



Edge vertex prime labeling of Cayley (di)graphs

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Abstract

In this paper we investigate the edge vertex prime labeling and super edge vertex prime labeling on Cayley graph $\text{Cay}(\Gamma, \Omega)$, and Cayley digraph $\text{Cay}_D(\Gamma, \Omega)$ where Ω is a generating subset of a finite group Γ .

Keywords

Edge vertex prime, super edge vertex prime, Cayley graph, Cayley digraph.

AMS Subject Classification

05C20, 05C25, 05C78, 05C80.

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1. Introduction

Graph labeling is the interesting technique, is applied on a graph such as assigning the labels to the vertices or edges or both subject to the certain constrains. It was contributed by Alex Rosa in 1967[6]. On the past decade, we studied different kind of labeling to variety of graphs such graphs are known as labeled graphs. In this paper one of such labeling technique, namely Edge vertex prime labeling is applied on the well known algebraic structured Cayley graphs. It was introduced by Arthur Cayley in 1878 [2] to illustrate the concept of abstract groups. The vertex and edge transitivity(high symmetric) and regularity make it to allot the same routing and communication schemes in networking. It act as a network models and provide the efficient fault handling capacity to networks. Algorithm type of problems on Cayley directed graphs act as an important application parameter such as X-ray diffraction, coding theory and in the defense sector as a missile activator. The identity free set Ω is a generating subset of a finite group Γ and symmetric (or

closed under inverses. The Cayley graph, $\mathbb{G} = \text{Cay}(\Gamma, \Omega)$ whose vertices are the elements of Γ is denoted by $V(\mathbb{G})$ and the edges corresponds to the operations on every element of Γ with generators as well as connectors, that is $E(\mathbb{G}) = \{(\gamma, \gamma\omega) : \gamma \in \Gamma, \omega \in \Omega\}$. The condition $e \notin \Omega$ imposed the loop free structure to graph. When symmetry condition of Ω does not hold we have a Cayley directed graph, is denoted by $\vec{\mathbb{G}} = \text{Cay}_D(\Gamma, \Omega)$.

2. Preliminaries

Prime labeling was originated by Roger Entringer and it was introduced in paper by Tout Dobboucy and Howalla[11]. Entringer conjectured that prime labeling admits for all trees after it was proved for large trees by Haxell, Pikhurko and Taraz[3]. In [1], A graph $G(V, E)$ is said to be a prime labeled graph there exist a bijective function on the vertex set $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ and for each edge $xy \in E(G)$, $f(x)$ and $f(y)$ are relatively prime. Where as in a vertex prime graph $G(V, E)$ [5], there exist a bijective function on the edge set such that $f : E(G) \rightarrow \{1, 2, \dots, |E(G)|\}$. The greatest common divisor of the labels on the incident edges for each vertex $v \in V(G)$ is one. R. Jagadesh and J. Basker Babujee [4] introduced the concept of edge vertex prime labeling and proved the existence of the same for paths P_n , cycles C_n and star $K_{1,n}$. In [7], M. Simaringa and S Muthukumaran proved the triangular and rectangular book, butterfly graph, drums graph, Jahangir graph are edge vertex prime. In this paper we are showing the edge vertex prime labeling(EVPL) and super edge vertex prime labeling(SEVPL) of Cayley graphs and the digraph of it. For further interpretation related to labeling on Cayley digraphs, one can refer [9, 10].

Lemma 2.1. *Let Γ be a finite group and Ω be a generating subset of Γ , where Γ is not a two regular and does not contains atleast two odd cycle. Then Cayley graph $\text{Cay}(\Gamma, \Omega)$ admits vertex prime labeling.*

Proof. Let Γ be any finite group and Ω be the generating subset of Γ and $\mathbb{G} = \text{Cay}(\Gamma, \Omega)$. Define a bijective function $f : E(\mathbb{G}) \rightarrow \{1, 2, \dots, |E|\}$ as follows.

$$f(e) = i \text{ if } e \in \Omega_j, \quad 1 \leq j \leq |\Gamma| - 1 \text{ and } 1 \leq i \leq |E|$$

Then for any vertex $u \in V(\mathbb{G})$, the number of incident edges $f(uv)$ are relatively prime. \square

3. EVPL of Cayley graphs

In this section, we investigate the existence of edge vertex prime labeling of Cayley graph depending upon the generating subset $|\Omega|$ is odd or even.

A graph $G = (V, E)$ is said to have edge vertex prime labeling if there exist a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \dots, |V(G) \cup E(G)|\}$ with the property that given any edge $uv \in E(G)$, the numbers $f(u)$, $f(v)$ and $f(uv)$ are pair wise relatively prime. A graph G is said to be edge vertex prime if it admits have edge vertex prime labeling [4]. In this section we are partitioning $E(\mathbb{G})$ into the cycles and matchings such that $\{C_1, C_2, \dots, C_s, M_1, M_2, \dots, M_t\}$. On the other hand we arranging the generating set as follows $\Omega = \{\sigma_1, \sigma_2, \dots, \sigma_{2s}, \tau_1, \tau_2, \dots, \tau_t\}$. Cycles of $E(\mathbb{G})$ is produced by the elements $\{\sigma_1, \sigma_2, \dots, \sigma_{2s}\}$ and matchings is $\{\tau_1, \tau_2, \dots, \tau_t\}$. Both are the non-self inverse elements and self inverse elements of Ω having the order more than two and exactly two respectively. For $1 \leq i \leq s$ and $1 \leq j \leq t$, suppose σ_i and σ_{i+s} generates the same cycle then $\sigma_i^{-1} = \sigma_{i+s}$ in addition $\tau_j^{-1} = \tau_j$ when \mathbb{G} is undirected. Without loss of generality, let $\Gamma = D_{16}$ be a dihedral group of order 16 and let $\Omega = \{r, r^2, r^6, r^7, s, sr, sr^2\}$ where $\{r, r^2, r^6, r^7\}$ are generates the cycles and $\{s, sr, sr^2\}$ generates the matchings. Here, number of $s = 2$ and $t = 3$. For the further references, on the classifications of generating set one can refer[8].

Theorem 3.1. *Let Γ be a finite group and Ω be a generating subset of Γ then the Cayley graph $\mathbb{G} = \text{Cay}(\Gamma, \Omega)$ admits a edge vertex prime labeling when $|\Omega|$ is odd.*

Proof. Let $\Omega = \{\sigma_1, \sigma_2, \dots, \sigma_{2s}, \tau_1, \tau_2, \dots, \tau_t\}$, since $|\Omega|$ is odd then t must be odd. Let $E(\mathbb{G}) = \{C_1, C_2, \dots, C_s, M_1, M_2, \dots, M_t\}$. Define a bijective function $f : V(\mathbb{G}) \cup E(\mathbb{G}) \rightarrow \{1, 2, \dots, |V| + |E|\}$ as follows,

Case 1: Suppose $t = 0$ and $s > 1$. Then

$$f(v_p) = 2p - 1; \quad 1 \leq p \leq |\Gamma| \tag{3.1}$$

and

$$f(e_\ell) = \begin{cases} 2\ell - 2 & \text{if } e \in C_1, & 2 \leq \ell \leq |\Gamma| + 1 \\ \ell & \text{if } e \in C_i, & 2 \leq i \leq s, \\ & & 2|\Gamma| + 1 \leq \ell \leq |V| + |E| \end{cases}$$

$$(3.2)$$

Case 2: Suppose $s = 0$ and $t > 1$. Then

$$f(v_p) = 2p - 1; \quad 1 \leq p \leq |\Gamma| \tag{3.3}$$

and

$$f(e_\ell) = \begin{cases} 2\ell - 2 & 2 \leq \ell \leq |\Gamma| + 1 \\ \ell & 2|\Gamma| + 1 \leq \ell \leq |V| + |E| \end{cases}$$

Case 3: Suppose $s > 0$ and $t \neq 0$. Then

$$f(v_p) = 2p - 1; \quad 1 \leq p \leq |\Gamma|$$

and

$$f(e_\ell) = \begin{cases} 2\ell - 2 & \text{if } e \in C_1, 2 \leq \ell \leq |\Gamma| + 1 \\ \ell & \text{if } e \in C_i, 2 \leq i \leq s, \\ & & 2|\Gamma| + 1 \leq \ell \leq (s + 1)|\Gamma| \\ \ell & \text{if } e \in M_j, \quad 1 \leq j \leq t, \\ & & (s + 1)|\Gamma| + 1 \leq \ell \leq |V| + |E| \end{cases}$$

Case 4: Suppose $t = 0$ and $s = 1$. Then

$$f(v_p) = 2p - 1; \quad 1 \leq p \leq |\Gamma| \text{ and}$$

$$f(e_\ell) = 2\ell - 2; \quad 2 \leq \ell \leq |\Gamma| + 1$$

Case 5: Suppose $t = 1$ and $s = 0$. Then

$$f(v_p) = \begin{cases} 2p - 1 & 1 \leq p \leq \frac{|\Gamma|}{2} \\ p & |\Gamma| + 1 \leq p \leq |V| + |E| \end{cases}$$

and

$$f(e_\ell) = 2\ell - 2; \quad 2 \leq \ell \leq \frac{|\Gamma|}{2} + 1$$

From the above defined cases 1, 2, 3, 4, 5 for every edge $uv \in E(\mathbb{G})$, the numbers $f(u)$, $f(v)$ and $f(uv)$ are relatively prime. Hence, \mathbb{G} admits a edge vertex prime labeling when $|\Omega|$ is odd. \square

Theorem 3.2. *Let Γ be a finite group and Ω be a generating subset of Γ then the Cayley graph $\mathbb{G} = \text{Cay}(\Gamma, \Omega)$ admits edge vertex prime labeling when $|\Omega|$ is even.*

Proof. Let $\Omega = \{\sigma_1, \sigma_2, \dots, \sigma_{2s}, \tau_1, \tau_2, \dots, \tau_t\}$, since $|\Omega|$ is even then t must be even. Let

$E(\mathbb{G}) = \{C_1, C_2, \dots, C_s, M_1, M_2, \dots, M_t\}$. Define a bijective function $f : V(\mathbb{G}) \cup E(\mathbb{G}) \rightarrow \{1, 2, \dots, |V| + |E|\}$ as follows,

Case 1: Suppose $s = 0$ and t is even. Then

$$f(v_p) = 2p - 1; \quad 1 \leq p \leq |\Gamma| \text{ and}$$

$$f(e_\ell) = 2\ell - 2; \quad 2 \leq \ell \leq |\Gamma| + 1$$



Case 2: Suppose $t = 0$ and s is even. Then

$$f(v_p) = 2p - 1; 1 \leq p \leq |\Gamma|$$

and

$$f(e_\ell) = \begin{cases} 2\ell - 2 & \text{if } e \in C_1, 2 \leq \ell \leq |\Gamma| + 1 \\ \ell & \text{if } e \in C_i, 2 \leq i \leq s, \\ & 2|\Gamma| + 1 \leq \ell \leq |V| + |E| \end{cases}$$

Case 3: Suppose $s > 0$ and $t \neq 0$. Then

$$f(v_p) = 2p - 1; 1 \leq p \leq |\Gamma|$$

and

$$f(e_\ell) = \begin{cases} 2\ell - 2 & \text{if } e \in C_1, 2 \leq \ell \leq (|\Gamma| + 1) \\ \ell & \text{if } e \in C_i, 2 \leq i \leq s, \\ & 2|\Gamma| + 1 \leq \ell \leq s|\Gamma| \\ \ell & \text{if } e \in M_j, 1 \leq j \leq t, \\ & s|\Gamma| + 1 \leq \ell \leq |V| + |E| \end{cases}$$

From the defined cases 1, 2, 3 for every edge $u, v \in E(\mathbb{G})$ the numbers $f(u), f(v)$ and $f(uv)$ are relatively prime. Hence, \mathbb{G} admits a edge vertex prime labeling when $|\Omega|$ is even. \square

4. SEVPL of Cayley graphs

A graph $G(p, q)$ with p vertices and q edges is said to have super edge vertex prime labeling if there exists a bijection $f(V) \rightarrow \{1, 2, \dots, p\}$ and $f(E) \rightarrow \{p + 1, p + 2, \dots, p + q\}$ such that for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pair wise relatively prime. A graph G is said to have a super edge vertex prime if it admits a super edge vertex prime labeling[4].

Theorem 4.1. Let Γ be a finite group and Ω be a generating subset of Γ then the Cayley graph $\mathbb{G} = \text{Cay}(\Gamma, \Omega)$ admits a super edge vertex prime labeling when $|\Omega|$ is odd.

Proof. Let $\Omega = \{\sigma_1, \sigma_2, \dots, \sigma_{2s}, \tau_1, \tau_2, \dots, \tau_t\}$, since $|\Omega|$ is odd then t must be odd. Let $E(\mathbb{G})$ has a partition $\{C_1, C_2, \dots, C_s, M_1, M_2, \dots, M_t\}$. Define a bijective function $f : V(\mathbb{G}) \rightarrow \{1, 2, \dots, |V|\}$ as follows:

$$f(v_p) = p; 1 \leq p \leq |\Gamma|$$

and a bijective function $f : E(\mathbb{G}) \rightarrow \{|V| + 1, |V| + 2, \dots, |V| + |E|\}$ is defined as follows:

Case 1: Suppose $t = 0$ and $t > 1$. Then

$$f(e_\ell) = \begin{cases} \ell & \text{if } e \in C_1, |\Gamma| + 1 \leq \ell \leq 2|\Gamma| \\ \ell & \text{if } e \in C_i, 2 \leq i \leq s, \\ & 2|\Gamma| + 1 \leq \ell \leq |V| + |E| \end{cases}$$

Case 2: Suppose $s = 0$ and $s > 1$. Then

$$f(e_\ell) = \ell \text{ if } e \in M_i, 1 \leq i \leq t, |\Gamma| + 1 \leq \ell \leq |V| + |E|$$

Case 3: Suppose $s > 0$ and $t \neq 0$. Then

$$f(e_\ell) = \begin{cases} \ell & \text{if } e \in C_1, |\Gamma| + 1 \leq \ell \leq 2|\Gamma| \\ \ell & \text{if } e \in C_i, 2 \leq i \leq s, \\ & 2|\Gamma| + 1 \leq \ell \leq (s + 1)|\Gamma| \\ \ell & \text{if } e \in M_j, 1 \leq j \leq t, \\ & (s + 1)|\Gamma| + 1 \leq \ell \leq |V| + |E| \end{cases}$$

Case 4: Suppose $t = 0$ and $s = 1$ (or) Suppose $t = 1$ and $s = 0$. Then

$$f(e_\ell) = \ell; |\Gamma| + 1 \leq \ell \leq |V| + |E|$$

From the defined cases 1, 2, 3 for every edge $uv \in E(\mathbb{G})$ the numbers $f(u), f(v)$ and $f(uv)$ are relatively prime. Hence, the graph \mathbb{G} is super edge vertex prime when $|\Omega|$ is odd. \square

Theorem 4.2. Let Γ be a finite group and Ω be a generating subset of Γ then the Cayley graph $\text{Cay}(\Gamma, \Omega)$ admits a super edge vertex prime labeling when $|\Omega|$ is even.

Proof. Let $\Omega = \{\sigma_1, \sigma_2, \dots, \sigma_{2s}, \tau_1, \tau_2, \dots, \tau_t\}$, since $|\Omega|$ is odd then t must be odd. Let $G = \text{Cay}(\Gamma, \Omega)$ has a partition $\{C_1, C_2, \dots, C_s, M_1, M_2, \dots, M_t\}$. Define a bijective function $f : V(\mathbb{G}) \rightarrow \{1, 2, \dots, |V|\}$ as follows:

$$f(v_p) = p; 1 \leq p \leq |\Gamma|$$

and a bijective function $f(E) \rightarrow \{(|V(G)| + 1), (|V(G)| + 2), \dots, |V(G)| \cup |E(G)|\}$ is defined as follows:

Case 1: Suppose $s = 0$ and t is even.

$$f(e_\ell) = \ell; |\Gamma| + 1 \leq \ell \leq |V| + |E|$$

Case 2: Suppose $t = 0$ and s is even.

$$f(e_\ell) = \begin{cases} \ell & \text{if } e \in C_1, |\Gamma| + 1 \leq \ell \leq 2|\Gamma| \\ \ell & \text{if } e \in C_i, 2 \leq i \leq s, 2|\Gamma| + 1 \leq \ell \leq |V| + |E| \end{cases}$$

Case 3: Suppose $s > 0$ and $t \neq 0$.

$$f(e_\ell) = \begin{cases} \ell & \text{if } e \in C_i, 2 \leq i \leq s, |\Gamma| + 1 \leq \ell \leq s|\Gamma| \\ \ell & \text{if } e \in M_j, s|\Gamma| + 1 \leq \ell \leq |V| + |E| 2 \leq j \leq t, \end{cases}$$

From the defined cases 1, 2, 3 for every edge $uv \in E(G)$ the numbers $f(u), f(v)$ and $f(uv)$ are relatively prime. Hence, the graph \mathbb{G} is super edge vertex prime when $|\Omega|$ is even. \square

5. EVPL of Cayley digraphs

In this section, we present an algorithms to construct a edge vertex prime labeling of the Cayley digraph.

Cayley digraphs $\tilde{\mathbb{G}} = \text{Cay}_D(\Gamma, \Omega)$ are defined with a abstract group Γ and a generating subset Ω . The vertices are the elements of the group, and its arcs are all the couples $(a, a\omega)$ with $a \in \Gamma$ and $\omega \in \Omega$. \mathbb{G} is symmetric if $\omega = \omega^{-1}, \forall \omega \in \Omega$. To avoid loops, we are forbidding the presence of the unit element in Γ . The graph \mathbb{G} is strongly



connected if and only if Ω generates Γ . Cayley digraphs corresponding to the finite group is said to be self inverse it reversing all the arcs and provides it be a isomorphic image such that $\text{Cay}(\Gamma, \Omega) \cong \text{Cay}(\Gamma, \Omega^{-1})$. It has the same size of positive and negative fragments, when Γ is abelian if not $\text{Cay}(\Gamma, \Omega)$ is isomorphic to $\text{Cay}(\Gamma, \Omega^{-1})$. Also \mathbb{G} is vertex transitive, has more utility in interconnection networks. Blooms and Hsu[9] extended the (graceful) labeling concept on digraphs. In this section we extend the investigation of edge vertex prime and super edge vertex prime labeling on Cayley digraphs, by Algorithmic way[10].

Algorithm 5.1. Input: The group Γ with the generating subset Ω .

Begin

Step 1: Construct the Cayley Digraph $\vec{\mathbb{G}} = \text{Cay}_D(\Gamma, \Omega)$.

Step 2: let $V(\vec{\mathbb{G}}) = \{v_1, v_2, \dots, v_n\}$ denote the vertex set of $\vec{\mathbb{G}}$.

Step 3: let $A(\vec{\mathbb{G}}) = (A_{\Omega_1}, A_{\Omega_2}, A_{\Omega_3}, \dots, A_{\Omega_{|\Omega|}})$ and the set $\{\vec{a}_1, \vec{a}_2, \dots, \vec{a}_{|\Omega|n}\}$ denote the arc set of $\vec{\mathbb{G}}$ and A_{Ω_j} is the set of all out going arcs of v_i generated by Ω_j , where $1 \leq i \leq |\Gamma|$, $1 \leq j \leq |\Omega|$.

Step 4: Define a function f on vertex of $\vec{\mathbb{G}}$, such that $f(v_i) = 2i - 1, 1 \leq i \leq |\Gamma|$.

Step 5: Define a function g on the arc set of $\vec{\mathbb{G}}$, such that $g(\vec{a}_{ij})$ as follows,

$$g(\vec{a}_{ij}) = \begin{cases} 2\ell - 2 & \text{if } \vec{a}_{ij} \in A_{\Omega_1}, 2 \leq \ell \leq |\Gamma| + 1 \\ \ell & \text{if } \vec{a}_{ij} \in A_{\Omega_j}, 2|\Gamma| + 1 \leq \ell \leq |V| + |E|, \\ & 2 \leq j \leq |\Omega|. \end{cases}$$

End

Output: $\vec{\mathbb{G}}$ is Edge Vertex Prime.

Theorem 5.2. The Cayley digraph $\text{Cay}_D(\Gamma, \Omega)$ admits Edge vertex prime labeling, where Ω is a generating subset of Γ .

Proof. From the construction of the Cayley digraph for a group of Γ , has n vertices and n times of $|\Omega|$ arcs. Let us denote the vertex set as $\{v_1, v_2, \dots, v_n\}$. To prove $\text{Cay}_D(\Gamma, \Omega)$ admits Edge vertex prime labeling. We have to show that for every outgoing arcs from each $v_i, 1 \leq i \leq n$ the numbers $f(u), f(v)$ and $f(uv)$ are relatively prime. By the construction of the Cayley digraph, we have that each vertex has exactly $|\Omega|$ outgoing arcs, out of which one arc is generated by $|\Omega|_i, 1 \leq i \leq |\Omega|$. Now we define a vertex mapping and arc mapping to the vertex set and the arc set as directed in the step 4 and step 5 in the Algorithm 5.1. We get the labels to the vertex set and the arc set of $\text{Cay}_D(\Gamma, \Omega)$. For any two integers i, j such that $i \neq j; f(u_i), f(v_i)$ and $f(u_i v_i)$ are relatively prime. Hence, the $\text{Cay}_D(\Gamma, \Omega)$ admits edge vertex prime labeling. \square

Algorithm 5.3. Input: The group Γ with the generating subset Ω .

Begin

Step 1: Construct the Cayley Digraph $\vec{\mathbb{G}} = \text{Cay}_D(\Gamma, \Omega)$.

Step 2: let $V(\vec{\mathbb{G}}) = \{v_1, v_2, \dots, v_n\}$ denote the vertex set of $\vec{\mathbb{G}}$.

Step 3: let $A(\vec{\mathbb{G}}) = (A_{\Omega_1}, A_{\Omega_2}, \dots, A_{\Omega_{|\Omega|}})$ and the set $\{\vec{a}_1, \vec{a}_2, \dots, \vec{a}_{|\Omega|n}\}$ denote the arc set of $\vec{\mathbb{G}}$ and A_{Ω_j} is the set of all out going arcs of v_i generated by Ω_j , where $1 \leq i \leq |\Gamma|$, $1 \leq j \leq |\Omega|$.

Step 4: Define a function f on vertex set of $\vec{\mathbb{G}}$, such that $f(v_i) = i, |\Gamma| \leq i \leq 1$.

Step 5: Define a function g on the arc set of $\vec{\mathbb{G}}$, such that $g(\vec{a}_{ij})$ as follows,

$$g(\vec{a}_{ij}) = \ell \text{ if } \vec{a}_{ij} \in \Omega_j, |\Gamma| + 1 \leq \ell \leq |V| + |E|, 1 \leq j \leq |\Omega|$$

End

Output: $\vec{\mathbb{G}}$ is Super Edge Vertex Prime.

Theorem 5.4. The Cayley digraph $\text{Cay}_D(\Gamma, \Omega)$ admits Super edge vertex prime labeling, where Ω is a generating subset of Γ .

Proof. From the construction of the Cayley digraph for a group of Γ , has n vertices and n times of $|\Omega|$ arcs. Let us denote the vertex set as $V = \{v_1, v_2, \dots, v_n\}$. To prove $\text{Cay}_D(\Gamma, \Omega)$ admits Super edge vertex prime labeling. By the definition, We have to show that the vertex set is labeled with in $|\Gamma|$ and for every arc(out going) from each $v_i, 1 \leq i \leq n$, the numbers $f(u), f(v)$ and $f(uv)$ are relatively prime. By the construction of the Cayley digraph, we have that each vertex has exactly $|\Omega|$ outgoing arcs, out of which one arc is generated by $|\Omega|_i, 1 \leq i \leq |\Omega|$. Now we define a vertex mapping and edge mapping to the vertex set and the edge set as directed in the step 4 and step 5 in the Algorithm 5.3. We get the labels to the vertex set and the edge set of $\text{Cay}_D(\Gamma, \Omega)$. For any two integers i, j such that $i \neq j; f(u_i), f(v_i)$ and $f(u_i v_i)$ are relatively prime. Hence, the $\text{Cay}_D(\Gamma, \Omega)$ admits Super edge vertex prime labeling. \square

6. Conclusion

In this paper, we have shown the existence of edge vertex(super edge vertex) prime labeling[4] on Cayley graphs. The algebraic property of Cayley graph has a significant contribution in computer science and computational theory. Moreover, primality[3] is basis of this labeling and we approached this labeling problem through algorithm. It will provide the more clarity on the applications of Cayley graphs. Based on the requirement, this analogue can be extended for some algebraic oriented graph as a Mathematical model.

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