Malaya Journal of Matematik, Vol. 8, No. 2, 522-526, 2020

https://doi.org/10.26637/MJM0802/0034

A short note on some of the fuzzy rough hyper-ideals in semihyper-groups

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Abstract

In this paper we apply some of the ideals in semihyper-group in terms of fuzzy rough set. Semihyper-group is an algebraic structure which is an extended structure of semigroup. We introduce the concept of fuzzy rough hyper-ideal and fuzzy rough bi hyper-ideal in semihyper-group. We define the $q-$level set in semihyper-group and study the relation of ideals between the fuzzy rough set and the $q-$level set of a fuzzy rough set.

Keywords

Fuzzy rough set, semihyper-group, Fuzzy rough hyper-ideals, Fuzzy rough subsemihyper-group, Fuzzy rough interior hyper-ideals, q-level set.

AMS Subject Classification

20M05, 08A72, 20M12.

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Article History: Received **12** November **2019**; Accepted **19** March **2020**

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1. Introduction

Marty(1934)[6] inspired from algebraic structures, he extended the concept to algebraic hyper-structures. The attraction of hyper-structure is its special property that the image of each pair of a cross product of two sets is lead to a set where in classical structures it is an element again. Davvaz(2002)[2], Kehayopulu[4][5], Corsini and Shabir[1] studied some of the fuzzy ideals in hyper-structures. In this paper we apply some of the ideals which was introduced by Subha et.al [7] in hyper-structures. In each section we discuss distinct fuzzy rough hyper-ideals and study its properties.

2. Preliminaries

In this section we recollect the basic definitions of ideals in hyper-structures such as hyper-ideal, subsemihyper-group and bi hyper-ideal.

Definition 2.1. *[4] Let $\mathscr{H}$ be a non-empty set and Θ be the hyper-operation on $\mathscr{H}$ is defined as*

$\Theta : \mathscr{H} \times \mathscr{H} \to \mathscr{F}(\mathscr{H})$

where $\mathscr{F}(\mathscr{H})$ is the set of all subsets of $\mathscr{H}$. Then the set $\mathscr{H}$ with the hyper-operation Θ is called hyper-groupoid(say $\mathscr{H}^{\Theta}$).

The image of the pair $(g_1,g_2) \in \mathscr{H} \times \mathscr{H}$ is denoted by $g_1\Theta g_2$, the hyper-product of the elements $g_1,g_2 \in \mathscr{H}$. Define hyper-operation

$\star : \mathscr{F}(\mathscr{H}) \times \mathscr{F}(\mathscr{H}) \to \mathscr{F}(\mathscr{H}).$

If X and Y are the subsets of $\mathscr{F}(\mathscr{H})$ then the hyper-product of the sets X and Y is defined by

$$X \star Y = \bigcup_{(g_1,g_2)\in X\times Y} g_1\Theta g_2 \tag{2.1}$$

We write the hyper-operation on a set X with an element

$'g_1'$ as $X \star \{g_1\}$ and an hyper-operation on an element $'g_1'$ with any set say X is defined by $\{g_1\} \star X$.

The hyper-operation between the elements is denoted by Θ. The hyper-operation between the sets is denoted by $\star$.

Definition 2.2. *[4] A hyper-groupoid $(\mathscr{H}^{\Theta})$ is called a semihyper-group(say $\mathscr{H}^{\star}$) if*
$(\{r\} \star \{s\}) \star \{t\} = \{r\} \star (\{s\} \star \{t\})$ *for all* $r,s,t \in \mathscr{H}$.

Example 2.3. *Let us consider the universe set $\mathscr{H}$ as $\mathscr{H} = \{r,s,t\}$. Define a hyper-operation Θ on $\mathscr{H}$ is defined by $r\Theta s = \{r,s\}$ $\forall$ $r,s \in \mathscr{H}$ which is a hyper-groupoid. Then we have*

$$\begin{aligned} r\Theta(s\Theta t) &= \{r\} \star \{s,t\} \\ &= (r\Theta s) \cup (r\Theta t) \\ &= \{r,s\} \cup \{r,t\} \\ &= \{r,s,t\} \end{aligned}$$

And also we have $(r\Theta s)\Theta t = \{r,s,t\}$ Hence $r\Theta(s\Theta t) = (r\Theta s)\Theta t$ holds for all $r,s,t \in \mathscr{H}$. Therefore $\mathscr{H}$ is a semihyper-group on the hyper-operation $\star$.

Definition 2.4. *[4] A non-empty subset F of a semihyper-group $\mathscr{H}^{\star}$ is called a left(right) hyper-ideal of $\mathscr{H}^{\star}$ if $\mathscr{H}^{\star}F \subseteq F (F\mathscr{H}^{\star} \subseteq F)$.*

If a subset F in $\mathscr{H}^{\star}$ is both a left and right hyper-ideal then F is called hyper-ideal of $\mathscr{H}^{\star}$.

3. Fuzzy rough hyper-ideals of $(\mathscr{H}^{\star}, \rho)$

In this section we introduce fuzzy rough hyper-ideals in semihyper-group.

Definition 3.1. *Let $(\mathscr{H}^{\star}, \rho)$ be a fuzzy approximation space where $\mathscr{H}^{\star}$ is a semihyper-group with hyper-operation $\star$ and ρ be a fuzzy equivalence relation. From the fuzzy set $\widetilde{F}$ in $\mathscr{H}^{\star}$ and the fuzzy equivalence relation ρ, we define a fuzzy rough(FR) set $\rho(\widetilde{F}) = (\underline{\rho}(\widetilde{F}), \overline{\rho}(\widetilde{F}))$ which is a pair of lower and upper approximations of a fuzzy set $\widetilde{F}$ and its membership function is given by*
$\mu_{\underline{\rho}(\widetilde{F})} : \mathscr{H}^{\star} \to [0,1]$ *and*
$\mu_{\overline{\rho}(\widetilde{F})} : \mathscr{H}^{\star} \to [0,1]$
with the property that $\mu_{\underline{\rho}(\widetilde{F})}(f) \le \mu_{\overline{\rho}(\widetilde{F})}(f)$ $\forall$ $f \in \mathscr{H}^{\star}$.

Definition 3.2. *A FR set $\rho(\widetilde{F}) = (\underline{\rho}(\widetilde{F}), \overline{\rho}(\widetilde{F}))$ in $(\mathscr{H}^{\star}, \rho)$ is said to be a fuzzy rough left hyper-ideal($FRHI_L$) of $(\mathscr{H}^{\star}, \rho)$ if*
(i) $\mu_{\underline{\rho}(\widetilde{F})}(n) \le \inf_{g \in m\Theta n} \mu_{\underline{\rho}(\widetilde{F})}(g)$
(ii) $\mu_{\overline{\rho}(\widetilde{F})}(n) \le \inf_{g \in m\Theta n} \mu_{\overline{\rho}(\widetilde{F})}(g)$ $\forall m,n,g \in \mathscr{H}^{\star}$.
A FR set $\rho(\widetilde{F}) = (\underline{\rho}(\widetilde{F}), \overline{\rho}(\widetilde{F}))$ in $(\mathscr{H}^{\star}, \rho)$ is said to be a fuzzy rough right hyper-ideal($FRHI_R$) of $(\mathscr{H}^{\star}, \rho)$ if
(i) $\mu_{\underline{\rho}(\widetilde{F})}(m) \le \inf_{g \in m\Theta n} \mu_{\underline{\rho}(\widetilde{F})}(g)$
(ii) $\mu_{\overline{\rho}(\widetilde{F})}(m) \le \inf_{g \in m\Theta n} \mu_{\overline{\rho}(\widetilde{F})}(g)$ $\forall$ $m,n,g \in \mathscr{H}^{\star}$.
A FR set $\rho(\widetilde{F}) = (\underline{\rho}(\widetilde{F}), \overline{\rho}(\widetilde{F}))$ which is both a $FRHI_L$ and $FRHI_R$ is called $FRHI$ of $(\mathscr{H}^{\star}, \rho)$.

Proposition 3.3. *If $\rho(\widetilde{X}) = (\underline{\rho}(\widetilde{X}), \overline{\rho}(\widetilde{X}))$ and $\rho(\widetilde{Y}) = (\underline{\rho}(\widetilde{Y}), \overline{\rho}(\widetilde{Y}))$ are the two $FRHI$s of $(\mathscr{H}^{\star}, \rho)$ then the intersection of $\rho(\widetilde{X})$ and $\rho(\widetilde{Y})$ is also a $FRHI$ of $(\mathscr{H}^{\star}, \rho)$.*

Proof: Let $\underline{\rho}(\widetilde{X})$ and $\underline{\rho}(\widetilde{Y})$ are $FRHI_L$. For $m,n,g \in (\mathscr{H}^{\star}, \underline{\rho})$ we have

$$\begin{aligned} \mu_{\underline{\rho}(\widetilde{X}) \cap \underline{\rho}(\widetilde{Y})}(n) &= \min\{\mu_{\underline{\rho}(\widetilde{X})}(n), \mu_{\underline{\rho}(\widetilde{Y})}(n)\} \\ &\le \min\{\inf_{g \in m\Theta n} \mu_{\underline{\rho}(\widetilde{X})}(g), \inf_{g \in m\Theta n} \mu_{\underline{\rho}(\widetilde{Y})}(g)\} \\ &\le \inf_{g \in m\Theta n} \{\min\{\mu_{\underline{\rho}(\widetilde{X})}(g), \mu_{\underline{\rho}(\widetilde{Y})}(g)\}\} \\ &\le \inf_{g \in m\Theta n} \{\mu_{\underline{\rho}(\widetilde{X}) \cap \underline{\rho}(\widetilde{Y})}(g)\}. \end{aligned}$$

Therefore $\underline{\rho}(\widetilde{X}) \cap \underline{\rho}(\widetilde{Y})$ is a $FRHI_L$. Similarly we can prove that $\overline{\rho}(\widetilde{X}) \cap \overline{\rho}(\widetilde{Y})$ is also a $FRHI_L$ of $(\mathscr{H}^{\star}, \rho)$. Likewise we can prove that $\rho(X) \cap \rho(Y)$ is a $FRHI_R$. Hence we conclude the theorem.

Definition 3.4. *Let $\rho(\widetilde{X}) = (\underline{\rho}(\widetilde{X}), \overline{\rho}(\widetilde{X}))$ be a FR set of $(\mathscr{H}^{\star}, \rho)$ then the $q-$ level set of $\rho(X)$ is defined by*
$X_L^q = \{g \in \mathscr{H}^{\star} : \mu_{\underline{\rho}(\widetilde{F})}(g) \ge q\}$ *and*
$X_U^q = \{g \in \mathscr{H}^{\star} : \mu_{\overline{\rho}(\widetilde{F})}(g) \ge q\}$ $\forall$ $g \in \mathscr{H}^{\star}$.

Theorem 3.5. *If $\rho(\widetilde{X}) = (\underline{\rho}(\widetilde{X}), \overline{\rho}(\widetilde{X}))$ is a $FRHI$ of $(\mathscr{H}^{\star}, \rho)$ then the $q-$ level set $X^q = (X_L^q, X_U^q)$ is a hyper-ideal.*

Proof: Let us assume that $\underline{\rho}(\widetilde{X})$ be a $FRHI_L$. By Def[3.4], it is clear that X_L^q is a crisp set. Now we have to show that X_L^q is a hyper-ideal. For that let us assume $m \in \mathscr{H}^{\star}$ and $n \in X_L^q$. which implies that

$$\mu_{\underline{\rho}(\widetilde{X})}(n) \ge q \tag{3.1}$$

Since $\mu_{\underline{\rho}(\widetilde{X})}(n) \le \inf_{g \in m\Theta n} \mu_{\underline{\rho}(\widetilde{X})}(g)$
$\implies \inf_{g \in m\Theta n} \mu_{\underline{\rho}(\widetilde{X})}(g) \ge q$
$\implies \mu_{\underline{\rho}(\widetilde{X})}(g) \ge q$ for $g \in m\Theta n$
$\implies m\Theta n \subseteq X_L^q$ for $m \in \mathscr{H}^{\Theta}$ and $n \in X_L^q$.
Hence from the Def[2.4] we can say that X_L^q is a left hyper-ideal. Similarly we can prove that X_U^q is a left hyper-ideal. Therefore X_q is a hyper-ideal of $\mathscr{H}^{\star}$.

Now let us assume that $\underline{\rho}(\widetilde{X})$ is a $FRHI_R$

$$i.e., \mu_{\underline{\rho}(\widetilde{X})}(m) = \inf_{g \in m\Theta n} \mu_{\underline{\rho}(\widetilde{X})}(g) \tag{3.2}$$

Let $m \in X_L^q$ and $n \in \mathscr{H}^{\star}$ which implies $\mu_{\underline{\rho}(\widetilde{X})}(m) \ge q$
Eq. (3.2) $\implies \inf_{g \in m\Theta n} \mu_{\underline{\rho}(\widetilde{X})}(m) \ge q$

$$\Longrightarrow \mu_{\underline{\rho}(\widetilde{X})}(m) \geq q \text{ for } g \in m\Theta n$$

then $m\Theta n \subseteq X_L^q$ for $m \in X_L^q$ and $n \in \mathscr{H}^\star$.
Therefore X_L^q is a right hyper-ideal. Similarly X_U^q is also a right hyper-ideal. Hence X^q is a right hyper-ideal of $(\mathscr{H}^\star, \rho)$.

It is clear that if $\rho(\widetilde{X})$ is a $FRHI$ then X^q is a left and right hyper-ideal. Therefore we conclude that X^q is a hyper-ideal of $\mathscr{H}^\star$.

4. Some properties on max-min hyper-product of fuzzy rough sets in $(\mathscr{H}^\star, \rho)$

In this section we introduce the concept of max-min hyper-product of fuzzy rough sets in semihyper-group and discuss some of the properties.

Definition 4.1. *The max-min hyper-product of FR sets $\rho(\widetilde{X})$ and $\rho(\widetilde{Y})$ is denoted by $\rho(\widetilde{X}) \circ \rho(\widetilde{Y})$ and is defined by for each $g \in \mathscr{H}^\star$, we have*

$$\mu_{\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Y})}(g) = \begin{cases} \max\limits_{g\in h_1\Theta h_2}\{\min(\mu_{\underline{\rho}(\widetilde{X})}(h_1), \mu_{\underline{\rho}(\widetilde{X})}(h_2))\} \\ \quad \text{if } g \in h_1\Theta h_2,\ \forall\, h_1, h_2 \in \mathscr{H}^\star \\ 0, \text{ if } g \notin h_1\Theta h_2,\ \forall\, h_1, h_2 \in \mathscr{H}^\star. \end{cases}$$

and

$$\mu_{\overline{\rho}(\widetilde{X})\circ\overline{\rho}(\widetilde{Y})}(g) = \begin{cases} \max\limits_{g\in h_1\Theta h_2}\{\min(\mu_{\overline{\rho}(\widetilde{X})}(h_1), \mu_{\overline{\rho}(\widetilde{X})}(h_2))\} \\ \quad \text{if } g \in h_1\Theta h_2,\ \forall\, h_1, h_2 \in \mathscr{H}^\star \\ 0, \text{ if } g \notin h_1\Theta h_2,\ \forall\, h_1, h_2 \in \mathscr{H}^\star. \end{cases}$$

Example 4.2. *Let $(\mathscr{H}^\star, \rho)$ be a fuzzy approximation space where $\mathscr{H}^\star = \{h_1, h_2, h_3, h_4\}$ with a hyper-operation Θ is defined below:*

Θ	h_1	h_2	h_3	h_4
h_1	h_1	h_2	h_3	h_4
h_2	h_2	$\{h_1, h_3\}$	$\{h_2, h_3\}$	h_4
h_3	h_3	$\{h_2, h_3\}$	$\{h_1, h_2\}$	h_4
h_4	h_4	h_4	h_4	$\mathscr{H}^\star$

Let us consider two fuzzy sets $\widetilde{X}, \widetilde{Y}$ and fuzzy equivalence relation ρ as
$\widetilde{X} = \{(h_1/0.2), (h_2/0.4), (h_3/0), (h_4/0.3)\}$
$\widetilde{Y} = \{(h_1/0.3), (h_2/0.6), (h_3/0.5), (h_4/0.1)\}$ *and*

$$\rho = \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{array}\begin{array}{c} \begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \end{array} \\ \begin{pmatrix} 1 & 0.8 & 0 & 0.4 \\ 0.8 & 1 & 0 & 0.4 \\ 0 & 0 & 1 & 0 \\ 0.4 & 0.4 & 0 & 1 \end{pmatrix} \end{array}$$

respectively.
Then we have the FR sets $\rho(\widetilde{X})$ and $\rho(\widetilde{Y})$ as

$$\rho(\widetilde{X}) = (\{(h_1/0.2), (h_2/0.2), (h_3/0), (h_4/0)\}, \{(h_1/0.9), (h_2/0.5), (h_3/0.5), (h_4/0.4)\})$$

$$\rho(\widetilde{Y}) = (\{(h_1/0.3), (h_2/0.3), (h_3/0.5), (h_4/0.1)\}, \{(h_1/0.6), (h_2/0.6), (h_3/0.5), (h_4/0.4)\}).$$

Then
$$\rho(\widetilde{X}) \circ \rho(\widetilde{Y}) = (\{(h_1/0.2), (h_2/0.2), (h_3/0.2), (h_4/0.1))\}, \{(h_1/0.4), (h_2/0.4), (h_3/0.4), (h_4/0.4)\}.$$

Proposition 4.3. *If $\rho(\widetilde{X}), \rho(\widetilde{Y})$ and $\rho(\widetilde{Z})$ are the FR sets of $(\mathscr{H}^\star, \rho)$ then*
(i) $\rho(\widetilde{X}) \circ (\rho(\widetilde{Y}) \cup \rho(\widetilde{Z})) \supseteq (\rho(\widetilde{X}) \circ \rho(\widetilde{Y})) \cup (\rho(\widetilde{X}) \circ \rho(\widetilde{Z}))$
(ii) $\rho(\widetilde{X}) \circ (\rho(\widetilde{Y}) \cap \rho(\widetilde{Z})) \subseteq (\rho(\widetilde{X}) \circ \rho(\widetilde{Y})) \cap (\rho(\widetilde{X}) \circ \rho(\widetilde{Z}))$ holds.

Proof: Let $g \in \mathscr{H}^\star$. Consider

$$\begin{aligned}
&\mu_{\underline{\rho}(\widetilde{X})\circ(\underline{\rho}(\widetilde{Y})\cup\underline{\rho}(\widetilde{Z}))}(g) \\
&= \max_{g\in m\Theta n}\{\min\{\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{(\underline{\rho}(\widetilde{Y})\cup\underline{\rho}(\widetilde{Z}))}(n)\}\} \\
&= \max_{g\in m\Theta n}\{\min\{\mu_{\underline{\rho}(\widetilde{X})}(m), \max\{\mu_{\underline{\rho}(\widetilde{Y})}(n), \mu_{\underline{\rho}(\widetilde{Z})}(n)\}\}\} \\
&\geq \max_{g\in m\Theta n}\{\max\{\min\{\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{\underline{\rho}(\widetilde{Y})}(n)\}, \\
&\qquad \min\{\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{\underline{\rho}(\widetilde{Z})}(n)\}\}\} \\
&\geq \max\{\max_{g\in m\Theta n}\{\min(\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{\underline{\rho}(\widetilde{Y})}(n)\}, \\
&\qquad \max_{g\in m\Theta n}\{\min(\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{\underline{\rho}(\widetilde{Z})}(n)\}\} \\
&\geq \max\{\mu_{(\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Y}))}(g), \mu_{(\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Z}))}(g)\} \\
&\geq \mu_{((\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Y}))\cup(\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Z})))}(g)
\end{aligned}$$

Therefore
$\underline{\rho}(\widetilde{X}) \circ (\underline{\rho}(\widetilde{Y}) \cup \underline{\rho}(\widetilde{C})) \supseteq (\underline{\rho}(\widetilde{X}) \circ \underline{\rho}(\widetilde{Y})) \cup (\underline{\rho}(\widetilde{X}) \circ \underline{\rho}(\widetilde{Z}))$.
Similarly $\overline{\rho}(\widetilde{X}) \circ (\overline{\rho}(\widetilde{Y}) \cup \overline{\rho}(\widetilde{Z})) \supseteq (\overline{\rho}(\widetilde{X}) \circ \overline{\rho}(\widetilde{Y})) \cup (\overline{\rho}(\widetilde{X}) \circ \overline{\rho}(\widetilde{Z}))$ holds.
(ii) Let $g \in \mathscr{H}^\star$. Consider

$$\begin{aligned}
&\mu_{\underline{\rho}(\widetilde{X})\circ(\underline{\rho}(\widetilde{Y})\cap\underline{\rho}(\widetilde{Z}))}(g) \\
&= \max_{g\in m\Theta n}\{\min\{\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{(\underline{\rho}(\widetilde{Y})\cap\underline{\rho}(\widetilde{Z}))}(n)\}\} \\
&= \max_{g\in m\Theta n}\{\min\{\mu_{\underline{\rho}(\widetilde{X})}(m), \min\{\mu_{\underline{\rho}(\widetilde{Y})}(n), \mu_{\underline{\rho}(\widetilde{Z})}(n)\}\}\} \\
&= \max_{g\in m\Theta n}\{\min\{\min\{\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{\underline{\rho}(\widetilde{X})}(n)\}, \\
&\qquad \min\{\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{\underline{\rho}(\widetilde{Z})}(n)\}\}\} \\
&\leq \min\{\max_{g\in m\Theta n}\{\min(\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{\underline{\rho}(\widetilde{Y})}(n)\}, \\
&\qquad \max_{g\in m\Theta n}\{\min(\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{\underline{\rho}(\widetilde{Z})}(n)\}\} \\
&\leq \min\{\mu_{(\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Y}))}(g), \mu_{(\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Z}))}(g)\} \\
&\leq \mu_{(\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Y}))\cap(\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Z}))}(g).
\end{aligned}$$

Therefore
$\underline{\rho}(\widetilde{X}) \circ (\underline{\rho}(\widetilde{Y}) \cap \underline{\rho}(\widetilde{Z})) \subseteq (\underline{\rho}(\widetilde{X}) \circ \underline{\rho}(\widetilde{Y})) \cap (\underline{\rho}(\widetilde{X}) \circ \underline{\rho}(\widetilde{Z})))$.
Similarly we can prove for upper approximation.

Proposition 4.4. *If $\rho(X)$, $\rho(Y)$ and $\rho(Z)$ are the $FRHI_R$ of $(\mathscr{H}^\star, \rho)$ then the following property holds:*

1. $\underline{\rho}(\widetilde{X}) \circ \underline{\rho}(\widetilde{Y}) \subseteq \underline{\rho}(\widetilde{X}) \cap \underline{\rho}(\widetilde{Y})$
2. $\overline{\rho}(\widetilde{X}) \circ \overline{\rho}(\widetilde{Y}) \subseteq \overline{\rho}(\widetilde{X}) \cap \overline{\rho}(\widetilde{Y})$

Proof: Let $m, n, g \in \mathscr{H}^\star$. Consider

$$\begin{aligned}
&\mu_{(\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Y}))}(g) \\
&= \max_{g\in m\Theta n}\{\min\{\mu_{\underline{\rho}(\widetilde{X})}(m), \mu_{\underline{\rho}(\widetilde{Y})}(m)\}\} \\
&\leq \max_{g\in m\Theta n}\{\min\{\inf_{g\in m\Theta n}\mu_{\underline{\rho}(\widetilde{X})}(g), \inf_{g\in m\Theta n}\mu_{\underline{\rho}(\widetilde{Y})}(g)\}\} \\
&\leq \max_{g\in m\Theta n}\{\inf_{g\in m\Theta n}\{\min(\mu_{\underline{\rho}(\widetilde{X})}(g), \mu_{\underline{\rho}(\widetilde{Y})}(g))\}\} \\
&\leq \max_{g\in m\Theta n}\{\inf_{g\in m\Theta n}\mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(g)\} \\
&\leq \max_{g\in m\Theta n}\mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(m\Theta n) \ \ for \ \ g \in m\Theta n
\end{aligned}$$

$\leq \mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(g)$

Therefore

$\underline{\rho}(\widetilde{X})\circ\underline{\rho}(\widetilde{Y})\subseteq\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})$. Similarly (2) holds.

5. Fuzzy rough bi hyper-ideal of $(\mathscr{H}^{\star},\rho)$

In this section we introduce the concept of fuzzy rough bi hyper-ideal in semihyper-groups and study its properties

Definition 5.1. *[8] A non-empty subset X of a semihyper-group $\mathscr{H}^{\star}$ is called a subsemihyper-group if $XX\subseteq X$.*

Definition 5.2. *A FR set $\rho(\widetilde{X})=(\underline{\rho}(\widetilde{X}),\overline{\rho}(\widetilde{X}))$ in $(\mathscr{H}^{\star},\rho)$ is said to be a FR subsemihyper-group of $(\mathscr{H}^{\star},\rho)$ if*

(i) $\min\{\mu_{\underline{\rho}(\widetilde{X})}(m),\mu_{\underline{\rho}(\widetilde{X})}(n)\}\leq\inf_{g\in m\Theta n}\mu_{\underline{\rho}(\widetilde{X})}(g)$

(ii) $\min\{\mu_{\overline{\rho}(\widetilde{X})}(m),\mu_{\overline{\rho}(\widetilde{X})}(n)\}\leq\inf_{g\in m\Theta n}\mu_{\overline{\rho}(\widetilde{X})}(g)$

$\forall\, m,n,g\in\mathscr{H}^{\star}$.

Proposition 5.3. *If $\rho(\widetilde{X})=(\underline{\rho}(\widetilde{X}),\overline{\rho}(\widetilde{X}))$ and $\rho(\widetilde{Y})=(\underline{\rho}(\widetilde{Y}),\overline{\rho}(\widetilde{Y}))$ are the two FR subsemihyper-group of $(\mathscr{H}^{\star},\rho)$ then the intersection of $\rho(\widetilde{X})$ and $\rho(\widetilde{Y})$ is also a FR subsemihyper-group of $(\mathscr{H}^{\star},\rho)$.*

Proof: Let $\underline{\rho}(\widetilde{X})$ and $\underline{\rho}(\widetilde{Y})$ be the FR subsemihyper-group. Let $m,n,g\in\mathscr{H}^{\star}$. Consider

$$\begin{aligned}
&\min\{\mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(m),\mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(n)\}\\
&=\min\{\min(\mu_{\underline{\rho}(\widetilde{X})}(m),\mu_{\underline{\rho}(\widetilde{Y})}(m)),\\
&\quad\min(\mu_{\underline{\rho}(\widetilde{X})}(n),\mu_{\underline{\rho}(\widetilde{Y})}(n))\}\\
&=\min\{\min(\mu_{\underline{\rho}(\widetilde{X})}(m),\mu_{\underline{\rho}(\widetilde{X})}(n)),\\
&\quad\min(\mu_{\underline{\rho}(\widetilde{Y})}(m),\mu_{\underline{\rho}(\widetilde{Y})}(n))\}\\
&\leq\min\{\inf_{g\in m\Theta n}\mu_{\underline{\rho}(\widetilde{X})}(g),\inf_{g\in m\Theta n}\mu_{\underline{\rho}(\widetilde{Y})}(g)\}\\
&\leq\inf_{g\in m\Theta n}\{\min(\mu_{\underline{\rho}(\widetilde{X})}(g),\mu_{\underline{\rho}(\widetilde{Y})}(g))\}\\
&\leq\inf_{g\in m\Theta n}\mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(g)
\end{aligned}$$

Therefore

$\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})$ is a FR subsemihyper-group. Similarly $\overline{\rho}(\widetilde{X})\cap\overline{\rho}(\widetilde{Y})$ is also a FR subsemihyper-group of $(\mathscr{H}^{\star},\rho)$.

Theorem 5.4. *If a FR set is a FR subsemihyper-group of $(\mathscr{H}^{\star},\rho)$ then the $q-$level set of $\rho(X)[X^{q}]$ is a subsemihyper-group.*

Proof: Let $\underline{\rho}(\widetilde{X})$ be a FR subsemihyper-group. Let $m,n,g\in\mathscr{H}^{\star}$ such that $m,n\in X_{L}^{q}$ and $g\in\mathscr{H}^{\star}$ which implies $\mu_{\underline{\rho}(\widetilde{X})}(m)\geq q$ and $\mu_{\underline{\rho}(\widetilde{X})}(n)\geq q$.

Since $\min\{\mu_{\underline{\rho}(\widetilde{X})}(m),\mu_{\underline{\rho}(\widetilde{X})}(n)\}\leq\inf_{g\in m\Theta n}\mu_{\underline{\rho}(\widetilde{X})}(g)$

$\Longrightarrow\inf_{g\in m\Theta n}\mu_{\underline{\rho}(\widetilde{X})}(g)\geq min\{q,q\}=q$

$\Longrightarrow\mu_{\underline{\rho}(\widetilde{X})}(g)\geq q$ for $g\in m\Theta n$

$\Longrightarrow m\Theta n\subseteq X_{L}^{q}$.

Therefore X_{L}^{q} is a subsemihyper-group. Similarly we can prove for upper approximation.

Definition 5.5. *[4] A subsemihyper-group X in $\mathscr{H}^{\star}$ is called a bi hyper-ideal if $X\mathscr{H}^{\star}X\subseteq X$.*

Definition 5.6. *A FR subsemihyper-group $\rho(\widetilde{X})=(\underline{\rho}(\widetilde{X}),\overline{\rho}(\widetilde{X}))$ of $(\mathscr{H}^{\star},\rho)$ is said to be a fuzzy rough bi hyper-ideal($FRHI_B$) of $(\mathscr{H}^{\star},\rho)$ if*

(i) $\min\{\mu_{\underline{\rho}(\widetilde{X})}(m),\mu_{\underline{\rho}(\widetilde{X})}(n)\}\leq\inf_{g\in m\Theta u\Theta n}\mu_{\underline{\rho}(\widetilde{X})}(g)$

(ii) $\min\{\mu_{\overline{\rho}(\widetilde{X})}(m),\mu_{\overline{\rho}(\widetilde{X})}(n)\}\leq\inf_{g\in m\Theta u\Theta n}\mu_{\overline{\rho}(\widetilde{X})}(g)$

$\forall\, m,n,u,g\in\mathscr{H}^{\star}$.

Proposition 5.7. *If $\rho(\widetilde{X})=(\underline{\rho}(\widetilde{X}),\overline{\rho}(\widetilde{X}))$ and $\rho(\widetilde{Y})=(\underline{\rho}(\widetilde{Y}),\overline{\rho}(\widetilde{Y}))$ are the two $FRHI_B$ of $(\mathscr{H}^{\star},\rho)$ then the intersection of $\rho(\widetilde{X})$ and $\rho(\widetilde{Y})$ is also a FR bi hyper-ideal of $(\mathscr{H}^{\star},\rho)$.*

Proof: Let $\underline{\rho}(\widetilde{X})$ and $\underline{\rho}(\widetilde{Y})$ be the $FRHI_B$. Let $m,n,u,g\in\mathscr{H}^{\star}$. Consider

$$\begin{aligned}
&\min\{\mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(m),\mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(n)\}\\
&=\min\{\min(\mu_{\underline{\rho}(\widetilde{X})}(m),\mu_{\underline{\rho}(\widetilde{Y})}(m)),\\
&\quad\min(\mu_{\underline{\rho}(\widetilde{X})}(n),\mu_{\underline{\rho}(\widetilde{Y})}(n))\}\\
&\leq\min\{\inf_{g\in m\Theta u\Theta n}\mu_{\underline{\rho}(\widetilde{X})}(g),\inf_{g\in m\Theta u\Theta n}\mu_{\underline{\rho}(\widetilde{Y})}(g)\}\\
&\leq\inf_{g\in m\Theta u\Theta n}\{\min(\mu_{\underline{\rho}(\widetilde{X})}(g)),\mu_{\underline{\rho}(\widetilde{Y})}(g)\}\\
&\leq\inf_{g\in m\Theta u\Theta n}\mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(g)
\end{aligned}$$

Similarly

$\min\{\mu_{\overline{\rho}(\widetilde{X})\cap\overline{\rho}(\widetilde{Y})}(m),\mu_{\overline{\rho}(\widetilde{X})\cap\overline{\rho}(\widetilde{Y})}(n)\}\leq\inf_{g\in m\Theta u\Theta n}\mu_{\overline{\rho}(\widetilde{X})\cap\overline{\rho}(\widetilde{Y})}(g)$

Then by the Proposition[5.3] we conclude that $\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})$ is also a $FRHI_B$ of $(\mathscr{H}^{\star},\rho)$.

Theorem 5.8. *If a FR set is a $FRHI_B$ of $(\mathscr{H}^{\star},\rho)$ then the q-level set of $\rho(X)[X^{q}]$ is a bi hyper-ideal.*

Proof: Let $\underline{\rho}(\widetilde{X})$ be a $FRHI_B$. Since by the Theorem[5.4], X_{L}^{q} is a subsemihyper-group. Now consider $m\Theta u\Theta n\in X_{L}^{q}\mathscr{H}^{\star}X_{L}^{q}$ such that $m,n\in X_{L}^{q}$ and $g\in\mathscr{H}^{\star}$ which implies $\mu_{\underline{\rho}(\widetilde{X})}(m)\geq q$ and $\mu_{\underline{\rho}(\widetilde{X})}(n)\geq q$.

Since $\min\{\mu_{\underline{\rho}(\widetilde{X})}(m),\mu_{\underline{\rho}(\widetilde{X})}(n)\}\leq\inf_{g\in m\Theta u\Theta n}\mu_{\underline{\rho}(\widetilde{X})}(g)$

$\Longrightarrow\inf_{g\in m\Theta u\Theta n}\mu_{\underline{\rho}(\widetilde{X})}(g)\geq min\{q,q\}=q$

$\Longrightarrow\mu_{\underline{\rho}(\widetilde{X})}(g)\geq q$ for $g\in m\Theta u\Theta n$

$\Longrightarrow m\Theta u\Theta n\subseteq X_{L}^{q}$.

Therefore X_{L}^{q} is a bi hyper-ideal. Similarly we can prove for upper approximation.

6. Fuzzy rough interior hyper-ideal of $(\mathscr{H}^{\star},\rho)$

In this section we study fuzzy rough interior hyper-ideal and its properties.

Definition 6.1. *A subsemihyper-group X in $\mathscr{H}^{\star}$ is called interior hyper-ideal if $\mathscr{H}^{\star}X\mathscr{H}^{\star}\subseteq X$.*

Definition 6.2. *A FR subsemihyper-group $\rho(\widetilde{X})=(\underline{\rho}(\widetilde{X}),\overline{\rho}(\widetilde{X}))$ of $(\mathscr{H}^{\star},\rho)$ is said to be a FR interior hyper-ideal of $(\mathscr{H}^{\star},\rho)$ if*

(i) $\mu_{\underline{\rho}(\widetilde{X})}(x) \leq \inf_{g\in m\Theta x\Theta n} \mu_{\underline{\rho}(\widetilde{X})}(g)$
(ii) $\mu_{\overline{\rho}(\widetilde{X})}(x) \leq \inf_{g\in m\Theta x\Theta n} \mu_{\overline{\rho}(\widetilde{X})}(g)\ \forall\ m,x,n,g \in \mathscr{H}^\star$.

Proposition 6.3. *If* $\rho(\widetilde{X}) = (\underline{\rho}(\widetilde{X}), \overline{\rho}(\widetilde{X}))$ *and* $\rho(\widetilde{Y}) = (\underline{\rho}(\widetilde{Y}), \overline{\rho}(\widetilde{Y}))$ *are the two* $FRHI_I$ *of* $(\mathscr{H}^\star, \rho)$ *then the intersection of* $\rho(\widetilde{X})$ *and* $\rho(\widetilde{Y})$ *is also a* $FRHI_I$ *of* $(\mathscr{H}^\star, \rho)$.

Proof: By the Proposition [5.3], $\rho(\widetilde{X}) \cap \rho(\widetilde{Y})$ is a FR subsemihyper-group. Let $\underline{\rho}(\widetilde{X})$, $\underline{\rho}(\widetilde{Y})$ be $FRHI_I$ and let $a,b,g,x \in \mathscr{H}^\star$. Consider

$$\begin{aligned}
\mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(x) &= \min\{\mu_{\underline{\rho}(\widetilde{X})}(x), \mu_{\underline{\rho}(\widetilde{Y})}(x)\} \\
&\leq \min\{\inf_{g\in a\Theta x\Theta b} \mu_{\underline{\rho}(\widetilde{X})}(g), \\
&\qquad \inf_{g\in a\Theta x\Theta b} \mu_{\underline{\rho}(\widetilde{Y})}(g)\} \\
&\leq \inf_{g\in a\Theta x\Theta b} \{\min(\mu_{\underline{\rho}(\widetilde{X})}(g), \mu_{\underline{\rho}(\widetilde{Y})}(g))\} \\
&\leq \inf_{g\in a\Theta x\Theta b} \mu_{\underline{\rho}(\widetilde{X})\cap\underline{\rho}(\widetilde{Y})}(g).
\end{aligned}$$

Therefore
$\underline{\rho}(\widetilde{X}) \cap \underline{\rho}(\widetilde{Y})$ is a $FRHI_I$ of $(\mathscr{H}^\star, \rho)$.
Similarly $\overline{\rho}(\widetilde{X}) \cap \overline{\rho}(\widetilde{Y})$ is a $FRHI_I$ of $(\mathscr{H}^\star, \rho)$.

Theorem 6.4. *If a* FR *set* $\rho(\widetilde{X})$ *is a* $FRHI_I$ *of* $(\mathscr{H}^\star, \rho)$ *then for* $q \in [0,1]$ *the q-level set of* X^q *is an interior hyper-ideal.*

Proof: Let $\underline{\rho}(\widetilde{X})$ be a $FRHI_I$.

$$\mu_{\underline{\rho}(\widetilde{X})}(x) \leq \inf_{g\in m\Theta x\Theta n} \mu_{\underline{\rho}(\widetilde{X})}(g) \tag{6.1}$$

Let us consider that $X_L^q \neq \emptyset$. For $q \in [0,1]$ and $m\Theta x\Theta n \in \mathscr{H}^\star$ such that $x \in X_L^q$ and $m,n \in \mathscr{H}^\star$, then $\mu_{\underline{\rho}(\widetilde{X})}(x) \geq q$.
Since by Equation (6.1), $\inf_{g\in m\Theta x\Theta n} \mu_{\underline{\rho}(\widetilde{X})}(g) \geq q$

$\implies \mu_{\underline{\rho}(\widetilde{X})}(g) \geq q$ for $g \in m\Theta x\Theta n$.

Thus it can be easily shown that $m\Theta x\Theta n \subseteq X_L^q$ for $g \in m\Theta x\Theta n$

Hence X_L^q is an interior hyper-ideal. In the same way we can prove that X_U^q is an interior hyper-ideal of $(\mathscr{H}^\star, \rho)$.

7. Conclusion

Fuzzy rough set is a basic key for this paper. We applied the new concept of fuzzy rough hyper-ideal, fuzzy rough bi hyper-ideal and fuzzy rough interior hyper-ideal in an algebraic structure of semihyper-group. We also gave some properties on fuzzy rough hyper-ideals by using the composition concept. In future we may extend the concept of quasi ideals in semihyper-group in terms of fuzzy rough set.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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