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Coefficient estimates for Bi-univalent functions with respect to symmetric conjugate points associated with Horadam Polynomials

Jayaraman Sivapalan[1], Nanjundan Magesh[2]* and Samy Murthy[3]

Abstract
In this investigation, we propose to make use of the Horadam polynomials, we introduce a class of bi-univalent functions with respect to symmetric conjugate points. For functions belonging to this class, the coefficient bounds and the Fekete-Szegö bounds are discussed. Some interesting remarks of the results presented here are also investigated.

Keywords
Analytic functions, bi-univalent functions, Horadam polynomial, Fekete-Szegö inequality.

AMS Subject Classification
11B39, 30C45, 33C45, 30C50, 33C05.

[1] *Department of Mathematics, Govt Arts and Science College, Hosur-635109, Tamil Nadu, India.*
[2] *Post-Graduate and Research Department of Mathematics, Government Arts College for Men, Krishnagiri-635001, Tamil Nadu, India.*
[3] *Post-Graduate and Research Department of Mathematics, Government Arts College for Men, Krishnagiri-635001, Tamil Nadu, India.*
***Corresponding author**: [1] jsskavya2007@gmail.com;[2] nmagi_2000@yahoo.co.in; [3] smurthy07@yahoo.co.in

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Contents

1. Introduction

Let $\mathscr{A}$ denote the class of functions of the form

$$f(z) = z + a_2 z^2 + \cdots \tag{1.1}$$

which are analytic in

$$\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}.$$

We denote $\mathscr{S}$ by the class of univalent functions in $\mathbb{U}$. Further, we know that every univalent function has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U})$$

and

$$f(f^{-1}(w)) = w \qquad (|w| < r_0(f);\ r_0(f) \geqq \frac{1}{4}),$$

where

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \ldots.$$

A function $f \in \mathscr{A}$ is said to be bi-univalent in $\mathbb{U}$ if both a function f and it's inverse f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1.1) (see [17]). For more details of bi-univalent functions one may refer [1–3, 5, 6, 8, 12, 14] For analytic functions f and g in $\mathbb{U}$, f is said to be subordinate to g if there exists an analytic function w such that

$$w(0) = 0, \qquad |w(z)| < 1 \quad \text{and} \quad f(z) = g(w(z)) \qquad (z \in \mathbb{U}).$$

It is denoted by

$$f \prec g \qquad (z \in \mathbb{U}) \qquad \text{that is} \qquad f(z) \prec g(z) \qquad (z \in \mathbb{U}).$$

In particular, when g is univalent in $\mathbb{U}$,

$$f \prec g \qquad (z \in \mathbb{U}) \Leftrightarrow f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

For a, b, p, $q\mathbb{R}$, the Horadam polynomials $\hbar_n(x, a, b; p, q) := \hbar_n(x)$ are given by the recurrence relation (see [10, 11])):

$$\hbar_n(x) = px\hbar_{n-1}(x) + q\hbar_{n-2}(x) \qquad (n \in \mathbb{N}) \tag{1.2}$$

here

$$\hbar_1(x) = a; \qquad \hbar_2(x) = bx. \tag{1.3}$$

The generating function of the Horadam polynomials $\hbar_n(x)$ (see [11]) is given by

$$\Pi(x,\, z) := \sum_{n=1}^{\infty} \hbar_n(x) z^{n-1} = \frac{a + (b - ap)xz}{1 - pxz - qz^2}\,. \tag{1.4}$$

Here, and in what follows, the argument $x \in \mathbb{R}$ is independent of the argument $z \in \mathbb{C}$; that is, $x \neq \Re(z)$.

It is observed that for special values of the parameters involved in this polynomial leads to different some other polynomials like the Lucas polynomials, the Fibonacci polynomials, the Pell polynomials, the Pell-Lucas polynomials and the Chebyshev polynomials for more details (see, [1, 2, 11, 16]).

A function $f \in \mathscr{S}$ is said to be starlike with respect to symmetric points ($\mathscr{S}^*_{sc}$) if

$$\Re\left(\frac{zf'(z)}{f(z) - \overline{f(-\bar{z})}}\right) > 0, \qquad z \in \mathbb{U}.$$

A function $f \in \mathscr{S}$ is convex with respect to symmetric conjugate points ($\mathscr{C}_{sc}$) if

$$\Re\left(\frac{(zf'(z))'}{\left(f(z) - \overline{f(-\bar{z})}\right)'}\right) > 0, \qquad z \in \mathbb{U}.$$

The classes $\mathscr{S}^*_{sc}$ and $\mathscr{C}_{sc}$ were studied by El-Ashwah and Thomas [9]. For more details of the above said classes and its subclasses one could refer [13, 15, 18**?** , 19].

A function $f \in \sigma$ is said to be in the class $\mathscr{P}_\Sigma(\alpha,\, x)$, if

$$\frac{2zf'(z)}{f(z) - \overline{f(-\bar{z})}} + \frac{2\,(zf'(z))'}{\left(f(z) - \overline{f(-\bar{z})}\right)'} - \frac{2\alpha z^2 f''(z) + 2zf'(z)}{\alpha z\left(f(z) - \overline{f(-\bar{z})}\right)' + (1-\alpha)\left(f(z) - \overline{f(-\bar{z})}\right)} \prec \Pi(x,\, z) + 1 - a,\ z \in \mathbb{U}$$

and

$$\frac{2wg'(w)}{g(w) - \overline{g(-\bar{w})}} + \frac{2\,(wg'(w))'}{\left(g(w) - \overline{g(-\bar{w})}\right)'} - \frac{2\alpha w^2 g''(w) + 2wg'(w)}{\alpha w\left(g(w) - \overline{g(-\bar{w})}\right)' + (1-\alpha)\left(g(w) - \overline{g(-\bar{w})}\right)} \prec \Pi(x,\, w) + 1 - a, \quad w \in \mathbb{U}$$

hold.

Various, results for the special values of the parameters involved given as follws:

1. In particular, when $\alpha = 1$, we have $\mathscr{P}_\Sigma(1,\, x) := \mathscr{S}_{sc,\,\Sigma}(x)$, if

$$\frac{2zf'(z)}{f(z) - \overline{f(-\bar{z})}} \prec \Pi(x,\, z) + 1 - a, \qquad z \in \mathbb{U}$$

and

$$\frac{2wg'(w)}{g(w) - \overline{g(-\bar{w})}} \prec \Pi(x,\, w) + 1 - a, \qquad w \in \mathbb{U}$$

hold.

2. When $\alpha = 0$, $\mathscr{P}_\Sigma(0,\, x) := \mathscr{C}_{sc,\,\Sigma}(x)$ if

$$\frac{2\,(zf'(z))'}{\left(f(z) - \overline{f(-\bar{z})}\right)'} \prec \Pi(x,\, z) + 1 - a, \qquad z \in \mathbb{U}$$

and

$$\frac{2\,(wg'(w))'}{\left(g(w) - \overline{g(-\bar{w})}\right)'} \prec \Pi(x,\, w) + 1 - a, \qquad w \in \mathbb{U}$$

hold.

3. If $a = p = x = 1$, $b = 2$ and $q = 0$, then we have

$$\frac{2zf'(z)}{f(z) - \overline{f(-\bar{z})}} \prec \frac{1+z}{1-z} \qquad z \in \mathbb{U}.$$

In the current paper, we estimate the coefficients $|a_2|$ and $|a_3|$ of functions in $\mathscr{P}_\Sigma(\alpha,\, x)$. Further, we investigate Fekete-Szegö inequality for this defined class.

2. Main Results

The initial estimates and Fekete-Szegö inequality for $f \in \mathscr{P}_\Sigma(\alpha,\, x)$ are discussed in the following theorem.

Theorem 2.1. *Let $f(z) = z + a_2 z^2 + \cdots$ be in $\mathscr{P}_\Sigma(\alpha,\, x)$. Then*

$$|a_2| \le \min\left\{\frac{b^2x^2}{4(2-\alpha)^2},\ \frac{|bx|^{3/2}}{\sqrt{|2b^2x^2(3-2\alpha) - 4(pbx^2+qa)(2-\alpha)^2|}}\right\},$$

and

$$|a_3| \le \min\left\{\frac{|bx|}{6-4\alpha} + \frac{b^2x^2}{4(2-\alpha)^2},\ \frac{|bx|}{6-4\alpha} + \frac{|bx|^3}{|2b^2x^2(3-2\alpha) - 4(pbx^2+qa)(2-\alpha)^2|}\right\}$$

and for $\mu \in \mathbb{R}$

$$|a_3 - \mu a_2^2| \le \begin{cases} \dfrac{|bx|}{2(3-2\alpha)}; & |\mu - 1| \le \dfrac{|b^2x^2(3-2\alpha) - 2(pbx^2+qa)(2-\alpha)^2|}{(3-2\alpha)\,|bx|^2} \\[2ex] \dfrac{|bx|^3\,|\mu-1|}{|2b^2x^2(3-2\alpha) - 4(pbx^2+qa)(2-\alpha)^2|}; & |\mu - 1| \ge \dfrac{|b^2x^2(3-2\alpha) - 2(pbx^2+qa)(2-\alpha)^2|}{(3-2\alpha)\,|bx|^2}. \end{cases}$$

Proof. Let $f \in \mathscr{P}_\Sigma(\alpha,\, x)$ be given by (1.1). Then, for some analytic functions φ and ψ such that

$$\varphi(0)=0;\ \psi(0)=0,\ |\varphi(z)|<1 \text{ and } |\psi(z)|<1 \quad (\forall\, z,\, w \in \mathbb{U}),$$

we can write

$$\frac{2zf'(z)}{f(z)-\overline{f(-\bar{z})}}+\frac{2\left(zf'(z)\right)'}{\left(f(z)-\overline{f(-\bar{z})}\right)'} - \frac{2\alpha z^2 f''(z)+2zf'(z)}{\alpha z\left(f(z)-\overline{f(-\bar{z})}\right)'+(1-\alpha)\left(f(z)-\overline{f(-\bar{z})}\right)} = \Pi(x,\ \varphi(z))+1-a \tag{2.1}$$

and

$$\frac{2wg'(w)}{g(w)-\overline{g(-\bar{w})}}+\frac{2\left(wg'(w)\right)'}{\left(g(w)-\overline{g(-\bar{w})}\right)'} - \frac{2\alpha w^2 g''(w)+2wg'(w)}{\alpha w\left(g(w)-\overline{g(-\bar{w})}\right)'+(1-\alpha)\left(g(w)-\overline{g(-\bar{w})}\right)} = \Pi(x,\ \psi(w))+1-a. \tag{2.2}$$

Or, equivalently,

$$\frac{2zf'(z)}{f(z)-\overline{f(-\bar{z})}}+\frac{2\left(zf'(z)\right)'}{\left(f(z)-\overline{f(-\bar{z})}\right)'} - \frac{2\alpha z^2 f''(z)+2zf'(z)}{\alpha z\left(f(z)-\overline{f(-\bar{z})}\right)'+(1-\alpha)\left(f(z)-\overline{f(-\bar{z})}\right)} = 1+\hbar_1(x)-a+\hbar_2(x)\varphi(z)+\hbar_3(x)[\varphi(z)]^2+\ldots \tag{2.3}$$

and

$$\frac{2wg'(w)}{g(w)-\overline{g(-\bar{w})}}+\frac{2\left(wg'(w)\right)'}{\left(g(w)-\overline{g(-\bar{w})}\right)'} - \frac{2\alpha w^2 g''(w)+2wg'(w)}{\alpha w\left(g(w)-\overline{g(-\bar{w})}\right)'+(1-\alpha)\left(g(w)-\overline{g(-\bar{w})}\right)} = 1+\hbar_1(x)-a+\hbar_2(x)\psi(w)+\hbar_3(x)[\psi(w)]^2+\ldots \tag{2.4}$$

From (2.3) and (2.4), we obtain

$$\frac{2zf'(z)}{f(z)-\overline{f(-\bar{z})}}+\frac{2\left(zf'(z)\right)'}{\left(f(z)-\overline{f(-\bar{z})}\right)'} - \frac{2\alpha z^2 f''(z)+2zf'(z)}{\alpha z\left(f(z)-\overline{f(-\bar{z})}\right)'+(1-\alpha)\left(f(z)-\overline{f(-\bar{z})}\right)} = 1+\hbar_2(x)\xi_1 z+[\hbar_2(x)\xi_2+\hbar_3(x)\xi_1^2]z^2+\ldots \tag{2.5}$$

and

$$\frac{2wg'(w)}{g(w)-\overline{g(-\bar{w})}}+\frac{2\left(wg'(w)\right)'}{\left(g(w)-\overline{g(-\bar{w})}\right)'} - \frac{2\alpha w^2 g''(w)+2wg'(w)}{\alpha w\left(g(w)-\overline{g(-\bar{w})}\right)'+(1-\alpha)\left(g(w)-\overline{g(-\bar{w})}\right)} = 1+\hbar_2(x)\tau_1 w+[\hbar_2(x)\tau_2+\hbar_3(x)\tau_1^2]w^2+\ldots \tag{2.6}$$

It is known that $|\varphi(z)| = \left|\xi_1 z+\xi_2 z^2+\ldots\right| < 1$ and $|\psi(z)| = \left|\tau_1 w+\tau_2 w^2+\ldots\right| < 1$, then

$$|\xi_k| \le 1 \qquad \text{and} \qquad |\tau_k| \le 1 \qquad (k\in\mathbb{N}).$$

Thus from (2.5) and (2.6), we get

$$2(2-\alpha)a_2 = \hbar_2(x)\xi_1 \tag{2.7}$$

$$2(3-2\alpha)a_3 = \hbar_2(x)\xi_2+\hbar_3(x)\xi_1^2 \tag{2.8}$$

$$-2(2-\alpha)a_2 = \hbar_2(x)\tau_1 \tag{2.9}$$

and

$$-2(3-2\alpha)\left(a_3-2a_2^2\right) = \hbar_2(x)\tau_2+\hbar_3(x)\tau_1^2. \tag{2.10}$$

From (2.7) and (2.9), we have

$$\xi_1 = -\tau_1, \tag{2.11}$$

and

$$\begin{aligned} 8(2-\alpha)^2 a_2^2 &= [\hbar_2(x)]^2(\xi_1^2+\tau_1^2) \\ a_2^2 &= \frac{[\hbar_2(x)]^2(\xi_1^2+\tau_1^2)}{8(2-\alpha)^2} \end{aligned} \tag{2.12}$$

this gives

$$|a_2^2| = \frac{b^2x^2}{4(2-\alpha)^2}\ .$$

By adding (2.8) to (2.10), we have

$$4\,(3-2\alpha)\,a_2^2 = \hbar_2(x)(\xi_2+\tau_2)+\hbar_3(x)(\xi_1^2+\tau_1^2). \tag{2.13}$$

From (2.12), (2.13), we get

$$a_2^2 = \frac{[\hbar_2(x)]^3\,(\xi_2+\tau_2)}{4\,(3-2\alpha)\,[\hbar_2(x)]^2-8\hbar_3(x)(2-\alpha)^2} \tag{2.14}$$

this gives

$$|a_2| \le \frac{|bx|\,\sqrt{|bx|}}{\sqrt{\left|2b^2x^2(3-2\alpha)-4(pbx^2+qa)(2-\alpha)^2\right|}} \tag{2.15}$$

By subtracting (2.10) from (2.8) and from (2.11), we get

$$\begin{aligned} 4(3-2\alpha)a_3-4(3-2\alpha)a_2^2 &= \hbar_2(x)\,(\xi_2-\tau_2)+\hbar_3(x)\left(\xi_1^2-\tau_1^2\right) \\ a_3 &= \frac{\hbar_2(x)\,(\xi_2-\tau_2)}{4(3-2\alpha)}+a_2^2. \end{aligned} \tag{2.16}$$

In view of (2.12), (2.16) reduces

$$a_3 = \frac{\hbar_2(x)(\xi_2-\tau_2)}{4(3-2\alpha)} + \frac{[\hbar_2(x)]^2(\xi_1^2+\tau_1^2)}{8(2-\alpha)^2}.$$

Using (1.3), we have

$$|a_3| \leq \frac{|bx|}{6-4\alpha} + \frac{b^2x^2}{4(2-\alpha)^2}.$$

Further, by using (2.14) in (2.16), we get

$$|a_3| \leq \frac{|bx|}{6-4\alpha} + \frac{|bx|^3}{|2b^2x^2(3-2\alpha)-4(pbx^2+qa)(2-\alpha)^2|}.$$

From (2.16), for $\mu \in \mathbb{R}$, we have

$$a_3 - \mu a_2^2 = \frac{\hbar_2(x)(\xi_2-\tau_2)}{4(3-2\alpha)} + (1-\mu)a_2^2. \tag{2.17}$$

By using (2.14) in (2.17), we get

$$\begin{aligned} a_3 - \mu a_2^2 &= \frac{\hbar_2(x)(\xi_2-\tau_2)}{4(3-2\alpha)} \\ &+ (1-\mu)\left(\frac{[\hbar_2(x)]^3(\xi_2+\tau_2)}{4(3-2\alpha)[\hbar_2(x)]^2 - 8\hbar_3(x)(2-\alpha)^2}\right) \\ &= \hbar_2(x)\left\{\left(\Lambda(\mu,x) + \frac{1}{4(3-2\alpha)}\right)\xi_2\right. \\ &+ \left.\left(\Lambda(\mu,x) - \frac{1}{4(3-2\alpha)}\right)\tau_2\right\}, \end{aligned} \tag{2.18}$$

where

$$\Lambda(\mu,x) = \frac{(1-\mu)[\hbar_2(x)]^2}{4(3-2\alpha)[\hbar_2(x)]^2 - 8\hbar_3(x)(2-\alpha)^2}.$$

Hence, in view of (1.3), we conclude that

$$|a_3-\mu a_2^2| \leq \begin{cases} \dfrac{|\hbar_2(x)|}{2(3-2\alpha)} & ; 0 \leq |\Lambda(\mu,x)| \leq \dfrac{1}{4(3-2\alpha)} \\ 2|\hbar_2(x)||\Lambda(\mu,x)| & ; |\Lambda(\mu,x)| \geq \dfrac{1}{4(3-2\alpha)}. \end{cases}$$

Thus this completes the proof. □

Corollary 2.2. *Let $f(z) = z + a_2z^2 + \cdots$ be in $\mathscr{S}_{sc}^*$. Then*

$$|a_2| \leq \min\left\{\frac{b^2x^2}{4}, \frac{|bx|^{3/2}}{\sqrt{|2b^2x^2-4(pbx^2+qa)|}}\right\},$$

and

$$|a_3| \leq \min\left\{\frac{|bx|}{2} + \frac{b^2x^2}{4}, \frac{|bx|}{2} + \frac{|bx|^3}{|2b^2x^2-4(pbx^2+qa)|}\right\}$$

and for $\mu \in \mathbb{R}$

$$|a_3-\mu a_2^2| \leq \begin{cases} \dfrac{|bx|}{2} & ; |\mu-1| \leq \dfrac{|b^2x^2-2(pbx^2+qa)|}{|bx|^2} \\ \dfrac{|bx|^3|\mu-1|}{|2b^2x^2-4(pbx^2+qa)|} & ; |\mu-1| \geq \dfrac{|b^2x^2-2(pbx^2+qa)|}{|bx|^2} \end{cases}.$$

Corollary 2.3. *Let $f(z) = z + a_2z^2 + \cdots$ be in $\mathscr{C}_{sc}^*$. Then*

$$|a_2| \leq \min\left\{\frac{b^2x^2}{16}, \frac{|bx|^{3/2}}{\sqrt{|6b^2x^2-16(pbx^2+qa)|}}\right\},$$

and

$$|a_3| \leq \min\left\{\frac{|bx|}{6} + \frac{b^2x^2}{16}, \frac{|bx|}{6} + \frac{|bx|^3}{|6b^2x^2-16(pbx^2+qa)|}\right\}$$

and for $\mu \in \mathbb{R}$

$$|a_3-\mu a_2^2| \leq \begin{cases} \dfrac{|bx|}{6} & ; |\mu-1| \leq \dfrac{|3b^2x^2-8(pbx^2+qa)|}{3|bx|^2} \\ \dfrac{|bx|^3|\mu-1|}{|6b^2x^2-16(pbx^2+qa)|} & ; |\mu-1| \geq \dfrac{|3b^2x^2-8(pbx^2+qa)|}{3|bx|^2} \end{cases}.$$

Remark 2.4. *Taking $a = 1$, $q = -1$, $b = 2$ and $p = 2$ and in Theorem 2.1 and Corollary 2.2 and Corollary 2.3, the results improve the estimates discussed in [20] .*

References

[1] C. Abirami, N. Magesh, J. Yamini and N. B. Gatti, Horadam Polynomial coefficient estimates for the classes of $\lambda-$bi-pseudo-starlike and Bi-Bazilevič Functions, *J. Anal.*, (2020), 1–10(https://doi.org/10.1007/s41478-020-00224-2).

[2] C. Abirami, N. Magesh and J. Yamini, Initial bounds for certain classes of bi-univalent functions defined by Horadam polynomials, *Abstract and Applied Analysis*, Art. ID 7391058 (2020), 1–10.

[3] R. M. Ali, S. K. Lee, V. Ravichandran, S. Supramanian, Coefficient estimates for bi-univalent Ma-Minda starlike and convex functions, *Appl. Math. Lett.,* 25(3)(2012), 344–351.

[4] F. M. Al-Oboudi, On Classes of Functions Related to Starlike Functions with Respect to Symmetric Conjugate Points Defined by a Fractional Differential Operator, *Complex Anal. Oper. Theory,* 5(2011), 647–658.

[5] Ş. Altınkaya and S. Yalçin, Fekete-Szegö Inequalities for Certain Classes of Biunivalent Functions, *International Scholarly Research Notices*, Volume 2014, Article ID 327962, 6 pages

[6] Ş. Altınkaya and S. Yalçin, Coefficient estimates for two new subclasses of bi-univalent functions with respect to symmetric points, *J. Funct. Spaces*, 2015 Art. ID 145242, (2015), 1–5.

[7] Ming-Po Chen, Zhwo-Ren Wu and Zhong-Zhu Zou, On functions starlike with respect to symmetric conjugate points, *J. Math. Anal. Appl.,* 201(1996), 25-34.

[8] O. Crisan, Coefficient Estimates for Certain Subclasses of Bi-Univalent Functions, *Gen. Math. Notes*, 16(2)(2013), 93-102. ISSN 2219-7184.

[9] R. M. El-Ashwah and D. K. Thomas, Some subclasses of close-to-convex functions, *J. Ramanujan Math. Soc.,* 2(1987), 86-100.

[10] A. F. Horadam, J. M. Mahon, Pell and Pell–Lucas polynomials, *Fibonacci Quart.*, 23(1985), 7—20.

[11] T. Hörçum, E. Gökçen Koçer, On some properties of Horadam polynomials, *Int Math Forum,* 4(2009), 1243–1252.

[12] A.Y. Lashin, On certain subclasses of analytic and bi-univalent functions, *Journal of the Egyptian Mathematical Society*, 24(2016), 220–225.

[13] V. Ravichandran, Starlike and convex functions with respect to conjugate points, *Acta Mathematica Academiae Paedagogicae Nyregyhaziensis*, 20(2004), 31-37.

[14] G. Singh, Coefficient estimates for bi-univalent functions with respect to symmetric points, *J. Nonlinear Funct. Anal.*, (2013), 1–9.

[15] J. Sokol, Function starlike with respect to conjugate point, *Zeszyty Nauk. Politech. Rzeszowskiej Mat. Fiz*, 12(1991), 53-64.

[16] H. M. Srivastava, Ş. Altınkaya and S. Yalçin, Certain subclasses of bi-univalent functions associated with the Horadam polynomials, *Iran J Sci Technol Trans Sci.*, (2018), 1–7.

[17] H. M. Srivastava, A. K. Mishra and P. Gochhayat, Certain subclasses of analytic and bi-univalent functions, *Appl. Math. Lett.,* 23(10)(2010), 1188–1192.

[18] H. Tang and G. Deng, Coefficient estimates for new subclasses of analytic functions with respect to other points, *Tamkang J. Math.,* 44(2)(2013), 141–148.

[19] H. Tang and G.-T. Deng, New subclasses of analytic functions with respect to symmetric and conjugate points, *J. Complex Anal.,* 2013, Art. ID 578036, 9 pp.

[20] A. K. Wanas, A. H. Majeed, Chebyshev polynomial bounded for analytic and bi-univalent functions with respect to symmetric conjugate points, *Applied Mathematics E-Notes*, 19(2019), 14-21.

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