\}\right\}\right\}$$
$$= \sup_{y=bec}\left\{min\left\{\tilde{\zeta}(p),\tilde{\zeta}(c)\right\}\right\} \geq [\rho_1,\rho_2]$$

Therefore, $\tilde{\zeta}(y) \geq \tilde{\mathcal{S}}\tilde{\zeta} \cap \tilde{\zeta}\tilde{\mathcal{S}}\tilde{\zeta} \geq [\rho_1,\rho_2]$. Hence, $\tilde{\zeta}(y) \geq [\rho_1,\rho_2]$ implies $y \in (\tilde{\zeta},[\rho_1,\rho_2])$. Similarly we prove for $r\mathcal{B}_i QI$.
Conversely $(\tilde{\zeta},[\rho_1,\rho_2])$ is a $\mathcal{B}_i QI$ of $\mathcal{S}$ for all $[\rho_1,\rho_2],[\nu_1,\nu_2] \in \mathscr{D}[0,1]$. Let $x,y \in \mathcal{S}$, $\tilde{\zeta}(x) = [\rho_1,\rho_2]$ and $\tilde{\zeta}(y) = [\nu_1,\nu_2]$ where $[\rho_1,\rho_2] \geq [\nu_1,\nu_2]$ implies $x,y \in (\tilde{\zeta},[\nu_1,\nu_2])$. We have $\tilde{\mathcal{S}}(\tilde{\zeta},[\varsigma_1,\varsigma_2]) \cap (\tilde{\zeta},[\varsigma_1,\varsigma_2])\tilde{\mathcal{S}}(\tilde{\zeta},[\varsigma_1,\varsigma_2]) \subseteq (\tilde{\zeta},[\varsigma_1,\varsigma_2])$ for all $[\varsigma_1,\varsigma_2] \in \mathscr{D}[0,1]$. Suppose $[\iota_1,\iota 2] = min\{\mathscr{D}[0,1]\}$. Then $\tilde{\mathcal{S}}(\tilde{\zeta},[\iota_1,\iota_2]) \cap (\tilde{\zeta},[\iota_1,\iota_2])\tilde{\mathcal{S}}(\tilde{\zeta},[\iota_1,\iota_2]) \subseteq (\tilde{\zeta},[\iota_1,\iota_2])$. Therefore $\tilde{\mathcal{S}}\tilde{\zeta} \cap \tilde{\zeta}\tilde{\mathcal{S}}\tilde{\zeta} \subseteq \tilde{\zeta}$. Hence $\tilde{\zeta}$ is an $I_{\mathcal{V}}\mathcal{F}\, l\mathcal{B}_i QI$ of $\mathcal{S}$. Similarly we can prove for $I_{\mathcal{V}}\mathcal{F}\, r\mathcal{B}_i QI$. Hence the theorem.

Theorem 4.7. *Let $\tilde{\zeta}$ be a non-empty $I_{\mathcal{V}}\mathcal{F}$ sub-set of $\mathcal{S}$. Then an $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$ of $\mathcal{S}$ $\Longleftrightarrow$ the $\langle(\rho_1,\rho_2)\rangle$-cut of $\tilde{\zeta}$ is a $\mathcal{B}_i QI$ of $\mathcal{S}$ $\forall$ $[\rho_1,\rho_2] \in \mathscr{D}[0,1]$.*

Proof: Similar to Theorem 4.6

Theorem 4.8. *Every $I_{\mathcal{V}}\mathcal{F}\, lI$ is an $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$*

Proof: Let $\tilde{\zeta}_l$ be an $I_{\mathcal{V}}\mathcal{F}\, lI$ of $\mathcal{S}$. Let $i_1,r_1,t_1 \in \mathcal{S}$.

$$\tilde{\mathcal{S}}\tilde{\zeta}_l i_1 = \sup_{i_1=r_1t_1}\left\{min\left\{\tilde{\mathcal{S}}(r_1),\tilde{\zeta}_l(t_1)\right\}\right\}$$
$$= \sup_{i_1=r_1t_1}\left\{min\left\{\tilde{1},\tilde{\zeta}_l(t_1)\right\}\right\}$$
$$= \sup_{i_1=r_1t_1}\left\{\tilde{\zeta}_l(t_1)\right\}$$
$$\leq \sup_{i_1=r_1t_1}\left\{\tilde{\zeta}_l(r_1t_1)\right\} = \tilde{\zeta}_l(i_1)$$

Consider,

$$\tilde{\zeta}\tilde{\mathcal{S}}\tilde{\zeta}_l(i_1) = \sup_{i_1=m_1n_1v_1}\left\{min\left\{\tilde{\zeta}_l(m_1),(\tilde{\mathcal{S}}\tilde{\zeta}_l)(n_1v_1)\right\}\right\}$$
$$\leq \sup_{i_1=m_1n_1v_1}\left\{min\left\{\tilde{\zeta}_l(m_1),\tilde{\zeta}_l(n_1v_1)\right\}\right\}$$
$$= \tilde{\zeta}_l(i_1)$$

Now, $\tilde{\zeta}_l\tilde{\mathcal{S}} \cap \tilde{\zeta}_l\tilde{\mathcal{S}}\tilde{\zeta}_l = min\left\{\tilde{\zeta}_l\tilde{\mathcal{S}}(i_1),\tilde{\zeta}_l\tilde{\mathcal{S}}\tilde{\zeta}_l(i_1)\right\} \leq \tilde{\zeta}_l(i_1)$
Hence $\tilde{\zeta}_l$ is an $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$ of $\mathcal{S}$.

Theorem 4.9. *Every $I_{\mathcal{V}}\mathcal{F}\, rI$ of $\mathcal{S}$ is an $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$ of $\mathcal{S}$.*

Proof: Assuming $\tilde{\zeta}_r$ be an $I_{\mathcal{V}}\mathcal{F}\, rI$ of $\mathcal{S}$ and $i_1,a_1,b_1 \in \mathcal{S}$.

$$\tilde{\zeta}_r\tilde{\mathcal{S}}i_1 = \sup_{i_1=a_1b_1}\left\{min\left\{\tilde{\zeta}_r(a_1),\tilde{\mathcal{S}}(b_1)\right\}\right\}$$
$$= \sup_{i_1=a_1b_1}\left\{min\left\{\tilde{\zeta}_r(a_1)\right\}\right\}$$
$$\leq \sup_{i_1=a_1b_1}\left\{\tilde{\zeta}_r(a_1b_1)\right\} = \tilde{\zeta}_r(i_1)$$

Also,

$$(\tilde{\zeta}_r\tilde{\mathcal{S}}\tilde{\zeta}_r)(i_1) = \sup_{i_1=u_1v_1r_1}\left\{min\left\{\tilde{\zeta}_r\tilde{\mathcal{S}}(u_1v_1),\tilde{\zeta}_r(r_1)\right\}\right\}$$
$$\leq \sup_{i_1=u_1v_1r_1}\left\{min\left\{\tilde{\zeta}_r(u_1v_1r_1),\tilde{\zeta}_r(r_1)\right\}\right\}$$
$$= \tilde{\zeta}_r(i_1)$$

Now, $(\tilde{\zeta}_r\tilde{\mathcal{S}} \cap \tilde{\zeta}_r\tilde{\mathcal{S}})\tilde{\zeta}_r = min\left\{\tilde{\zeta}_r\tilde{\mathcal{S}}(i_1),\tilde{\zeta}_r\tilde{\mathcal{S}}\tilde{\zeta}_r(i_1)\right\}$
$\leq \tilde{\zeta}_r(i_1)$
Thus $\tilde{\zeta}_r$ is an $I_{\mathcal{V}}\mathcal{F}\, r\mathcal{B}_i QI$ of $\mathcal{S}$. Similarly we verify for $\tilde{\mathcal{S}}\tilde{\zeta}_r \cap \tilde{\zeta}_r\tilde{\mathcal{S}}\tilde{\zeta}_r \subseteq \tilde{\zeta}_r$. Hence $\tilde{\zeta}_r$ is an $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$ of $\mathcal{S}$.

Theorem 4.10. *Intersection of two $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$ is $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$*

Proof: Let us take $\tilde{\zeta}_i$ and $\tilde{\zeta}_k$ are $I_{\mathcal{V}}\mathcal{F}\mathcal{B}_i QI$ of $\mathcal{S}$.
Let $x,a,b \in \mathcal{S}$. Consider

$$\tilde{\mathcal{S}}(\tilde{\zeta}_i \cap \tilde{\zeta}_k)(x) = \sup_{x=ab}\left\{min\left\{\tilde{\mathcal{S}}(a),(\tilde{\zeta}_i \cap \tilde{\zeta}_k)(b)\right\}\right\}$$
$$= \sup_{x=ab}\left\{min\left\{\tilde{\mathcal{S}}(a),min\left\{\tilde{\zeta}_i(b),\tilde{\zeta}_k)(b)\right\}\right\}\right\}$$
$$= \sup_{x=ab}\left\{min\left\{min\left\{\tilde{\mathcal{S}}(a),\tilde{\zeta}_i(b)\right\},min\left\{\tilde{\mathcal{S}}(a),\tilde{\zeta}_k)(b)\right\}\right\}\right\}$$
$$= min\left\{\tilde{\mathcal{S}}\tilde{\zeta}_i(x),\tilde{\mathcal{S}}\tilde{\zeta}_k(x)\right\}$$
$$= \tilde{\mathcal{S}}\tilde{\zeta}_i \cap \tilde{\mathcal{S}}\tilde{\zeta}_k(x)$$

Again consider,

$$(\tilde{\zeta}_1 \cap \tilde{\zeta}_k)\tilde{\mathcal{S}}(\tilde{\zeta}_i \cap \tilde{\zeta}_k)(x)$$
$$= \sup_{x=abc}\left\{min\left\{(\tilde{\zeta}_i \cap \tilde{\zeta}_k)(a),\tilde{\mathcal{S}}(\tilde{\zeta}_i \cap \tilde{\zeta}_k)(bc)\right\}\right\}$$
$$= \sup_{x=abc}\left\{min\left\{(\tilde{\zeta}_i \cap \tilde{\zeta}_k)(a),\tilde{\mathcal{S}}\tilde{\zeta}_i \cap \tilde{\mathcal{S}}\tilde{\zeta}_k)(bc)\right\}\right\}$$
$$= min\left\{\zeta_i\tilde{\mathcal{S}}\tilde{\zeta}_i(x),\tilde{\zeta}_k\tilde{\mathcal{S}}\tilde{\zeta}_k(x)\right\}$$
$$= \tilde{\zeta}_i\tilde{\mathcal{S}}\tilde{\zeta}_i \cap \tilde{\zeta}_k\tilde{\mathcal{S}}\tilde{\zeta}_k(x).$$

Also, $\tilde{\mathcal{S}}(\tilde{\zeta}_i \cap \tilde{\zeta}_k)\tilde{\mathcal{S}} = (\tilde{\mathcal{S}}\tilde{\zeta}_i\tilde{\mathcal{S}}) \cap (\tilde{\mathcal{S}}\tilde{\zeta}_k\tilde{\mathcal{S}})$

$\tilde{S}(\tilde{\zeta}_i \cap \tilde{\zeta}_k) \cap (\tilde{\zeta}_i \cap \tilde{\zeta}_k) S (\tilde{\zeta}_i \cap \tilde{\zeta}_k)$
$= (\tilde{S}\tilde{\zeta}_i) \cap (\tilde{\zeta}_i \cap \tilde{\zeta}_k) \cap (\tilde{S}\tilde{\zeta}_k) \cap (\tilde{\zeta}_i \tilde{S} \tilde{\zeta}_k) \subseteq \tilde{\zeta}_i \cap \tilde{\zeta}_k$
Thence $\tilde{\zeta}_i \cap \tilde{\zeta}_k$ is a $I_{\mathcal{V}}\mathcal{F}l\mathcal{B}_iQI$. Consequently we verify for $I_{\mathcal{V}}\mathcal{F}r\mathcal{B}_iQI$. Therefore $\tilde{\zeta}_i \cap \tilde{\zeta}_k$ is a $I_{\mathcal{V}}\mathcal{F}\,\mathcal{B}_iQI$.

Theorem 4.11. *Intersection of $I_{\mathcal{V}}\mathcal{F}r\mathcal{B}_iQI$ and $I_{\mathcal{V}}\mathcal{F}l\mathcal{B}_iQI$ of $\mathcal{S}$ is an $I_{\mathcal{V}}\mathcal{F}\,\mathcal{B}_iQI$of $\mathcal{S}$.*

Proof: We take $\tilde{\zeta}_r$ and $\tilde{\zeta}_l$ be $I_{\mathcal{V}}\mathcal{F}r\mathcal{B}_iQI$ and $I_{\mathcal{V}}\mathcal{F}l\mathcal{B}_iQI$ of $\mathcal{S}$ respectively. Then by Theorem 4.10
$\tilde{S}(\tilde{\zeta}_r \cap \tilde{\zeta}_l) = \tilde{S}\tilde{\zeta}_r \cap \tilde{S}\tilde{\zeta}_l$ and $(\tilde{\zeta}_r \cap \tilde{\zeta}_l)\tilde{S}(\tilde{\zeta}_r \cap \tilde{\zeta}_l) = \tilde{\zeta}_r \tilde{S} \tilde{\zeta}_r \cap \tilde{\zeta}_l \tilde{S} \tilde{\zeta}_l$
$\tilde{S}(\tilde{\zeta}_r \cap \tilde{\zeta}_l) \cap (\tilde{\zeta}_r \cap \tilde{\zeta}_l) S (\tilde{\zeta}_r \cap \tilde{\zeta}_l)$
$= (\tilde{S}\tilde{\zeta}_r) \cap (\tilde{\zeta}_r \cap \tilde{\zeta}_l) \cap (\tilde{S}\tilde{\zeta}_l) \cap (\tilde{\zeta}_l \tilde{S} \tilde{\zeta}_l) \subseteq \tilde{\zeta}_r \cap \tilde{\zeta}_l$
Hence $\tilde{\zeta}_r \cap \tilde{\zeta}_l$ is a $I_{\mathcal{V}}\mathcal{F}l\mathcal{B}_iQI$ of $\mathcal{S}$. In a similar fashion we prove $\tilde{\zeta}_r \cap \tilde{\zeta}_l$ is a $I_{\mathcal{V}}\mathcal{F}r\mathcal{B}_iQI$.

In section 5,6,7 and 8 we denote $\wp$ as a complete congruence relation over $\mathcal{S}$.

5. $\mathcal{RBI}_nI$ of a semigroup

In this section we introduce the concept of rough bi-interior-ideal ($\mathcal{RBI}_nI$) in semigroup. It is proved that a bi-interior-ideal($\mathcal{BI}_nI$) of $\mathcal{S}$ is a rough bi-interior-ideal ($\mathcal{RBI}_nI$).

Definition 5.1. *A sub-set of $\mathcal{S}$ is said to be $\mathcal{RBI}_nI$ of $\mathcal{S}$ if it is both $\wp$-upper and $\wp$-lower $\mathcal{RBI}_nI$.*

An upper (lower) approximation of a sub-set of $\mathcal{S}$ is $\mathcal{BI}_nI$ then it is called an $\wp$-upper ($\wp$-lower) $\mathcal{RBI}_nI$.

Theorem 5.2. *Prove that a $\mathcal{BI}_nI$ of $\mathcal{S}$ is both $\wp$-upper and $\wp$-lower $\mathcal{RBI}_nI$ of $\mathcal{S}$.*

Proof: Suppose that κ be a $\mathcal{BI}_nI$ of $\mathcal{S}$ then $\mathcal{S}\kappa\mathcal{S} \cap \kappa\mathcal{S}\kappa \subseteq \kappa$. Let $x \in \mathcal{S}\overline{\wp}(\kappa)\mathcal{S}$ then $x = s_1 p s_2$ with $p \in \overline{\wp}(\kappa)$ and $s_1, s_2 \in \mathcal{S}$. $p \in \overline{\wp}(\kappa)$ then $[p]_\wp \cap \kappa \neq \phi$ $\exists$ $q \in [p]_\wp \cap \kappa$ such that $q \in [p]_\wp$ and $q \in \kappa$ implies $(p,q) \in \wp$ and $q \in \kappa$. Since κ is a $\mathcal{BI}_nI$ of $\mathcal{S}$ then $s_1 q s_2 \in \kappa$ and $r s_1 t \in A$ where $s_1, s_2 \in \mathcal{S}$ and $q, r, t \in \kappa$.

$$\begin{aligned}(p,q) \in \wp &\Rightarrow (s_1 p s_2, s_1 q s_2) \in \wp \\ &\Rightarrow s_1 q s_2 \in [s_1 p s_2]_\wp \\ &\Rightarrow [s_1 p s_2]_R \cap \kappa \neq \phi \\ &\Rightarrow [x]_\wp \cap \kappa \neq \phi \\ &\Rightarrow x \in \overline{\wp}(\kappa)\end{aligned}$$

Thus, $\mathcal{S}\overline{\wp}(\kappa)\mathcal{S} \subseteq \overline{\wp}(\kappa)$ (1)
Consider, $\mathcal{S}\overline{\wp}(\kappa)\mathcal{S} \cap \overline{\wp}(\kappa)\mathcal{S}\overline{\wp}(\kappa) \subseteq \mathcal{S}\overline{\wp}(\kappa)\mathcal{S} \subseteq \overline{\wp}(\kappa) by (1)$
Hence, κ is $\wp$-upper $\mathcal{RBI}_nI$ of $\mathcal{S}$.
(ii) $\mathcal{S}\underline{\wp}(\kappa)\mathcal{S} \cap \underline{\wp}(\kappa)\mathcal{S}\underline{\wp}(\kappa)$

$$\begin{aligned}&= \underline{\wp}(\mathcal{S})\underline{\wp}(\kappa)\underline{\wp}(\mathcal{S}) \cap \underline{\wp}(\kappa)\underline{\wp}(\mathcal{S})\underline{\wp}(\kappa) \\ &\subseteq \underline{\wp}(\mathcal{S}\kappa\mathcal{S}) \cap \underline{\wp}(\kappa\mathcal{S}\kappa) \\ &\subseteq \underline{\wp}((\mathcal{S}\kappa\mathcal{S}) \cap (\kappa\mathcal{S}\kappa)) \\ &\subseteq \underline{\wp}(\kappa)\end{aligned}$$

Hence, κ is $\wp$-lower $\mathcal{RBI}_nI$ of $\mathcal{S}$.

Corollary 5.3. *A $\mathcal{BI}_nI$ of $\mathcal{S}$ is $\mathcal{RBI}_nI$.*

Proof: Applying Theorem 5.2 we get the proof.

6. $\mathcal{RB}_iQI$ of semigroup

In this section we apply the roughness to bi-quasi-ideal($\mathcal{B}_iQI$).

Definition 6.1. *A sub-set of $\mathcal{S}$ is said to be $\mathcal{RB}_iQI$ of $\mathcal{S}$ if it is both $\wp$-upper and $\wp$-lower $\mathcal{RB}_iQI$.*

An upper (lower) approximation of a sub-set of $\mathcal{S}$ is $\mathcal{B}_iQI$ then it is called an $\wp$-upper ($\wp$-lower) $\mathcal{RB}_iQI$.

Theorem 6.2. *Prove that a $\mathcal{B}_iQI$ of $\mathcal{S}$ is both $\wp$-upper and $\wp$-lower $\mathcal{RB}_iQI$ of $\mathcal{S}$.*

Proof: Suppose assume that η be a $l\mathcal{B}_iQI$ of $\mathcal{S}$ then $\mathcal{S}\eta \cap \eta\mathcal{S}\eta \subseteq \eta$. Let $y \in \mathcal{S}\overline{\wp}(\eta)$then $y = s_1 f$ with $f \in \overline{\wp}(\eta)$ and $s_1 \in \mathcal{S}$. Since $f \in \overline{\wp}(\eta)$ then $[f]_\wp \cap \eta \neq \phi$ $\exists$ $r \in [f]_\wp \cap \eta$ such that $r \in [f]_\wp$ and $r \in \eta$ implies $(f,r) \in \wp$ and $r \in \eta$. Since η is a $\mathcal{B}_iQI$ of $\mathcal{S}$ then $s_1 a \in \eta$ and $l s_2 m \in \eta$ where $s_1 s_2 \in \mathcal{S}$ and $a, l, m \in \eta$.

$$\begin{aligned}(f,p) \in \wp &\Rightarrow (s_1 f, s_1 r) \in \wp \\ &\Rightarrow s_1 r \in [s_1 f]_\wp \\ &\Rightarrow [s_1 r]_\wp \cap \eta \neq \phi \\ &\Rightarrow [x]_\wp \cap \eta \neq \phi \\ &\Rightarrow x \in \overline{\wp}(\eta)\end{aligned}$$

Thus, $\mathcal{S}\overline{\wp}(\eta) \subseteq \overline{\wp}(\eta)$ (1)
Consider,$\mathcal{S}\overline{\wp}(\eta) \cap \overline{\wp}(\eta)\mathcal{S}\overline{\wp}(\eta) \subseteq \mathcal{S}\overline{\wp}(\eta) \subseteq \overline{\wp}(\eta)$ by (1).
Thus, η is $\wp$-upper $\mathcal{R}l\mathcal{B}_iQI$ of $\mathcal{S}$. In a similar way we prove $\wp$-upper $\mathcal{R}r\mathcal{B}_iQI$ of $\mathcal{S}$. Hence η is $\wp$-upper $\mathcal{RB}_iQI$ of $\mathcal{S}$.
(ii)$\mathcal{S}\underline{\wp}(\eta)\mathcal{S} \cap \underline{\wp}(\eta)\mathcal{S}\underline{\wp}(\eta)$

$$\begin{aligned}&= \underline{\wp}(\mathcal{S})\underline{\wp}(\eta)\underline{\wp}(\mathcal{S}) \cap \underline{\wp}(\eta)\underline{\wp}(\mathcal{S})\underline{\wp}(\eta) \\ &\subseteq \underline{\wp}(\mathcal{S}\eta) \cap \underline{\wp}(\eta\mathcal{S}\eta) \\ &\subseteq \underline{\wp}(\mathcal{S}\eta \cap \eta\mathcal{S}\eta) \\ &\subseteq \underline{\wp}(\eta)\end{aligned}$$

Thus, η is a $\wp$-lower $\mathcal{R}l\mathcal{B}_iQI$ of $\mathcal{S}$. In similar fashion we prove for $\wp$-lower $\mathcal{R}r\mathcal{B}_iQI$. Hence η is $\wp$-lower $\mathcal{RB}_iQI$ of $\mathcal{S}$.

Corollary 6.3. *A $\mathcal{B}_iQI$ is $\mathcal{RB}_iQI$ of $\mathcal{S}$.*

Proof: Applying Theorem 6.2 result is obvious.

7. $I_{\mathcal{V}}\mathcal{RFBI}_nI$ of semigroup

We now extend the idea of interval valued fuzzy bi-interior-ideal($I_{\mathcal{V}}\mathcal{FBI}_nI$) of $\mathcal{S}$ by interval valued rough fuzzy bi-interior-ideal($I_{\mathcal{V}}\mathcal{RFBI}_nI$) of $\mathcal{S}$.

Definition 7.1. *An $I_{\mathcal{V}}\mathcal{RFBI}_nI$ of $\mathcal{S}$ is defined as if it is both $\wp$-upper and $\wp$-lower $I_{\mathcal{V}}\mathcal{RFBI}_nI$ of $\mathcal{S}$.*

An $I_{\mathcal{V}}\mathcal{FBI}_nI$ $\tilde{\tau}$ of $\mathcal{S}$ is called an $\wp$-upper ($\wp$-lower) $I_{\mathcal{V}}\mathcal{RFBI}_nI$ of $\mathcal{S}$ if its upper(lower) approximation is an $I_{\mathcal{V}}\mathcal{FBI}_nI$ of $\mathcal{S}$.

Theorem 7.2. *An $I_{\mathcal{V}}\mathcal{FBI}_nI$ of $\mathcal{S}$ is an $I_{\mathcal{V}}\mathcal{RFBI}_nI$ of $\mathcal{S}$.*

Proof: Applying Theorem 3.3 $\langle[\rho_1, \rho_2]\rangle$-cut of $\tilde{\tau}$ is $\mathcal{BI}_nI$ of $\mathcal{S}$. Assume that if lower approximation of $\langle[\rho_1, \rho_2]\rangle$-cut of $\tilde{\tau}$ is non-empty, then By Theorem 5.2 $\underline{\wp}(\tilde{\tau}, [\rho_1, \rho_2])$ is $\mathcal{BI}_nI$ of $\mathcal{S}$. By Theorem 4.6 in [8] $\left(\underline{\wp}(\tilde{\tau}), [\rho_1, \rho_2]\right)$ is $\mathcal{BI}_nI$ of $\mathcal{S}$. Again By Theorem 3.3 lower approximation of $\tilde{\tau}$ is an $I_{\mathcal{V}}\mathcal{FBI}_nI$ of $\mathcal{S}$. Consequently, we prove the other case. Thence theorem.

Theorem 7.3. *Lower approximation of* $\tilde{\tau}$ *is* $\mathcal{I_V F B I_n I}$ $\iff$ *lower approximation of* $\langle[\rho_1,\rho_2]\rangle$*-cut of* $\tilde{\tau}$ *is* $\mathcal{BI_n I}$

Proof: Theorem 3.3 and Theorem4.6 (i) in [8].

Theorem 7.4. *Upper approximation of* $\tilde{\tau}$ *is* $\mathcal{I_V F B I_n I}$ $\iff$ *upper approximation of* $\langle(\rho_1,\rho_2)\rangle$*-cut of* $\tilde{\tau}$ *is* $\mathcal{BI_n I}$.

Proof: Theorem 3.4 and Theorem4.6(ii) in [8].

8. $\mathcal{I_V RF B_i QI}$ of a semigroup

In this section we extend the roughness to interval valued rough fuzzy set ($\mathcal{I_V RF}$). In particular we extend bi-quasi-ideal ($\mathcal{B_i QI}$) to interval valued rough fuzzy bi-quasi-ideal ($\mathcal{I_V RF B_i QI}$) of semigroup.

Definition 8.1. $\mathcal{I_V RF B_i QI}$ *of* $\mathcal{S}$ *is defined as if it is both* $\wp$*-upper and* $\wp$*-lower* $\mathcal{I_V RF B_i QI}$ *of* $\mathcal{S}$.
An $\mathcal{I_V F B_i QI}$ $\tilde{\tau}$ *of* $\mathcal{S}$ *is called an* $\wp$*-upper (*$\wp$*-lower)* $\mathcal{I_V RF B_i QI}$ *of* $\mathcal{S}$ *if its upper(lower) approximation is an* $\mathcal{I_V F B_i QI}$ *of* $\mathcal{S}$.

Theorem 8.2. *An* $\mathcal{I_V F B_i QI}$ *of* $\mathcal{S}$ *is an* $\mathcal{I_V RF B_i QI}$ *of* $\mathcal{S}$.

Proof: Applying Theorem 4.6 $\langle[\rho_1,\rho_2]\rangle$-cut of $\tilde{\tau}$ is $\mathcal{B_i QI}$ of $\mathcal{S}$. Assume that if lower approximation of $\langle[\rho_1,\rho_2]\rangle$-cut of $\tilde{\tau}$ is non-empty, then By Theorem 6.2 $\underline{\wp}(\tilde{\tau},[\rho_1,\rho_2])$ is $\mathcal{B_i QI}$ of $\mathcal{S}$. By Theorem 4.6 in [8] $\left(\underline{\wp}(\tilde{\tau}),[\rho_1,\rho_2]\right)$ is $\mathcal{BI_n I}$ of $\mathcal{S}$. Again Theorem 4.6 lower approximation of $\tilde{\tau}$ is an $\mathcal{I_V F B I_n I}$ of $\mathcal{S}$. Consequently, we prove the other case. Thence theorem.

Theorem 8.3. *Lower approximation of* $\tilde{\tau}$ *is* $\mathcal{I_V F B_i QI}$ $\iff$ *lower approximation of* $\langle[\rho_1,\rho_2]\rangle$*- cut of* $\tilde{\tau}$ *is* $\mathcal{BI_n I}$.

Theorem 8.4. *Upper approximation of* $\tilde{\tau}$ *is* $\mathcal{I_V F B_i QI}$ $\iff$ *upper approximation of* $\langle(\rho_1,\rho_2)\rangle$*- cut of* $\tilde{\tau}$ *is* $\mathcal{B_i QI}$.

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