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On the k-distant total labeling of graphs

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Abstract

A labeling of a graph is a mapping that maps some set of graph elements to a set of numbers. In this paper, two new variations of labeling named k-distant edge total labeling and k-distant vertex total labeling are introduced. Moreover, the study of two new graph parameters, called k-distant edge chromatic number (γ'_{kd}) and k-distant vertex chromatic number (γ_{kd}) related this labeling are initiated. The k-distant vertex total labeling for paths, cycles, complete graphs, stars, bi-stars and friendship graphs are studied and the value of the parameter γ_{kd} determined for these graph classes. Then k-distant edge total labeling for paths, cycles and stars are studied. Also, an upper bound of γ_{kd} and a lower bound of γ'_{kd} are presented for general graphs.

Keywords

Graph Labeling, total labeling, k-distant vertex total labeling, k-distant edge total labeling.

AMS Subject Classification

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Contents

1. Introduction

All graph considered in this paper are simple, connected and undirected. A labeling of a graph $G=(V,E)$ is a function from the vertex set V or the edge set E or both to the set of integers subject to certain restrictions. If the domain is the vertex-set (edge-set) the labeling is called vertex-labeling (edge labeling). Graph labeling has a variety of applications oriented areas like coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing, etc. Despite a tremendous amount of research done, there are still a lot of open problems in this area and many researchers working on various aspects of graph labeling. The several graphs labeling problems has been widely studied for many particular classes of graphs. The purpose of this paper is to introduce to new variations of labeling, called k-distant vertex total labeling and k-distant edge total labeling on graphs. Now, the notion of k-distant vertex total labeling and k-distant edge total labeling are defined more precisely as follows.

Given an undirected connected graph $G(V,E)$ with vertex set V and edge set E. For $x,y \in V$, let $(x,y) \in E$, i.e., the vertices x and y are adjacent. A labeling $f: V \cup E \to \{1,2,...,r\}$ is called k-distant vertex total labeling if $|w(x)-w(y)| \geq k$ for $(x,y) \in E$ where $w(x) = f(x) + \sum\limits_{(x,y)\in E} f(x,y)$, is the weight of the vertex x. The minimum r for which a graph G has k-distant vertex total labeling is called the total k-distant chromatic number of G and is denoted by $\gamma_{kd}(G)$.

A labeling $f: V \cup E \to \{1,2,...,r\}$ is called k-distant edge total labeling, if for any two adjacent edges $e_1, e_2 \in E$, $|w(e_1)-w(e_2)| \geq k$ where $w(e) = f(e)+f(x)+f(y)$ and $x,y \in V$ are two end vertices of the edge $e \in E$. The minimum r for which a graph G has k-distant edge total labeling is called the total k-distant edge chromatic number of G and is denoted by $\gamma'_{kd}(G)$.

From the above definitions, it follows that for the k-distant vertex (edge) total labeling of a graph, the weights of two adjacent vertices (edges) must differ by at least k. It is easily observed that the parameters $\gamma_{kd}(G)$ and $\gamma'_{kd}(G)$ exist for every connected graph having at least two vertices and two edges respectively.

Baca et. al. introduced edge irregular total k-labeling in [2]. A labeling $f: V(G) \cup E(G) \to \{1,2,...,k\}$ is said to be a edge irregular total k -labeling if for every two different edges e_1 and e_2, $w(e_1) \neq w(e_2)$, where $w(e) = f(e) + f(u) + f(v)$ and u, v are two end vertices of the edge e. The minimum k for which G has an edge irregular total k-labeling is defined as the total edge irregularity strength of G and is denoted by tes(G). Again, a labeling $f : V(G) \cup E(G) \to \{1, 2, ..., k\}$ is said to be a vertex irregular total k -labeling if for every two different vertices u and v, $w(u) \neq w(v)$, where $w(u) = f(u) + \sum_{(u,v)\in E} f(u,v)$. The minimum k for which G has a vertex irregular total k-labeling is defined as the total vertex irregularity strength of G and is denoted by tvs(G).

From the definition of vertex (edge) irregular total labeling, it follows that $w(x) \neq w(y)$ for all $x, y \in V$ $(w(e_1) \neq w(e_2)$ for all $e_1, e_2 \in E)$. Therefore, every vertex (edge) irregular total labeling is a 1-distant vertex (edge) total labeling but the converse may not be true. Clearly, $tvs(G) \geq \gamma_{1d}(G)$ and $tes(G) \geq \gamma'_{1d}(G)$.

In [1], Arumugam et. al. extended irregular total labeling and introduced a new labeling scheme for graph where adjacent vertices (edges) get different weights. Such a vertex (edge) labeling is called chromatic vertex (edge) labeling. So, k-distant vertex total labeling and k-distant edge total labeling are natural extension of the chromatic vertex labeling and chromatic edge labeling respectively. Here, the weights of adjacent edges are not only distinct but also the absolute difference of weights of two adjacent edges must be at least k.

The k-distant total labeling might be used to set up various service center, so that the minimum number of service center results most extreme advantages. Let each vertex represents a city an edge is the road between two cities. We want to set up minimum number of service center in cities and in between two cities such that there are back up of at least k service center which can serve during any disaster. This issue can be demonstrated as k-distant total labeling problem of graphs.

1.1 Review

The study of graphs labeling was initiated by Sadlacek in 1964 [7]. This study was continued by Stewart [9] and then Rosa in 1967 and by Kotzig and Rosa [4] in 1970. In the intervening years, many new directions and new results of graph labeling have been developed. Most graph labeling methods find their origin that was introduced by Rosa [6]. A labeling of a graph G is a mapping that carries a set of graph elements into a set of integers, called labels [10]. The labeling is called a vertex labeling or edge labeling or total labeling, if the domain of the mapping is a vertex set or an edge set or a union of vertex and edge sets respectively. A lots of variation of labeling on graphs are available in literature [3]. In the recent book "A Dynamic Survey of Graph labeling" by J. A. Gallian lists 2643 papers on over 200 variations of graph labeling. Irregular total labeling is one among the kinds of labeling that appeared in the literature which is defined by Baca et.al. [2]. For an extended and up-to-date survey of graph labeling one can see [3].

In this paper, the study of two new variations of labeling, that is, k-distant vertex total labeling and k-distant edge total labeling are initiated. Also, two new graph parameters γ_{kd} and γ'_{kd} are introduced. The value of γ_{kd} and γ'_{kd} for some classes of graphs are determined and bounds for these parameter of general graph are presented.

The paper is organized as follows. In Section 2, some results on total k-distant vertex chromatic number (γ_{kd}) are presented. Section 3 presents some results on total k-distant vertex chromatic number (γ'_{kd}). An upper bounds of γ_{kd} and a lower bound of γ'_{kd} for general graph are determined in Section 4. Finally, Section 5 contains some concluding remarks.

2. Some Results on γ_{kd}

This section presents the exact values of γ_{kd} for paths, cycles, complete graphs, star graphs, bi-star and friendship graphs.

Lemma 2.1. *For a path P_n,* $\gamma_{kd}(P_n) = k$ $(n > 2)$.

Proof: Let P_n be a path with n vertices $v_1, v_2, \ldots, v_n$. Then the vertex set of P_n is $V = \{v_i : 1 \leq i \leq n\}$ and edge set is $E = \{v_i v_{i+1} : 1 \leq i \leq n-1\}$.

Let us define a function $f : V \cup E \to \{1, \ldots, k\}$ as follows: $f(v_1) = 1$, $f(v_1v_2) = 1$, $f(v_2) = 1$, $f(v_2v_3) = k$, $f(v_3) = k$, $f(v_3v_4) = 2$, $f(v_4) = 1$ and so on.

Then $|w(v_i) - w(v_j)| \geq k$ for all $1 \leq i, j \leq n, i \neq j$. Therefore f is a k-distant vertex total labeling of P_n.

Observe that k is the least positive integer such that f is a k-distant vertex total labeling and consequently $\gamma_{kd}(P_n) = k$. □

It is easy to see that for P_2, $\gamma_{kd}(P_2) = k+1$.

A cycle graph C_n, $n \geq 3$ of n vertices is a graph on n vertices containing a single cycle through all the vertices.

Lemma 2.2. *For a cycle C_n with n vertices,* $\gamma_{kd}(C_n) = k+1$.

Proof: Let C_n be a cycle of n vertices v_1, v_2, …, v_n.

If n be even, then define a mapping $f : V \cup E \to \{1, \ldots, k+1\}$ as $f(v_n v_1) = 1$, $f(v_1) = 1$, $f(v_1v_2) = 1$, $f(v_2) = k+1$, $f(v_2v_3) = 1$, …, $f(v_{n-1}v_n) = 1$, $f(v_n) = k+1$. Thus label the vertices of C_n with the integers 1 and $k+1$ alternately and all the edges with the label 1. Then f satisfies the conditions of k-distant vertex total labeling and $k+1$ is the least positive integer. Therefore $\gamma_{kd}(C_n) = k+1$.

If n be odd , then define a mapping $\phi : V \cup E \to \{1, \ldots, k+1\}$ as $\phi(v_n v_1) = 1$, $\phi(v_1) = 1$, $\phi(v_1v_2) = 1$, $\phi(v_2) = k+1$, $\phi(v_2v_3) = 1, \ldots$, $\phi(v_{n-2}) = 1$, $\phi(v_{n-2}v_{n-1}) = 1$, $\phi(v_{n-1}) = 1$, $\phi(v_{n-1}v_n) = k+1$, $\phi(v_n) = k+1$. Thus, label the vertices of C_n with the integers 1 and $k+1$ alternately but both the vertices v_{n-2} and v_{n-1} with 1 and all the edges with the label 1 but the last edge with $k+1$. Then ϕ satisfies the conditions of k-distant vertex total labeling and $k+1$ is the least positive integer. Therefore $\gamma_{kd}(C_n) = k+1$.

A complete graph is a simple graph in which every pair of vertices is connected by an edge. A complete graph of n vertices is denoted by K_n.

Lemma 2.3. *For a complete graph K_n with n vertices, $\gamma_{kd}(K_n) = k+1$*

Proof: Let K_n be a complete graph of n vertices with vertex set $V = \{v_1, ..., v_n\}$.

Then the edge set of the K_n is $E = \{v_iv_j : 1 \le i, j \le n, i \neq j\}$.

Let us define $f : V \cup E \to \{1, k+1\}$ as $f(v_1) = 1, f(v_1v_i) = 1, 1 < i \le n$.

$f(v_2) = k+1, f(v_2v_i) = 1, 2 < i \le n$

$f(v_3) = k+1, f(v_3v_i) = 1, 3 < i \le n-1, f(v_3v_n) = k+1$

$f(v_4) = k+1, f(v_4v_i) = 1, 4 < i \le n-2, f(v_4v_{n-1}) = k+1, f(v_4v_n) = k+1$

$\dots, \dots, \dots$

$f(v_{n-1}) = k+1, f(v_{n-1}v_n) = k+1$

$f(v_n) = k+1$

Therefore $\gamma_{kd}(K_n) = k+1$. □

A star $K_{1,n}$ is the complete bipartite graph, a tree with one internal vertex and n pendant vertices.

Lemma 2.4. *For a star graph $K_{1,n}$ with n vertices,* $\gamma_{kd}(K_{1,n}) = \begin{cases} \mathbf{1}, & k < n \\ \mathbf{k-n+2}, & k \ge n \end{cases}.$

Proof: Let $K_{1,n}$ be a star graph of $n+1$ vertices with vertex set $V = \{v_0, v_1, ..., v_n\}$ where v_0 is the internal vertex and all other vertices are pendant vertex. The edge set of $K_{1,n}$ is $E = \{v_0v_i : 1 \le i \le n\}$.

Two cases may arise.

Case 1: $k \le n-1$

Let us define a mapping f from the domain $V \cup E$ as $f(v_0) = 1$, $f(v_0v_i) = 1$ and $f(v_i) = 1$ $i = 1, 2, \dots, n$. Then $w(v_0) = n+1$ and $w(v_i) = 2, 1 \le i \le n$. Therefore $|w(v_0) - w(v_i)| = n-1 \ge k$ for all $1 \le i \le n$. Hence f is a k-distant vertex total labeling of $K_{1,n}$ and $\gamma_{kd}(K_{1,n}) = 1$.

Case 2: $k \ge n-1$

In this case, let us define a mapping ϕ from the domain $V \cup E$ as $f(v_0) = k-n+2$, $f(v_0v_i) = 1$ and $f(v_i) = 1$ $i = 1, 2, \dots, n$. Then $w(v_0) = k-n+2+n = k+2$ and $w(v_i) = 2, 1 \le i \le n$. Therefore $|w(v_0) - w(v_i)| = k$ for all $1 \le i \le n$. Hence ϕ is a k-distant vertex total labeling of $K_{1,n}$. Clearly, $k-n+2$ is the least positive integer such that ϕ is a k-distant vertex total labeling of $K_{1,n}$ and therefore $\gamma_{kd}(K_{1,n}) = k-n+2$. □

A graph obtained by joining two internal vertices of two copies of star $K_{1,n}$ is called a bi-star and is denoted by $B_{n,n}$. So a bi-star $B_{n,n}$ has $2n+2$ vertices and $2n+1$ edges.

Lemma 2.5. *For a bistar $B_{n,n}$,* $\gamma_{kd}(B_{n,n}) = \begin{cases} \mathbf{2}, & k \le n-1 \\ \lceil \frac{\mathbf{2k+2}}{\mathbf{n+1}} \rceil, & k > n-1 \end{cases}$

Proof: Let $V = \{u, v, u_i, v_i : 1 \le i \le n\}$ and $E = \{uv, uu_i, vv_i\}$ are the vertex set and edge set of $B_{n,n}$ respectively, where u and v are two internal vertices of two stars. **Case 1:** $k \le n-1$ We assign the label 1 to each of the vertices $u_i, 1 \le i \le n$, edges $uu_i, 1 \le i \le n$ and uv. Then $w(u_i) = 2$ and $w(u) = n+1$. Again, we assign the label 1 to each of the vertices

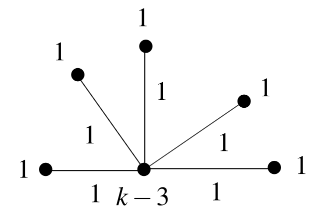

Figure 1. Star graph $K_{1,5}$

$v_i, 1 \le i \le n$, the label 2 to each of the edges $vv_i, 1 \le i \le n$ and the vertex v. Then $w(v_i) = 3$ and $w(v) = 2n+1$. Therefore $|w(x) - w(y)| \ge k$ for all $xy \in E$ and hence $\gamma_{kd}(B_{n,n}) = 2$.

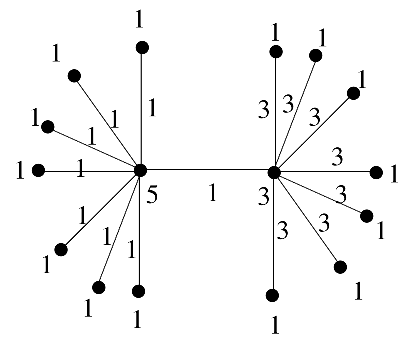

Figure 2. $\gamma_{11d}(B_{7,7}) = 3$

Case 2: $k > n-1$

Let us define a mapping f from the domain set $V \cup E$ as $f(u_i) = 1$ for $1 \le i \le n$, $f(v_i) = 1$ for $1 \le i \le n$, $f(uu_i) = 1$ for $1 \le i \le n$, $f(vv_i) = \lceil \frac{2k+2}{n+1} \rceil$ for $1 \le i \le n$, $f(u) = k-n+1$, $f(uv) = 1$ and $f(v) = \lceil \frac{2k+2}{n+1} \rceil$. Then $w(u_i) = 2$, $w(u) = k+2$, $w(v) = 2k+2$ and $w(v_i) = \lceil \frac{2k+2}{n+1} \rceil + 1$. Then $|w(x) - w(y)| \ge k$ for all $xy \in E$ and hence $\gamma_{kd}(B_{n,n}) = \lceil \frac{2k+2}{n+1} \rceil$. □

A friendship graph F_n is copies of n triangle with a common vertex. The vertex set of F_n is $V(F_n) = \{v_{00}, v_{i1}, v_{i2} : i = 1, 2, \dots, n\}$ and edge set is $E(F_n) = \{v_{00}v_{i1}, v_{00}v_{i2}, v_{i1}v_{i2} : i = 1, 2, \dots, n\}$. So $|V(F_n)| = 2n+1$ and $|E(F_n)| = 3n$. The exact value of k-distant chromatic number is obtained in the next lemma.

Lemma 2.6. $\gamma_{kd}(F_n) = \begin{cases} \mathbf{2k}, & k \ge n \\ \mathbf{2n}, & k < n \end{cases}.$

Proof: Let F_n be a friendship graph and v be the common vertex of n 3-cycle graphs $C_{3i}, i = 1, 2, \dots, n$. Let C_{3i} has vertex set $\{v, v_{i1}, v_{i2}\}$. Then V has vertex set $V = \{v, v_{i1}, v_{i2} : 1 \le i \le n\}$ and edge set $E = \{vv_{i1}, vv_{i2}, v_{i1}v_{i2}\}$

Let us define a mapping f from the domain $V \cup E$ as $f(v) = 1, f(vv_{i1}) = 1, f(vv_{i1}) = 1, f(v_{i1}) = k, f(vv_{i2}) = 2k$ and $f(v_{i1}v_{i2}) = 2n$ for $1 \le i \le n$. Then f is a k-distant vertex total labeling of F_n and therefore $\gamma_{kd}(F_n) = \begin{cases} \mathbf{2k}, & k \ge n \\ \mathbf{2n}, & k < n \end{cases}.$ □

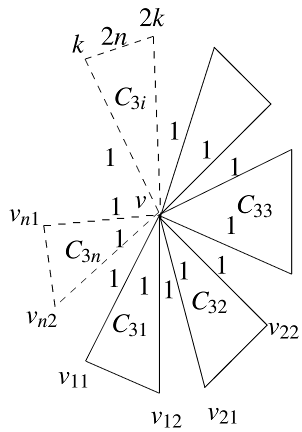

Figure 3. Friendship graph F_n

3. Some Results on $\gamma'_{kd}(G)$

In this section the boundary values of γ'_{kd} presented for paths, cycles and star graphs.

Lemma 3.1. *For a path P_n of $n(>3)$ vertices,* $\gamma'_{kd}(P_n) = \lceil \frac{3k}{4} \rceil + 1$.

Proof: Let P_n be a path with n vertices $v_1, v_2, \ldots, v_n$. Then the vertex set of P_n is $V = \{v_i : 1 \leq i \leq n\}$ and edge set is $E = \{v_i v_{i+1} : 1 \leq i \leq n-1\}$.

Let us define a function $g : V \cup E \to \{1, \ldots, \lceil \frac{3k}{4} \rceil + 1\}$ as follows: $g(v_1) = 1, g(v_1v_2) = 1, g(v_2) = 1, g(v_2v_3) = \lceil \frac{k}{2} \rceil + 1$, $g(v_3) = \lceil \frac{k}{2} \rceil + 1$, $g(v_3v_4) = \lceil \frac{3k}{4} \rceil + 1$, $g(v_4) = \lceil \frac{3k}{4} \rceil + 1$, $g(v_4v_5) = \lceil \frac{k}{4} \rceil + 1$. Then repeat the mapping for the remaining vertices and edges of P_n.

Then $|w(v_iv_j) - w(v_iv_k)| \geq k$ for all i, j and k. Therefore f is a k-distant edge total labeling of P_n.

Also, $\lceil \frac{3k}{4} \rceil + 1$ is the least positive integer such that $|w(v_iv_j) - w(v_iv_k)| \geq k$ for all i, j and k. Therefore $\gamma'_{kd}(P_n) = \lceil \frac{3k}{4} \rceil + 1$. $\square$

It should be noted that, $\gamma'_{kd}(P_3) = \lceil \frac{k}{2} \rceil + 1$.

Lemma 3.2. *For a cycle C_n with $n(\geq 3)$ vertices,*

$$\gamma'_{kd}(C_n) = \begin{cases} \mathbf{\frac{2^{\frac{n}{2}-1}}{2^{\frac{n}{2}}}k + 1}, & n \ \text{is even} \\ \mathbf{k+1}, & n \ \text{is odd} \end{cases}.$$

Proof: Let C_n be a cycle of n vertices $v_1, v_2, \ldots, v_n$.

Two cases which may arise.

Case 1: n is even. First, we shall show that the theorem is true for $n = 4$.

Let C_4 be a cycle of four vertices v_1, v_2, v_3, v_4. Let us define a mapping g such that $g(v_1) = 1, g(v_1v_2) = 1, g(v_2) = 1, g(v_2v_3) = \lceil \frac{k}{2} \rceil + 1$, $g(v_3) = \lceil \frac{k}{2} \rceil + 1$, $g(v_3v_4) = \lceil \frac{3k}{4} \rceil + 1$, $g(v_4) = \lceil \frac{3k}{4} \rceil + 1$, $g(v_4v_1) = \lceil \frac{k}{4} \rceil + 1$. Then g satisfy the condition of k-distant total edge labeling and $\lceil \frac{3k}{4} \rceil + 1$ is the least positive integer. So, $\gamma'_{kd}(C_4) = \lceil \frac{3k}{4} \rceil + 1 = \frac{2^{\frac{4}{2}-1}}{2^{\frac{4}{2}}}k + 1$. Therefore the lemma is true for $n = 4$.

Let it be true for m, where $m = 2k$. Then $2k$ vertices and $2k$ edges of C_{2k} can be labeled by using not greater than $\frac{2^{\frac{m}{2}-1}}{2^{\frac{m}{2}}}k + 1$ integers.

When $m = 2k + 2$, extra two vertices and three edges will be generated. These two vertices are v_{m-1}, v_m and three edges are $v_{m-2}v_{m-1}$, $v_{m-1}v_m$, v_mv_1. Then define $g(v_{m-2}v_{m-1}) = 1, g(v_{m-1}) = \frac{1}{2^{\frac{m}{2}-1}}k, g(v_{m-1}v_m) = \frac{2^{\frac{m}{2}-1}}{2^{\frac{m}{2}}}k + 1, g(v_m) = \frac{2^{\frac{m}{2}-1}}{2^{\frac{m}{2}}}k + 1$, $g(v_mv_1) = \frac{1}{2^{\frac{m}{2}}}k$. Other vertices and edges get the same label as in the case for $m = 2k$. Then g satisfy the condition of k-distant total edge labeling and $\frac{2^{\frac{m}{2}-1}}{2^{\frac{m}{2}}}k + 1$ is the least positive integer.

Therefore, by principle of mathematical induction, the lemma is true for all even positive integers.

Case 2: n is odd.

Let C_n has the vertex set $\{v_1, v_2, \ldots, v_n\}$ where $n = 2m + 1$. Let us define a function g from the set $V \cup E$ to the set of natural numbers such that $w(v_1v_2) = 3$, $w(v_2v_3) = k + 3$, $w(v_3v_4) = 2k + 3$, $w(v_4v_5) = k + 3$ and so on.

In this case the weight of the last edge i.e., v_nv_1 must be $2k + 3$. Since vertex 1 got label 1, the sum of the labels of the vertex v_n and the edge v_nv_1 must be $2k + 2$. This is achieved by using the least positive integer $k + 1$. Hence the lemma follows. $\square$

Lemma 3.3. *For a star graph $K_{1,n}$ with n vertices,* $\gamma'_{kd}(K_{1,n}) = \lceil \frac{(n-1)k}{2} \rceil + 1$.

Proof: Let $K_{1,n}$ be a star graph of $n+1$ vertices with vertex set $V = \{v_0, v_1, ..., v_n\}$ where v_0 is the internal vertex and all other vertices are pendant vertex. The edge set of $K_{1,n}$ is $E = \{v_0v_i : 1 \leq i \leq n\}$.

A mapping g from the vertex set and edge set to the set of natural numbers is defined as follows.

$g(v_0) = 1, g(v_0v_1) = 1, g(v_1) = 1$
$g(v_0v_2) = 1 + \lceil \frac{k}{2} \rceil, g(v_2) = 1 + \lceil \frac{k}{2} \rceil$
$g(v_0v_3) = 1 + \lceil \frac{2k}{2} \rceil, g(v_3) = 1 + \lceil \frac{2k}{2} \rceil$
......
$g(v_0v_n) = 1 + \lceil \frac{(n-1)k}{2} \rceil, g(v_n) = 1 + \lceil \frac{(n-1)k}{2} \rceil$

Then $|w(v_0v_i) - w(v_0v_j)| \geq k$ and $1 + \lceil \frac{(n-1)k}{2} \rceil$ is the least positive integer such that this inequality holds.

Therefore $\gamma'_{kd}(K_{1,n}) = \lceil \frac{(n-1)k}{2} \rceil + 1$. $\square$

4. Bounds of $\gamma_{kd}(G)$ and $\gamma'_{kd}(G)$ for general graphs

Theorem 4.1. *For an arbitrary graph G with maximum degree Δ,*

$$\gamma_{kd}(G) \leq k + \Delta.$$

Proof: Let G be an arbitrary graph with maximum degree Δ and v be a vertex of degree Δ. Then minimum weight of the vertex v is Δ. Observe that, if there exists any leaf vertex adjacent to v then to satisfy k-distant vertex total labeling, minimum label of that vertex will be $k+\Delta$. Again, if there does not exist any leaf vertex adjacent to v then minimum label of that vertex will be not greater than $k+\Delta$. Therefore, $\gamma_{kd}(G) \leq k+\Delta$. □

Theorem 4.2. *For an arbitrary graph G with n vertices,*

$$\gamma'_{kd}(G) \geq \lceil \frac{k}{2} \rceil + 1.$$

Proof: Observe that, the value of $\gamma'_{kd}(G)$ is minimum, when it is possible to label the vertices and edges of G in such a way that weight of each edge is least as well as difference of weight between two adjacent edges not less than k. Now, the least possible weight of an edge is 3. In that case each of the end vertices of the edge and edge itself get label 1. So the minimum weight of any adjacent edge must be at least $k+3$. Since, one end vertex of the edge already labeled by 1, the sum of the labels of another end vertex and the edge must be $k+2$.

It is easy to check that, $\lceil \frac{k}{2} \rceil + 1$ is the least positive integer so that sum of two numbers is $k+2$ and hence $\gamma'_{kd}(G) \geq \lceil \frac{k}{2} \rceil + 1$. □

Note that, the value of $\gamma'_{kd}(G)$ is maximum, if there exist en edge between every pair of vertices of G.

Conjecture 4.3. *For an arbitrary graph G with n vertices,*

$$\gamma'_{kd}(G) \leq \lceil \frac{(n+2)k}{2} \rceil + 1.$$

5. Conclusion

In this paper the study of a new labeling of graph, called k-distant total labeling is initiated. Then two new graph pamperers k-distant chromatic number (γ_{kd}) and k-distant edge chromatic number (γ'_{kd}) are introduced as the natural extensions of the vertex irregularity strength and the vertex irregularity strength of a graph respectively. Also, the values of γ_{kd} and γ'_{kd} for some particular classes of graphs namely, paths, cycles, complete graphs, stars, bistars and friendship graphs are estimated. An upper bound of γ_{kd} and a lower bound of γ'_{kd} for general graph is provided. Many real life problems can be modeled using k-distant total labeling of graphs. Future study can be done to find the value of γ_{kd} and γ'_{kd} for more general graph classes like trees, wheel graphs etc.. □

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