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A generalized fractional integral transform with exponential type kernel

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Abstract
We introduce a fractional integral transform which is a generalization of many integral transforms having exponential type kernel and discuss some of its properties. We also ensure the efficiency of the newly introduced fractional integral transform in solving some differential equations of fractional order.

Keywords
Mittag-Leffler function, Integral transforms, Fractional integral transforms.

AMS Subject Classification
26A33, 65R10, 34A08.

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1. Introduction

As fractional calculus is the generalization of notions of integer-order differentiation and n-fold integration, we can accomplish what integer-order calculus cannot. Because of wide range of applications in science and engineering, it will lead to great future developments in different fields.

There are many integral transforms that are used in different fields of science and engineering. Particularly, to solve differential equations of fractional order, integral transforms are widely used and so many research works are carried out on the theory and applications of Laplace transform, Fourier transform and Mellin transform. The most popular integral transform with exponential type kernel is the Laplace transform. Laplace transform has proved its dominancy in the applications of engineering and applied science. In the recent years, many integral transforms with exponential type kernel have been introduced.

The following table explores various integral transforms with exponential type kernel.

Table 1. Integral transforms with exponential type kernel

S.No	Name of the integral transform	Kernel
1	Laplace transform(1749-1827)	$k(x,v)=e^{-vx}$
2	Sumudu transform(1993)	$k(x,v)=\frac{1}{v}e^{\frac{-x}{v}}$
3	Laplace-Carson transform (1886-1940)	$k(x,v)=ve^{-vx}$
4	N-transform(2008),zz-transform(2016)	$k(x,s,v)=\frac{1}{v}e^{\frac{-x}{v}};s=1$
5	Elzaki transform(2011)	$k(x,v)=ve^{\frac{-x}{v}}$
6	Tarig transform(2013),New integral transform(2013)	$k(x,v)=\frac{1}{v}e^{\frac{-x}{v^2}}$
7	Aboodh transform(2016)	$k(x,v)=\frac{1}{v}e^{-vx}$
8	Kamal transform(2016),Yang transform(2018)	$k(x,v)=e^{\frac{-x}{v}}$
9	Mahgoub transform((2018),New transform(2018)	$k(x,v)=ve^{-vx}$

Definition 1.1. *Let f(x) be sectionally continuous on the interval $0 \leq x \leq T$ for any $T > 0$ and $|f(x)| \leq me^{bx}$ when $x \geq N$*

for any $b\varepsilon\Re$ and $m,N\varepsilon\Re^+$. We define Sadik transform of f(x) as

$$U[f(x)] = F(v^{\alpha,\beta}) = \frac{1}{v^\beta}\int_0^\infty e^{-xv^\alpha} f(x)dx \qquad (1.1)$$

where v is a complex variable, $\alpha > 0$ and $\beta\varepsilon\Re$.

All the transforms in the table 1 become particular cases of Sadik transform.
By changing the values of α and β, the Sadik transform is not only converted into the Laplace, Sumudu, Elzaki, Tarig, Kamal, Laplace-Carson, Aboodh but will also be converted into those integral transforms which are actually not present in the literature.

By fixing values of α and β, the following table shows how Sadik transform is converted into the different integral transforms.

Table 2. Conversion of Sadik transform to various integral transforms

S.No	Values of α and β	Sadik transform converts into
1	$\beta = 0$ and $\alpha = 1$	Laplace transform
2	$\beta = 1$ and $\alpha = -1$	Sumudu transform
3	$\beta = -1$ and $\alpha = 1$	Laplace-Carson transform
4	$\beta = 1$ and $\alpha = -1$	N-transform, zz-transform
5	$\beta = -1$ and $\alpha = -1$	Elzaki transform
6	$\beta = 1$ and $\alpha = -2$	Tarig transform, New integral transform
7	$\beta = 1$ and $\alpha = 1$	Aboodh transform
8	$\beta = 0$ and $\alpha = -1$	Kamal transform, Yang transform
9	$\beta = -1$ and $\alpha = 1$	Mahgoub transform, New transform

The function $f(x)$ so involved in (1.1) is usually continuous and continuously differentiable and the question is what happens when it is continuous but with a fractional derivative of order α, $0 < \alpha < 1$ only. There are two cases. In the first case, $f(x)$ has both a continuous derivative and a fractional derivative. In the second case, $f(x)$ has a derivative of order α, $0 < \alpha < 1$ but has no derivative. For the second case we have to find an alternative. The main goal of this article is to provide a possible approach to this alternative.
Recently, some fractional integral transforms as generalizations of the classical integral transforms have been introduced. The main purpose of fractional integral transforms is to solve fractional differential equations arising in engineering as well as in science fields. For example, the optics problems can also be interpreted by fractional fourier transform.
The following is a fractional integral transform which is a generalization of some existing fractional integral transforms with exponential type kernel as well as many classical integral transforms.

Definition 1.2. *Let f(x) be a function vanishing for negative values of x. It's generalized integral transform of fractional order α, $0 < \alpha < 1$ with exponential type kernel, is defined by*

$$S_\alpha\{f(x)\} = A_\alpha(v^{\beta,\gamma}) = \left(\frac{1}{v^\beta}\right)^\alpha \int_0^\infty E_\alpha(-xv^\gamma)^\alpha \times f(x)(dx)^\alpha$$

$$= \lim_{M\uparrow\infty}\left(\frac{1}{v^\beta}\right)^\alpha \int_0^M E_\alpha(-xv^\gamma)^\alpha f(x)(dx)^\alpha \qquad (1.2)$$

where v is a complex variable, γ is any non-zero real number, β is any real number and $E_\alpha(u)$ is the Mittag-Leffler function $\sum\frac{u^k}{(k\alpha)!}$

The following table explores how the generalized fractional integral transform with exponential type kernel is converted into fractional order integral transforms by assigning particular values of β and γ.

Table 3. Conversion of the generalized fractional integral transform with exponential type kernel into existing fractional integral transforms

S.No	Values of β and γ	The generalized fractional integral transform with exponential type kernel converts into
1	$\beta = 0$ and $\gamma = 1$	Fractional Laplace transform
2	$\beta = 1$ and $\gamma = -1$	Fractional Sumudu transform
3	$\beta = 1$ and $\gamma = -1$	Fractional Natural transform
4	$\beta = -1$ and $\gamma = -1$	Fractional Elzaki transform

2. Preliminaries

In this section, we recall some definitions and basic results of fractional calculus which will be used throughout the paper.

Definition 2.1. *A function f(x) is of fractional exponential order α,$0 < \alpha < 1$ if there are positive real constants c,M and A such that $E_\alpha(-xc^\gamma)|f(x)| < M$ for all $x > A$ where $E_\alpha(u)$ is the Mittag-Leffler function $\sum\frac{u^k}{(k\alpha)!}$ and γ is any non-zero real number.*

2.1 Sufficient conditions for the existence of the generalized fractional integral transform with exponential type kernel

Theorem 2.2. *Let a function f(x) be sectionally continuous in each finite subinterval of the intervals $0 \le x \le A$ and $x > A$ for any positive real constant A. If f(x) is of fractional exponential* *order α, $0<\alpha<1$ then the generalized fractional integral transform with exponential type kernel of f(x) defined by (1.2) exists.*

Proof. For any $A>0$,

$$S_\alpha\{f(x)\}=\left(\frac{1}{v^\beta}\right)^\alpha\int_0^A E_\alpha(-xv^\gamma)^\alpha f(x)(dx)^\alpha +\left(\frac{1}{v^\beta}\right)^\alpha\int_A^\infty E_\alpha(-xv^\gamma)^\alpha f(x)(dx)^\alpha \tag{2.1}$$

As $f(x)$ is sectionally continuous in $0\leq x\leq A$, the first integral in (2.1) exists
Now,

$$\begin{aligned}\left|\int_A^\infty E_\alpha(-xv^\gamma)^\alpha f(x)(dx)^\alpha\right| &\leq\int_A^\infty |E_\alpha(-xv^\gamma)^\alpha f(x)|(dx)^\alpha\\ &\leq\int_0^\infty E_\alpha(-xv^\gamma)^\alpha|f(x)|(dx)^\alpha\\ &\leq\int_0^\infty E_\alpha(-xv^\gamma)^\alpha\\ &\quad\times ME_\alpha(xc^\gamma)^\alpha(dx)^\alpha\\ \left|\int_A^\infty E_\alpha(-xv^\gamma)^\alpha f(x)(dx)^\alpha\right| &\leq\int_0^\infty ME_\alpha(-x(v^\gamma-c^\gamma))^\alpha\\ &\quad\times(dx)^\alpha\end{aligned}$$

Taking $t=x(v^\gamma-c^\gamma)$ and using

$$\Gamma_\alpha(x)=\frac{1}{\Gamma(\alpha+1)}\int_0^\infty E_\alpha(-t)^\alpha t^{(x-1)\alpha}(dt)^\alpha,$$

we have

$$\begin{aligned}\left|\int_A^\infty E_\alpha(-xv^\gamma)^\alpha f(x)(dx)^\alpha\right| &\leq\frac{M}{(v^\gamma-c^\gamma)^\alpha}\Gamma_\alpha(1)\Gamma(\alpha+1)\\ &=\frac{M}{(v^\gamma-c^\gamma)^\alpha}\Gamma(\alpha+1)\end{aligned}$$

Hence the second integral in (2.1) exists and so the generalized fractional integral transform with exponential type kernel of $f(x)$ defined by (1.2) exists. □

Definition 2.3. *Let us consider a class $\mathscr{A}$ of functions $f(x)$ as follows*

$$\mathscr{A}=\{f(x)|\exists M,k_1 \text{ and/or } k_2>0 \text{ such that } |f(x)|<ME_\alpha(|x|k_j{}^\gamma),\ \text{if } x\in(-1)^j\times[0,\infty)\}$$

where the constant M must be finite, while k_1 and k_2 may be finite or infinite, $0<\alpha<1$ and γ is any non-zero real number.

2.2 Duality relation between the fractional laplace transform and the generalized fractional integral transform with exponential type kernel

Theorem 2.4. *Let $F_\alpha(v)$ be the laplace transform of fractional order α of a function f(x) and $A_\alpha(v^{\beta,\gamma})$ be the generalized integral transform of fractional order α with exponential type kernel of the same function f(x). Then*

$$A_\alpha(v^{\beta,\gamma})=\left(\frac{1}{v^\beta}\right)^\alpha F_\alpha(v^\gamma)$$

Proof. Let $f(x)\in\mathscr{A}$. By the generalized fractional integral transform with exponential type kernel, we have

$$\begin{aligned}A_\alpha(v^{\beta,\gamma}) &=\left(\frac{1}{v^\beta}\right)^\alpha\int_0^\infty E_\alpha(-xv^\gamma)^\alpha f(x)(dx)^\alpha\\ &=\left(\frac{1}{v^\beta}\right)^\alpha F_\alpha(v^\gamma)\end{aligned}$$

□

3. Operational Properties

(i)

$$S_\alpha\{t^n\}=\frac{1}{(v^\beta)^\alpha}\frac{1}{(v^\gamma)^{(n+\alpha)}}\Gamma_\alpha\left(\frac{n}{\alpha}+1\right)\Gamma(\alpha+1)$$

Proof.

$$S_\alpha\{t^n\}=\left(\frac{1}{v^\beta}\right)^\alpha\int_0^\infty E_\alpha(-tv^\gamma)^\alpha t^n(dt)^\alpha$$

By taking $tv^\gamma=x$, and using

$$\Gamma_\alpha(x)=\frac{1}{\Gamma(\alpha+1)}\int_0^\infty E_\alpha(-t)^\alpha t^{(x-1)\alpha}(dt)^\alpha, \tag{3.1}$$

we have,

$$S_\alpha\{t^n\}=\frac{1}{(v^\beta)^\alpha}\frac{1}{(v^\gamma)^{(n+\alpha)}}\Gamma_\alpha\left(\frac{n}{\alpha}+1\right)\Gamma(\alpha+1)$$

□

(ii)

$$S_\alpha\{t^{n\alpha}\}=\left(\frac{1}{v^\beta}\right)^\alpha\frac{1}{(v^\gamma)^{(n+1)\alpha}}\Gamma^{(n+1)}(\alpha+1)\Gamma(n+1)$$

Proof.

$$S_\alpha\{t^{n\alpha}\}=\left(\frac{1}{v^\beta}\right)^\alpha\int_0^\infty E_\alpha(-tv^\gamma)^\alpha t^{n\alpha}(dt)^\alpha$$

By taking $tv^\gamma=x$,and using (3.1), we have

$$S_\alpha\{t^{n\alpha}\}=\left(\frac{1}{v^\beta}\right)^\alpha\frac{1}{(v^\gamma)^{(n+1)\alpha}}\Gamma_\alpha(n+1)\Gamma(\alpha+1)$$

Since

$$\Gamma_\alpha(n+1) = \Gamma^n(\alpha+1)\Gamma(n+1), n \in N,$$

We have,

$$S_\alpha\{t^{n\alpha}\} = \left(\frac{1}{v^\beta}\right)^\alpha \frac{1}{(v^\gamma)^{(n+1)\alpha}} \Gamma^{(n+1)}(\alpha+1)\Gamma(n+1)$$

□

(iii)

$$S_\alpha\{f(ax)\} = \frac{1}{a^{2\alpha}} A_\alpha(v^{\gamma,\beta})$$

Proof.

$$S_\alpha\{f(ax)\} = \left(\frac{1}{v^\beta}\right)^\alpha \int_0^\infty E_\alpha(-xv^\gamma)^\alpha f(ax)(dx)^\alpha$$

$$S_\alpha\{f(ax)\} = \alpha\left(\frac{1}{v^\beta}\right)^\alpha \lim_{M\uparrow\infty} \int_0^M [M-x]^{\alpha-1} E_\alpha(-xv^\gamma)^\alpha \times f(ax)(dx)$$

By taking ax=u, we have

$$\begin{aligned} S_\alpha\{f(ax)\} &= \alpha\left(\frac{1}{v^\beta}\right)^\alpha \lim_{M\uparrow\infty} \int_0^{aM} \left[M - \frac{u}{a}\right]^{\alpha-1} \\ &\quad \times E_\alpha\left(-\frac{u}{a}v^\gamma\right)^\alpha f(u)\frac{du}{a} \\ &= \left(\frac{1}{a^{2\alpha}}\right)\left(\frac{1}{v^\beta}\right)^\alpha \lim_{M\uparrow\infty} \int_0^{aM} E_\alpha(-uv^\gamma)^\alpha \\ &\quad \times f(u)(du)^\alpha \\ S_\alpha\{f(ax)\} &= \frac{1}{a^{2\alpha}} S_\alpha\{f(x)\} \end{aligned}$$

□

(iv)

$$S_\alpha\{f(x-b)\} = E_\alpha(-b^\alpha v^{\gamma\alpha}) S_\alpha\{f(u)\}$$

Proof.

$$\begin{aligned} S_\alpha\{f(x-b)\} &= \left(\frac{1}{v^\beta}\right)^\alpha \int_0^\infty E_\alpha(-xv^\gamma)^\alpha f(x-b)(dx)^\alpha \\ &= \alpha\left(\frac{1}{v^\beta}\right)^\alpha \lim_{M\uparrow\infty} \int_0^M E_\alpha(-xv^\gamma)^\alpha (M-x)^{\alpha-1} \\ &\quad \times f(x-b)dx \end{aligned}$$

By taking x-b=u, we have

$$\begin{aligned} S_\alpha\{f(x-b)\} &= \alpha\left(\frac{1}{v^\beta}\right)^\alpha \lim_{M\uparrow\infty} \int_0^{M-b} (M-b-u)^{\alpha-1} \\ &\quad \times E_\alpha(-(v^\gamma)^\alpha (b+u)^\alpha) f(u)du \\ S_\alpha\{f(x-b)\} &= E_\alpha(-b^\alpha v^{\gamma\alpha}) S_\alpha\{f(u)\} \end{aligned}$$

□

(v)

$$S_\alpha\{E_\alpha(-c^\alpha x^\alpha)f(x)\} = S_\alpha\{f(x)\}_{(v^\gamma \to c+v^\gamma)}$$

Proof.

$$\begin{aligned} S_\alpha\{E_\alpha(-c^\alpha x^\alpha)f(x)\} &= \left(\frac{1}{v^\beta}\right)^\alpha \int_0^\infty E_\alpha(-xv^\gamma)^\alpha \\ &\quad \times E_\alpha(-c^\alpha x^\alpha) f(x)(dx)^\alpha \\ &= \left(\frac{1}{v^\beta}\right)^\alpha \int_0^\infty E_\alpha(-x(v^\gamma+c))^\alpha \\ &\quad \times f(x)(dx)^\alpha \\ S_\alpha\{E_\alpha(-c^\alpha x^\alpha)f(x)\} &= S_\alpha\{f(x)\}_{(v^\gamma \to c+v^\gamma)} \end{aligned}$$

□

(vi)

$$S_\alpha\{f^{(\alpha)}(x)\} = (v^\gamma)^\alpha S_\alpha\{f(x)\} - \left(\frac{1}{v^\beta}\right)^\alpha \Gamma(1+\alpha)f(0)$$

Proof.

$$S_\alpha\{f^{(\alpha)}(x)\} = \left(\frac{1}{v^\beta}\right)^\alpha \int_0^\infty E_\alpha(-xv^\gamma)^\alpha f^{(\alpha)}(x)(dx)^\alpha$$

By integration by parts and using

$$D_x^\alpha\{E_\alpha(\lambda x^\alpha)\} = \lambda E_\alpha(\lambda x^\alpha)$$

we have

$$\begin{aligned} S_\alpha\{f^{(\alpha)}(x)\} &= \left(\frac{1}{v^\beta}\right)^\alpha \Big\{\Gamma(1+\alpha)[f(x)E_\alpha(-xv^\gamma)^\alpha]_0^\infty \\ &\quad + v^{\gamma\alpha} \int_0^\infty E_\alpha(-xv^\gamma)^\alpha f(x)(dx)^\alpha\Big\} \\ &= -\left(\frac{1}{v^\beta}\right)^\alpha \Gamma(1+\alpha)f(0) \\ &\quad + \frac{v^{\gamma\alpha}}{(v^\beta)^\alpha} \int_0^\infty E_\alpha(-xv^\gamma)^\alpha f(x)(dx)^\alpha \end{aligned}$$

$$S_\alpha\{f^{(\alpha)}(x)\} = (v^\gamma)^\alpha S_\alpha\{f(x)\} - \left(\frac{1}{v^\beta}\right)^\alpha \Gamma(1+\alpha)f(0) \tag{3.2}$$

□

(vii)

$$S_\alpha\left\{\int_0^x f(x)(dx)^\alpha\right\} = v^{-\gamma\alpha}\Gamma(\alpha+1)S_\alpha\{f(x)\}$$

Proof. From (3.2),

$$(v^{\gamma\alpha})S_\alpha\{f(x)\} = S_\alpha\{f^{(\alpha)}(x)\} + \left(\frac{1}{v^\beta}\right)^\alpha \Gamma(1+\alpha)f(0)$$

Let $g(x) = \int_0^x f(x)(dx)^\alpha$. Then $g(0) = 0$

$$\begin{aligned}(v^{\gamma\alpha})S_\alpha\left\{\int_0^x f(x)(dx)^\alpha\right\} =& S_\alpha\{g^{(\alpha)}(x)\}\\ =& S_\alpha\left\{D_x^\alpha \int_0^x f(x)(dx)^\alpha\right\}\\ =& S_\alpha\{\Gamma(\alpha+1)f(x)\}\\ =& \Gamma(\alpha+1)S_\alpha\{f(x)\}\\ S_\alpha\left\{\int_0^x f(x)(dx)^\alpha\right\} =& v^{-\gamma\alpha}\Gamma(\alpha+1)S_\alpha\{f(x)\}\end{aligned}$$

□

4. Convolution theorem

Theorem 4.1. *If the convolution of order α of the two functions f(x) and g(x) is given by the expression*

$$(a(x)*b(x))_\alpha = \int_0^x a(x-u)b(u)(du)^\alpha$$

then one has the equality

$$S_\alpha\{a(x)*b(x)\} = (v^\beta)^\alpha S_\alpha\{a(x)\}S_\alpha\{b(x)\}$$

Proof.

$$\begin{aligned}S_\alpha\{(a*b)_\alpha\} &= \left(\frac{1}{v^\beta}\right)^\alpha \int_0^\infty E_\alpha(-x^\alpha v^{\gamma\alpha})\\ &\quad\times \int_0^x a(x-u)b(u)(du)^\alpha(dx)^\alpha\\ &= \left(\frac{1}{v^\beta}\right)^\alpha \int_0^\infty E_\alpha(-v^{\gamma\alpha}(x-u)^\alpha)\\ &\quad\times E_\alpha(-v^{\gamma\alpha}u^\alpha)\int_0^x a(x-u)b(u)\\ &\quad\times (du)^\alpha(dx)^\alpha\end{aligned}$$

By changing the variables x-u=t, u=s, we obtain

$$\begin{aligned}S_\alpha\{(a*b)_\alpha\} &= \left(\frac{1}{v^\beta}\right)^\alpha \int_0^\infty E_\alpha(-v^{\gamma\alpha}t^\alpha)a(t)(dt)^\alpha\\ &\quad\times \int_0^\infty E_\alpha(-v^{\gamma\alpha}s^\alpha)b(s)(ds)^\alpha\\ &= (v^\beta)^\alpha S_\alpha\{a(t)\}S_\alpha\{b(s)\}\end{aligned}$$

□

5. Inversion theorem

Theorem 5.1. *For $0<\alpha<1$, the generalized fractional integral transform with exponential type kernel*

$$A_\alpha(v^{\beta,\gamma}) = \left(\frac{1}{v^\beta}\right)^\alpha \int_0^\infty E_\alpha(-xv^\gamma)^\alpha f(x)(dx)^\alpha$$

has the inversion formula,

$$f(x) = \frac{1}{(M_\alpha)^\alpha}\int_{-i\infty}^{+i\infty}(v^\beta)^\alpha E_\alpha(v^{\gamma\alpha}x^\alpha)A_\alpha(v^{\beta,\gamma})(dv)^\alpha$$

where M_α is the period of the complex-valued Mittag-Leffler function defined by the equality $E_\alpha(i(M_\alpha)^\alpha) = 1$

Proof. The laplace transform of fractional order α of $f(x)$,i.e.,

$$F_\alpha(v) = \int_0^\infty E_\alpha(-v^\alpha x^\alpha)f(x)(dx)^\alpha, 0<\alpha<1$$

has the inversion formula

$$f(x) = \frac{1}{(M_\alpha)^\alpha}\int_{-i\infty}^{+i\infty} E_\alpha(v^\alpha x^\alpha)F_\alpha(v)(dv)^\alpha$$

By the duality relation between the laplace transform of fractional order and the generalized fractional integral transform with exponential type kernel, we arrive the desired inversion formula.

□

6. Applications

Example 6.1. *Suppose $A_\alpha(v^{\beta,\gamma})$ is the generalized fractional integral transform with exponential type kernel of a function $f(x)$. Then the solution of fractional differential equation*

$$y^{(\alpha)} + \lambda y = f(x), y(0) = 0, 0<\alpha<1 \tag{6.1}$$

is given by

$$y(x) = \frac{1}{(M_\alpha)^\alpha}\int_{-i\infty}^{+i\infty}\frac{(v^\beta)^\alpha}{\lambda+(v^\gamma)^\alpha}E_\alpha(v^{\gamma\alpha}x^\alpha)A_\alpha(v^{\beta,\gamma})_f(dv)^\alpha \tag{6.2}$$

where λ is a constant.

Now we solve (6.1). By taking the generalized fractional integral transform with exponential type kernel on the both sides of (6.1), we get

$$(v^\gamma)^\alpha A_\alpha\{v^{\beta,\gamma}\}_y + \lambda A_\alpha\{v^{\beta,\gamma}\}_y = A_\alpha\{v^{\beta,\gamma}\}_f$$

$$A_\alpha\{v^{\beta,\gamma}\}_y = \frac{1}{\lambda+(v^\gamma)^\alpha} A_\alpha\{v^{\beta,\gamma}\}_f \tag{6.3}$$

By substituting (6.3) in the inversion formula of the generalized fractional integral transform with exponential type kernel, we arrive the required solution (6.2).

Example 6.2. *The current in a circuit with inductance L, resistance R and capacitance C with an applied voltage $E(t)$ is governed by the equation*

$$L\frac{dI}{dt}+RI+\frac{1}{C}\int_0^t Idt = E(t)$$

where L,R and C are constants and the current $I(t)$ having the relation

$$I(t) = \frac{dQ}{dt}$$

with accumulated charge Q on the condenser at time t is

$$Q(t) = \int_0^t I(t)dt$$

Let us consider the fractional equation of current in the circuit, that is

$$LI^{(\alpha)}(t)+RI+\frac{1}{C}\int_0^t I(t)(dt)^\alpha = E(t), \tag{6.4}$$

with t=0, I(0)=0, Q(0)=0and $0<\alpha<1$

Now we solve (6.4). By taking the generalized fractional integral transform with exponential type kernel on both sides of (6.4), we get,

$$A_\alpha(v^{\beta,\gamma})_I = \frac{1}{\frac{v^{2\gamma\alpha}\Gamma(\alpha+1)}{c}+Rv^\alpha+L} A_\alpha(v^{\beta,\gamma})_E \tag{6.5}$$

where $A_\alpha(v^{\beta,\gamma})_I$ and $A_\alpha(v^{\beta,\gamma})_E$ are the generalized fractional integral transforms of $I(t)$ and $E(t)$ respectively. Then characteristic equation of (6.5) is given by

$$v^{2\gamma\alpha}+\frac{R}{L}v^{\gamma\alpha}+\frac{\Gamma(\alpha+1)}{CL} = 0$$

The roots of the above characteristic equation are $v^{\gamma\alpha} = -k\pm in$, where $k=\frac{R}{2L}$ and $n^2=\frac{\Gamma(\alpha+1)}{CL}-\frac{R^2}{4L^2}$ having the negative real part. This shows that the system is stable. Now by substituting (6.5) in the inversion formula of the generalized fractional integral transform with exponential type kernel, we arrive

$$I(t) = \frac{1}{L(M_\alpha)^\alpha}\int_{-i\infty}^{+i\infty} \frac{(v^\beta)^\alpha E_\alpha(x^\alpha v^{\gamma\alpha})}{v^{2\gamma\alpha}+\frac{R}{L}v^{\gamma\alpha}+\frac{\Gamma(\alpha+1)}{CL}} A_\alpha(v^{\beta,\gamma})_E(dv)^\alpha$$

as a solution of (6.4).

7. Conclusion

The generalized fractional integral transform with exponential type kernel is thus a very powerful tool because we have a choice whether we wish to proceed by the generalized fractional integral transform with exponential type kernel itself or any other existing, non existing fractional integral transforms according to our convenience and situation of the problems. Sufficient conditions for the existence of the generalized fractional integral transform with exponential type kernel ensure the existence of many fractional integral transforms. It is also seen that the generalized fractional integral transform with exponential type kernel is an efficient and useful technique for solving many fractional differential equations.

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