



H_∞ feedback sampled-data control of T-S fuzzy systems via a novel integral inequality approach

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Abstract

This work addresses the result of sampled-data-based H_∞ control of Takagi-Sugeno (T-S) fuzzy systems. The sampling period is assumed to be varying within an interval. In order to construct a less delay dependent stability condition, a Lyapunov-Krasovskii functional (LKF) containing new integral terms is imported. Approach of convex is applied to determine a less stability conditions in the form of linear matrix inequalities (LMIs) without any free-weighting matrices approach which increase badly the computational anxiety of the stability analysis. Through the use of the derived inequality and by constructing a suitable LKF, improved stability criteria are shown in the form of LMIs. Two simulation examples are carried out to demonstrate that the results out perform the state of the art in the literature.

Keywords

T-S fuzzy system; Feedback control, Sampled-data scheme, H_∞ performance, Time-delay.

AMS Subject Classification

54C05, 54C08, 54C10.

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Article History: Received 24 January 2020; Accepted 19 April 2020

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1. Introduction

In real-world, nonlinearities exist in most of the physical systems which make it complex. To represent these complex nonlinear systems, Takagi-Sugeno fuzzy systems have emerged to be an effective and conceptually simple tool. This class of systems is described by a set of if-then rules which gives linear representation of the considered system which is then easy to analyze [1, 13]. Hence the fuzzy representation has turned out to be a significant way to approach the complex nonlinear systems. The study of T-S fuzzy systems has received considerable attention and many pronounced advances have been achieved ([4, 7–9]). Besides, the rate

of change of physical models depends not only on current time but also on the previous time instants. Hence the system stability explicitly depends on the time delay also. At the same time, the existence of time delay may result in instability and undesirable performance of the system. Hence the time delay in dynamical systems has received substantial attention and popularity. In addition to this, external disturbances and nonlinearities make the modelling and formulation of dynamical systems sophisticated. As a consequence, the study on the stability and stabilization of dynamical systems subject to time delay, uncertainties, disturbances and nonlinearities is exceptionally demanding and worth of further research ([9, 14]).

The concept of sampled-data feedback control is a practical and useful tool to implement some complicated control schemes and it has been applied in many areas of science and engineering. A sampled-data scheme is driven by a periodic clock and on each clock edge, it samples its inputs, changes state and updates its outputs. Moreover, its technologies has been constructed relatively well in control theory, especially sampled-data scheme result for complex dynamical control systems has paid small attention according to the mathematical complexity. Hence, under the rapid con-

struction of computer hardware, the sampled-data scheme has explicit superiority over another feedback control techniques ([6, 10, 12]). Uncertain disturbance design in H_∞ ambience have good gain and easily represented that the H_∞ control is associated to the competence of perturbation elimination in complex dynamical models [2]. Then, the H_∞ performances guarantee the covet achievement by rejecting the effect of alikeness errors, parametric perturbations and sound perturbations in the dynamical models ([3, 5]). To the best of our knowledge, there are no works on H_∞ feedback sampled-data control of T-S fuzzy systems via a novel integral inequality approach. Motivated by the above discussions, in this work, we consider the result of feedback based H_∞ control for T-S fuzzy systems via a novel double integral inequality approach.

2. Preliminaries

Consider a nonlinear time-delayed system which can be expressed by T-S fuzzy model with r plant rules:

Fuzzy rule i : **IF** $x_1(t)$ is F_{i1} and $x_2(t)$ is F_{i2} and ... and $x_g(t)$ is F_{ig} , **THEN**

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{hi} x(t-h(t)) + B_i u(t) + B_{wi} w(t), \\ z(t) = C_i x(t) + D_i w(t), \\ x(t) = \phi(t), t \in [h, 0] \end{cases} \quad (2.1)$$

where $i = 1, 2, \dots, r$, r denotes the number of **IF-THEN** rules; $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input signal; $z(t) \in \mathbb{R}^l$ is the output; $w(t) \in \mathbb{R}^v$ is the external disturbance which belongs to $L_2[0, \infty)$; $x_a(t)$ ($a = 1, 2, \dots, g$) are the premise variables; F_{ia} ($i = 1, 2, \dots, r$, $a = 1, 2, \dots, g$) are the fuzzy sets; $A_i, A_{hi}, B_i, C_i, B_{wi}$ and D_i are known constant matrices. Further, $h(t)$ is time-varying delay that obeys the condition $0 < h(t) \leq h, \dot{h}(t) \leq \mu < 1$ in which h is the upper bound of time-varying delay.

Now, based on fuzzy blending, the above formulated T-S fuzzy model (2.1) can be expressed as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r v_i(x(t)) [A_i x(t) + A_{hi} x(t-h(t)) + B_i u(t) + B_{wi} w(t)], \\ z(t) = \sum_{i=1}^r v_i(x(t)) [C_i x(t) + D_i w(t)], \end{cases} \quad (2.2)$$

where $v_i(x(t))$ is the normalized membership function defined by $v_i(x(t)) = v_i(x(t)) / \sum_{i=1}^r v_i(x(t))$ with $v_i(x(t)) =$

$\prod_{a=1}^g F_{ia}(x_a(t))$ and $F_{ia}(x_a(t))$ is the degree of membership of $x_a(t)$ in F_{ia} . Then, it is considered that $v_i(x(t)) \geq 0$ and $\sum_{i=1}^r v_i(x(t)) = 1$ for all $t > 0$. Moreover, it is seen that $v_i(x(t)) \geq 0$ and $\sum_{i=1}^r v_i(x(t)) = 1$ for all $t > 0$.

Control rule i : **IF** $x_1(t)$ is F_{i1} and $x_2(t)$ is F_{i2} and ... and $x_g(t)$ is F_{ig} , **THEN**

$$u(t) = K_i x(i_k h), \quad t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}], \quad (2.3)$$

where $u(t) = u(i_k h) = K_i x(i_k h)$, K_i ($i = 1, 2, \dots, r$) are the controller gains, i_k ($k = 1, 2, \dots$) represent some non-negative values and $i_k h$ denotes the sensor sampling instant. Then, it is seen that $\cup_{k=1}^\infty [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}) = [0, \infty)$. Further, for some interval $[i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$, define $\gamma(t) = (t - i_k h)$ which implies that $i_k h = t - (t - i_k h) = t - \gamma(t)$. Based on the analysis of $\gamma(t)$, we obtain that $\dot{\eta}(t) = 1$ and $0 \leq \tau_{i_k} \leq \eta(t) \leq (i_{k+1} - i_k)h + \tau_{i_{k+1}} \leq \bar{\eta}$.

Then, the fuzzy controller (2.3) is expressed in the following form

$$u(t) = \sum_{j=1}^r v_j(x(t)) K_j x(t - \gamma(t)), \quad (2.4)$$

where $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}]$.

Combining (2.4) with (2.2), we get the upcoming closed-loop fuzzy model:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r v_i(x(t)) v_j(x(t)) [A_i x(t) + A_{hi} x(t-h(t)) + B_i K_j x(t - \gamma(t)) + B_{wi} w(t)], \\ z(t) = \sum_{i=1}^r \sum_{j=1}^r v_i(x(t)) v_j(x(t)) [C_i x(t) + D_i w(t)] \end{cases} \quad (2.5)$$

Lemma 2.1. (Schur Complement Lemma)

Given appropriate dimensioned matrices S_{11}, S_{12} and S_{22} with $S_{11}^T = S_{11}, S_{22}^T = S_{22}$, then the inequality $S_{11} + S_{12}^T S_{22}^{-1} S_{12} < 0$ holds if and only if $\begin{bmatrix} S_{11} & S_{12}^T \\ * & -S_{22} \end{bmatrix} < 0$ or $\begin{bmatrix} -S_{22} & S_{12} \\ * & S_{11} \end{bmatrix} < 0$.

Lemma 2.2. [11] For any matrix $S > 0$, the following inequality satisfies for all continuously differential function x in $[a, b] \rightarrow \mathbb{R}^n$:

$$\int_a^b \dot{x}^T(s) S \dot{x}(s) ds \geq \frac{1}{b-a} \omega_1^T S \omega_1 + \frac{3}{b-a} \omega_2^T S \omega_2 + \frac{5}{b-a} \omega_3^T S \omega_3,$$

where

$$\begin{aligned} \omega_1 &= x(b) - x(a), \\ \omega_2 &= x(b) + x(a) - \frac{2}{b-a} \int_a^b x(s) ds, \\ \omega_3 &= x(b) - x(a) + \frac{6}{b-a} \int_a^b x(s) ds \\ &\quad - \frac{12}{(b-a)^2} \int_a^b \int_u^b x(s) ds du. \end{aligned}$$

Lemma 2.3. [11] For any matrix $S > 0$, the following inequality satisfies for all continuously differential function x in $[a, b] \rightarrow \mathbb{R}^n$:

$$\int_a^b \int_u^b \dot{x}^T(s) S \dot{x}(s) ds du \geq 2\omega_4^T S \omega_4 + 4\omega_5^T S \omega_5 + 6\omega_6^T S \omega_6,$$



where

$$\begin{aligned} \omega_4 &= x(b) - \frac{1}{b-a} \int_a^b x(s)ds, \\ \omega_5 &= x(b) + \frac{2}{b-a} \int_a^b x(s)ds \\ &\quad - \frac{6}{(b-a)^2} \int_a^b \int_u^b x(s)dsdu, \\ \omega_6 &= x(b) - \frac{3}{b-a} \int_a^b x(s)ds \\ &\quad + \frac{24}{(b-a)^2} \int_a^b \int_u^b x(s)dsdu \\ &\quad - \frac{60}{(b-a)^3} \int_a^b \int_u^b \int_s^b x(r)drdsdu. \end{aligned}$$

Definition 2.4. [3] T-S fuzzy system (2.5) is said to be asymptotically stable with a prescribed H_∞ performance $\eta > 0$ if $w(t) \in L_2[0, \infty)$, output $z(t)$ satisfies under the zero initial condition

$$\int_0^\infty z^T(t)z(t)dt \leq \eta^2 \int_0^\infty w^T(t)w(t)dt. \quad (2.6)$$

3. Main results

Theorem 3.1. Given positive scalars μ, h, γ , symmetric matrix \mathcal{N} and the gain matrix K_j , the considered system (2.5) is asymptotically stable if there exist positive definite matrices $P_1, Q_1, Q_2, Q_3, Q_4, R_1, R_2, S_1, S_2$, such that the upcoming matrix inequalities satisfied for $1 \leq i \leq j \leq r$:

$$\begin{aligned} [\Phi_{ii}] &< 0, \quad i = j, \\ [\Phi_{ij}] + [\Phi_{ji}] &< 0, \quad i < j, \end{aligned} \quad (3.1)$$

where,

$$\begin{aligned} \Phi_{1,1} &= Q_1 + Q_2 + Q_3 + Q_4 - 9R_1 - 9R_2 - 12S_1 - 12S_2 \\ &\quad + \text{sym}(\mathcal{N}A_i), \\ \Phi_{1,2} &= \mathcal{N}Ah_i, \\ \Phi_{1,3} &= \mathcal{N}B_iK_j, \\ \Phi_{1,4} &= R_1 + 3R_1, \\ \Phi_{1,5} &= 3R_2, \\ \Phi_{1,6} &= -\frac{24R_1}{h} + \frac{12S_1}{h} - \frac{54S_1}{h^2}, \\ \Phi_{1,7} &= -\frac{24R_2}{\gamma} + \frac{12S_2}{\gamma} - \frac{54S_2}{\gamma^2}, \\ \Phi_{1,8} &= \frac{60R_1}{h^2} + \frac{24S_1}{h^2} + \frac{432S_1}{h^3} - \frac{144S_1}{h^2}, \\ \Phi_{1,9} &= \frac{60R_2}{h^2} + \frac{24S_2}{\gamma^2} - \frac{144S_2}{\gamma^2}, \\ \Phi_{1,10} &= \frac{360S_1}{h^3} - \frac{1080S_1}{h^4}, \\ \Phi_{1,11} &= \frac{360S_2}{\gamma^3}, \\ \Phi_{1,12} &= \mathcal{N}B_{wi}, \\ \Phi_{1,13} &= 2P - \mathcal{N} + A_i^T \mathcal{N}^T, \\ \Phi_{2,2} &= -(1 - \mu)Q_1, \\ \Phi_{2,14} &= A_{hi}^T \mathcal{N}, \\ \Phi_{3,3} &= -(1 - \gamma(t))Q_2, \\ \Phi_{3,13} &= K_j^T B_i^T \mathcal{N}, \\ \Phi_{4,4} &= -5R_1 + Q_3, \end{aligned}$$

$$\begin{aligned} \Phi_{4,5} &= 32 \frac{R_1}{h}, \\ \Phi_{4,6} &= \frac{60R_1}{h^2}, \\ \Phi_{5,5} &= -9R_2 + Q_4, \\ \Phi_{5,7} &= \frac{2R_2}{\gamma} + \frac{30R_2}{\gamma^2}, \\ \Phi_{5,9} &= \frac{60R_2}{\gamma}, \\ \Phi_{6,6} &= -\frac{192R_1}{h^2} - \frac{18S_1}{h^2}, \\ \Phi_{6,8} &= \frac{360R_1}{h^3} + \frac{48S_1}{h^3}, \\ \Phi_{7,7} &= -\frac{192R_2}{\gamma^2} - \frac{70S_2}{\gamma^2}, \\ \Phi_{7,9} &= \frac{360R_2}{\gamma^2} + \frac{480S_2}{\gamma^3}, \\ \Phi_{7,11} &= -\frac{1080S_2}{\gamma^4}, \\ \Phi_{8,8} &= -\frac{720R_1}{h^4} - \frac{144S_1}{h^4} - \frac{3456S_1}{h^4}, \\ \Phi_{8,10} &= \frac{8640S_1}{h^5}, \\ \Phi_{9,9} &= -\frac{720R_2}{\gamma^4} - \frac{144S_2}{\gamma^4} - \frac{3456S_2}{\gamma^4}, \\ \Phi_{9,11} &= \frac{8640S_2}{\gamma^5}, \\ \Phi_{10,10} &= -\frac{21600S_1}{h^6}, \\ \Phi_{11,11} &= -\frac{21600S_2}{\gamma^6}, \\ \Phi_{13,13} &= (h^2R_1 + \gamma^2R_2 + h^2S_1 + \gamma^2S_2 - \mathcal{N}). \end{aligned}$$

Proof. We consider the following L-K functional:

$$V(x(t)) = \sum_{s=1}^4 V_s(x(t)), \quad (3.2)$$

where

$$\begin{aligned} V_1(x(t)) &= x^T(t)P_1x(t), \\ V_2(x(t)) &= \int_{t-h(t)}^t x^T(s)Q_1x(s)ds + \int_{t-\gamma(t)}^t x^T(s)Q_2x(s)ds \\ &\quad + \int_{t-h}^t x^T(s)Q_3x(s)ds + \int_{t-\gamma}^t x^T(s)Q_4x(s)ds, \\ V_3(x(t)) &= h \int_{t-h}^t \int_u^t \dot{x}^T(s)R_1\dot{x}(s)dsdu \\ &\quad + \gamma \int_{t-\gamma}^t \int_u^t \dot{x}^T(v)R_2\dot{x}(v)dsdu, \\ V_4(x(t)) &= \frac{h^2}{2} \int_{t-h}^t \int_u^t \int_s^t \dot{x}^T(r)S_1\dot{x}(r)drdsdu \\ &\quad + \frac{\gamma^2}{2} \int_{t-\gamma}^t \int_u^t \int_s^t \dot{x}^T(r)S_2\dot{x}(r)drdsdu. \end{aligned}$$

Calculating the derivatives of $V(x(t))$ along the trajectories of



the closed-loop system (2.5), we obtain

$$\dot{V}_1(x(t)) = x^T(t)2P_1\dot{x}(t), \tag{3.3}$$

$$\begin{aligned} \dot{V}_2(x(t)) = & x^T(t)(Q_1 + Q_2 + Q_3 + Q_4)x(t) \\ & - (1 - \mu)x^T(t-h(t))Q_1x(t-h(t)) \\ & - x^T(t-h)Q_3x(t-h) \\ & - x^T(t-\bar{\gamma})Q_4x(t-\bar{\gamma}), \end{aligned} \tag{3.4}$$

$$\begin{aligned} \dot{V}_3(x(t)) = & h^2\dot{x}^T(t)R_1\dot{x}(t) + \bar{\gamma}^2\dot{x}^T(t)R_2\dot{x}(t) \\ & - h \int_{t-h}^t \dot{x}^T(s)R_1\dot{x}(s)ds \\ & - \bar{\gamma} \int_{t-\bar{\gamma}}^t \dot{x}^T(s)R_2\dot{x}(s)ds, \end{aligned} \tag{3.5}$$

$$\begin{aligned} \dot{V}_4(x(t)) = & \frac{\bar{\gamma}^4}{4}\dot{x}^T(t)S_1\dot{x}(t) + \frac{h^4}{4}\dot{x}^T(t)S_2\dot{x}(t) \\ & - \frac{h^2}{2} \int_{t-h}^t \int_s^t \dot{x}^T(s)S_1\dot{x}(s)dsdu \\ & - \frac{\bar{\gamma}^2}{2} \int_{t-\bar{\gamma}}^t \int_s^t \dot{x}^T(s)S_2\dot{x}(s)dsdu. \end{aligned}$$

Using Lemma 2.2, the integral terms in (3.5) can be expressed as
 $-\int_{t-h}^t \dot{x}^T(s)R_1\dot{x}(s)ds$

$$\begin{aligned} & \leq \frac{-1}{h}[x(t) - x(t-h)]^T R_1 [x(t) - x(t-h)] \\ & - \frac{3}{h} \left[x(t) + x(t-h) - \frac{2}{h} \int_{t-h}^t x(s)ds \right]^T R_1 \\ & \times \left[x(t) + x(t-h) - \frac{2}{h} \int_{t-h}^t x(s)ds \right] \\ & - \frac{5}{h} \left[x(t) - x(t-h) \right. \\ & \left. + \frac{6}{h} \int_{t-h}^t x(s)ds - \frac{12}{h^2} \int_{t-h}^t \int_u^t x(s)dsdu \right]^T \\ & \times \left[x(t) - x(t-h) \right. \\ & \left. + \frac{6}{h} \int_{t-h}^t x(s)ds - \frac{12}{h^2} \int_{t-h}^t \int_u^t x(s)dsdu \right] \end{aligned} \tag{3.6}$$

Similarly, for the integral term in R_2

$$\begin{aligned} & - \int_{t-\bar{\gamma}}^t \dot{x}^T(s)R_2\dot{x}(s)ds \\ & \leq \frac{-1}{\bar{\gamma}}[x(t) - x(t-\bar{\gamma})]^T R_2 \\ & \times [x(t) - x(t-\bar{\gamma})] - \frac{3}{\bar{\gamma}} \left[x(t) + x(t-\bar{\gamma}) \right. \\ & \left. - \frac{2}{\bar{\gamma}} \int_{t-\bar{\gamma}}^t x(s)ds \right]^T R_2 \left[x(t) + x(t-\bar{\gamma}) - \frac{2}{\bar{\gamma}} \int_{t-\bar{\gamma}}^t x(s)ds \right] \\ & - \frac{5}{\bar{\gamma}} \left[x(t) - x(t-\bar{\gamma}) + \frac{6}{\bar{\gamma}} \int_{t-\bar{\gamma}}^t x(s)ds - \frac{12}{\bar{\gamma}^2} \int_{t-\bar{\gamma}}^t \int_u^t x(s)dsdu \right]^T \\ & \times \left[x(t) - x(t-\bar{\gamma}) + \frac{6}{\bar{\gamma}} \int_{t-\bar{\gamma}}^t x(s)ds \right. \\ & \left. - \frac{12}{\bar{\gamma}^2} \int_{t-\bar{\gamma}}^t \int_u^t x(s)dsdu \right] \end{aligned} \tag{3.7}$$

Using Lemma 2.3, the integral terms in (3.5) can be expressed as

$$\begin{aligned} & - \int_{t-h}^t \int_u^t \dot{x}^T(s)S_1\dot{x}(s)dsdu \\ & \leq -2 \left[x(t) - \frac{1}{h} \int_{t-h}^t x(s)ds \right]^T S_1 \left[x(t) - \frac{1}{h} \int_{t-h}^t x(s)ds \right] \\ & - 4 \left[x(t) + \frac{2}{h} \int_{t-h}^t x(s)ds - \frac{6}{h^2} \int_{t-h}^t \int_u^t x(s)dsdu \right]^T S_1 \\ & \times \left[x(t) + \frac{2}{h} \int_{t-h}^t x(s)ds - \frac{6}{h^2} \int_{t-h}^t \int_u^t x(s)dsdu \right] \\ & - 6 \left[x(t) - \frac{3}{h} \int_{t-h}^t x(s)ds \right. \\ & \left. + \frac{24}{h^2} \int_{t-h}^t \int_u^t x(s)dsdu - \frac{60}{h^3} \int_{t-h}^t \int_u^t \int_s^t x(r)drdsdu \right]^T \\ & S_2 \times \left[x(t) - \frac{3}{h} \int_{t-h}^t x(s)ds \right. \\ & \left. + \frac{24}{h^2} \int_{t-h}^t \int_u^t x(s)dsdu - \frac{60}{h^3} \int_{t-h}^t \int_u^t \int_s^t x(r)drdsdu \right]^T \end{aligned} \tag{3.8}$$

Similarly, we can obtain for the integral term in S_2 as



$$\begin{aligned}
 & - \int_{t-\bar{\gamma}}^t \int_u^t \dot{x}^T(s) S_2 \dot{x}(s) ds du \\
 & \leq -2 \left[x(t) - \frac{1}{\bar{\gamma}} \int_{t-\bar{\gamma}}^t x(s) ds \right]^T S_2 \left[x(t) - \frac{1}{\bar{\gamma}} \int_{t-\bar{\gamma}}^t x(s) ds \right] \\
 & - 4 \left[x(t) + \frac{2}{\bar{\gamma}} \int_{t-\bar{\gamma}}^t x(s) ds - \frac{6}{\bar{\gamma}^2} \int_{t-\bar{\gamma}}^t \int_u^t x(s) ds du \right]^T S_2 \\
 & \times \left[x(t) + \frac{2}{\bar{\gamma}} \int_{t-\bar{\gamma}}^t x(s) ds - \frac{6}{\bar{\gamma}^2} \int_{t-\bar{\gamma}}^t \int_u^t x(s) ds du \right] \\
 & - 6 \left[x(t) - \frac{3}{\bar{\gamma}} \int_{t-\bar{\gamma}}^t x(s) ds + \frac{24}{\bar{\gamma}^2} \int_{t-\bar{\gamma}}^t \int_u^t x(s) ds du \right. \\
 & \left. - \frac{60}{\bar{\gamma}^3} \int_{t-\bar{\gamma}}^t \int_u^t \int_s^t x(r) dr ds du \right]^T S_2 \left[x(t) - \frac{3}{\bar{\gamma}} \int_{t-\bar{\gamma}}^t x(s) ds \right. \\
 & \left. + \frac{24}{\bar{\gamma}^2} \int_{t-\bar{\gamma}}^t \int_u^t x(s) ds du - \frac{60}{\bar{\gamma}^3} \int_{t-\bar{\gamma}}^t \int_u^t \int_s^t x(r) dr ds du \right]^T \tag{3.9}
 \end{aligned}$$

On the other side, for any matrix \mathcal{N} , the upcoming equality holds:

$$\begin{aligned}
 & 2[x^T(t) \dot{x}^T(t)] \mathcal{N} \left[\sum_{i=1}^r \sum_{j=1}^r v_i(x(t)) v_j(x(t-\gamma(t))) \right. \\
 & \times [A_i x(t) + A_{hi} x(t-h(t)) + B_i u(t) \\
 & \left. + B_{wi} w(t) - \dot{x}(t)] \right] = 0 \tag{3.10}
 \end{aligned}$$

Define

$$\begin{aligned}
 \mathcal{X}^T = & \left[x^T(t), x^T(t-h(t)), x^T(t-\gamma(t)), x^T(t-h), \right. \\
 & x^T(t-\bar{\gamma}), \int_{t-h}^t x^T(s) ds, \int_{t-\bar{\gamma}}^t x^T(s) ds, \\
 & \int_{t-h}^t \int_u^t x^T(s) ds du, \int_{t-\bar{\gamma}}^t \int_u^t x^T(s) ds du, \\
 & \int_{t-h}^t \int_u^t \int_s^t x^T(r) dr ds du, \int_{t-\bar{\gamma}}^t \int_u^t \int_s^t x^T(r) dr ds du, \\
 & \left. w^T(t), \dot{x}^T(t) \right].
 \end{aligned}$$

Adding the equations from (3.3)-(3.10), then we have

$$\dot{V}(x(t)) \leq \mathcal{X}^T(t) \Phi_{ij} \mathcal{X}(t), \tag{3.11}$$

where Φ_{ij} are defined in theorem statement. Therefore it observes that $\Phi_{ij} < 0$. Due to the definition of stability, the considered system (2.5) is asymptotically stable. Hence, complete the proof. \square

Theorem 3.2. Given positive scalars μ, h, γ , symmetric matrix X , the considered system (2.5) is asymptotically stable and

satisfies the H_∞ performance condition if there exist positive definite matrices $\bar{P}_1, \bar{Q}_1, \bar{Q}_2, \bar{Q}_3, \bar{Q}_4, \bar{R}_1, \bar{R}_2, \bar{S}_1, \bar{S}_2$, such that the upcoming matrix inequalities satisfied for $1 \leq i \leq j \leq r$:

$$\begin{aligned}
 & [\bar{\Phi}_{ii}] < 0, \quad i = j, \\
 & [\bar{\Phi}_{ij}] + [\bar{\Phi}_{ji}] < 0, \quad i < j, \tag{3.12}
 \end{aligned}$$

where, $\bar{\Phi}_{1,1} = \bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 + \bar{Q}_4 - 9\bar{R}_1 - 9\bar{R}_2 - 12\bar{S}_1 - 12\bar{S}_2 + \text{sym}(A_i X)$,

$$\bar{\Phi}_{1,2} = A h_i X, \bar{\Phi}_{1,3} = B_i Y_j,$$

$$\bar{\Phi}_{1,4} = \bar{R}_1 + 3\bar{R}_1,$$

$$\bar{\Phi}_{1,5} = 3\bar{R}_2,$$

$$\bar{\Phi}_{1,6} = -\frac{24\bar{R}_1}{h} + \frac{12\bar{S}_1}{h} - \frac{54\bar{S}_1}{h^2},$$

$$\bar{\Phi}_{1,7} = -\frac{24\bar{R}_2}{\bar{\gamma}} + \frac{12\bar{S}_2}{\bar{\gamma}} - \frac{54\bar{S}_2}{\bar{\gamma}^2},$$

$$\bar{\Phi}_{1,8} = \frac{60\bar{R}_1}{h^2} + \frac{24\bar{S}_1}{h^2} + \frac{432\bar{S}_1}{h^3} - \frac{144\bar{S}_1}{h^2},$$

$$\bar{\Phi}_{1,9} = \frac{60\bar{R}_2}{h^2} + \frac{24\bar{S}_2}{\bar{\gamma}^2} - \frac{144\bar{S}_2}{\bar{\gamma}^2},$$

$$\bar{\Phi}_{1,10} = \frac{360\bar{S}_1}{h^3} - \frac{1080\bar{S}_1}{h^4},$$

$$\bar{\Phi}_{1,11} = \frac{360\bar{S}_2}{\bar{\gamma}^3},$$

$$\bar{\Phi}_{1,12} = X B_{wi}, \bar{\Phi}_{1,13} = 2\bar{P} - X + X A_i^T,$$

$$\bar{\Phi}_{1,14} = C^T$$

$$\bar{\Phi}_{2,2} = -(1-\mu)\bar{Q}_1,$$

$$\bar{\Phi}_{2,14} = X A_{hi}^T,$$

$$\bar{\Phi}_{3,3} = -(1-\gamma(t))\bar{Q}_2,$$

$$\bar{\Phi}_{3,13} = Y_j^T B_i^T,$$

$$\bar{\Phi}_{4,4} = -5\bar{R}_1 + \bar{Q}_3,$$

$$\bar{\Phi}_{4,6} = 32 \frac{\bar{R}_1}{h},$$

$$\bar{\Phi}_{4,6} = \frac{60\bar{R}_1}{h^2},$$

$$\bar{\Phi}_{5,5} = -9\bar{R}_2 + \bar{Q}_4,$$

$$\bar{\Phi}_{5,7} = \frac{2\bar{R}_2}{\bar{\gamma}} + \frac{30\bar{R}_2}{\bar{\gamma}^2},$$

$$\bar{\Phi}_{5,9} = \frac{60\bar{R}_2}{\bar{\gamma}},$$

$$\bar{\Phi}_{6,6} = -\frac{192\bar{R}_1}{h^2} - \frac{18\bar{S}_1}{h^2},$$

$$\bar{\Phi}_{6,8} = \frac{360\bar{R}_1}{h^3} + \frac{48\bar{S}_1}{h^3},$$

$$\bar{\Phi}_{7,7} = -\frac{192\bar{R}_2}{\bar{\gamma}^2} - \frac{70\bar{S}_2}{\bar{\gamma}^2},$$

$$\bar{\Phi}_{7,9} = \frac{360\bar{R}_2}{\bar{\gamma}^2} + \frac{480\bar{S}_2}{\bar{\gamma}^2},$$

$$\bar{\Phi}_{7,11} = -\frac{1080\bar{S}_2}{\bar{\gamma}^4},$$

$$\bar{\Phi}_{8,8} = -\frac{720\bar{R}_1}{h^4} - \frac{144\bar{S}_1}{h^4} - \frac{3456\bar{S}_1}{h^4},$$

$$\bar{\Phi}_{8,10} = \frac{8640\bar{S}_1}{h^5},$$

$$\bar{\Phi}_{9,9} = -\frac{720\bar{R}_2}{\bar{\gamma}^4} - \frac{144\bar{S}_2}{\bar{\gamma}^4} - \frac{3456\bar{S}_2}{\bar{\gamma}^4},$$

$$\bar{\Phi}_{9,11} = \frac{8640\bar{S}_2}{\bar{\gamma}^6},$$

$$\bar{\Phi}_{10,10} = -\frac{21600\bar{S}_1}{h^6},$$

$$\bar{\Phi}_{11,11} = -\frac{21600\bar{S}_2}{\bar{\gamma}^6},$$

$$\bar{\Phi}_{11,11} = -X,$$

$$\bar{\Phi}_{12,12} = -\eta^2,$$

$$\bar{\Phi}_{12,14} = D^T,$$

$$\bar{\Phi}_{13,13} = (h^2 \bar{R}_1 + \bar{\gamma}^2 \bar{R}_2 + h^2 \bar{S}_1 + \bar{\gamma}^2 \bar{S}_2 - X),$$

$$\bar{\Phi}_{14,14} = -I. \text{ Further, the control values are estimated as }$$

$$K_j = Y_j X^{-1}.$$



Proof. To discuss the H_∞ feedback control performance of the system, we have

$$Z_n(t) = \int_0^\infty [z^T(t)z(t) - \eta^2 w^T(t)w(t)]dt. \quad (3.13)$$

By applying H_∞ Definition 2.4, initial and boundary condition, we have $V(0) = 0$ and $V(\infty) \geq 0$. Then

$$\dot{V}(x(t)) + Z_n(t) \leq \chi^T(t)\Phi_{ij}\chi(t), \quad (3.14)$$

where $\Phi_{1,1} = Q_1 + Q_2 + Q_3 + Q_4 - 9R_1 - 9R_2 - 12S_1 - 12S_2 + \text{sym}(\mathcal{N}A_i) + C^T C$, $\Phi_{12,12} = -\eta^2$, $\Phi_{12,14} = D^T D$ and the remaining parameters same in Φ_{ij} . The above inequality is not linear. To make linear matrix inequality, mention $\mathcal{N}^{-1} = X$, $\bar{P}_1 = X P_1 X$, $\bar{Q}_1 = X Q_1 X$, $\bar{Q}_2 = X Q_2 X$, $\bar{Q}_3 = X Q_3 X$, $\bar{Q}_4 = X Q_4 X$, $\bar{R}_1 = X R_1 X$, $\bar{R}_2 = X R_2 X$, $\bar{S}_1 = X S_1 X$ and $\bar{S}_2 = X S_2 X$, then pre and post multiply (3.14), through $\text{diag}\{X, X, X, X, X, X, X, X, X, X, X, X, X\}$ and using Schur Complement Lemma, it is easy to get the LMI (3.12). Thus finishes the proof. \square

4. Numerical examples

We consider numerical simulations to show the effectiveness and advantages of the proposed method.

Example 4.1. Consider the T-S fuzzy system (2.5) with two fuzzy rules are provided as below:

$$\begin{aligned} A_1 &= \begin{bmatrix} -2 & -1.4 \\ 1 & 0 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.4 & 0 \\ 0.4 & -0.7 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 1.7 \\ 0 \end{bmatrix}, B_{w1} = \begin{bmatrix} 1.6 \\ 0 \end{bmatrix}, C_1 = [0.6 \ 0], \\ A_2 &= \begin{bmatrix} -1.76 & -2.155 \\ 2 & 0 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.9123 & 0.5 \\ 0 & -1.2 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 0.7 \\ 0 \end{bmatrix}, B_{w2} = \begin{bmatrix} 1.23 \\ 0 \end{bmatrix}, C_2 = [0.7 \ 0] \end{aligned}$$

$$D_1 = 0.3, D_2 = 0.6.$$

The membership functions are chosen as

$$v_1(x) = \frac{1}{1 + \exp(x_1 + 0.5)} \quad \text{and} \quad v_2(x) = 1 - v_1(x).$$

Some scalar values included in the simulation are chosen as: $\eta = 0.2$, $h = 1.8$, $\bar{\gamma} = 2.8$. We take the external disturbance and initial condition as $w(t) = 0.2t \sin(0.1 \exp(0.8t))$, $x(0) = [-2, 2]^T$ for the simulation purpose respectively. Fig.1 represents the state trajectories of the unforced system. By Fig.1, it is seen that the unforced system is not stable. To make this unstable system, using Theorem 2 provides the upcoming control gain matrices:

$$\begin{aligned} K_1 &= Y_1 X^{-1} = \begin{bmatrix} -0.0058 & -0.0781 \end{bmatrix}, \\ K_2 &= Y_2 X^{-1} = \begin{bmatrix} -0.0048 & -0.0489 \end{bmatrix}. \end{aligned}$$

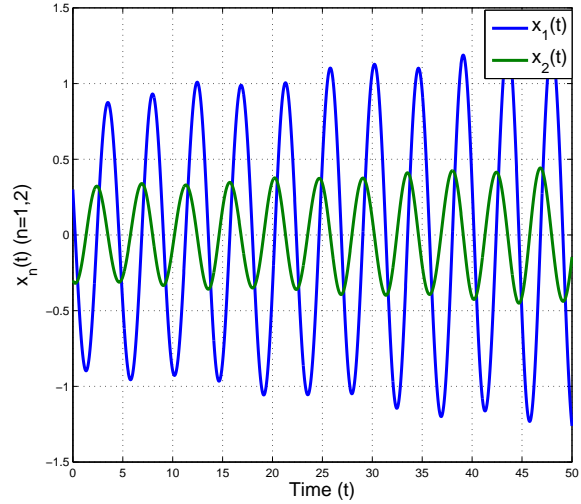


Figure 1. State responses of the unforced system (2.5)

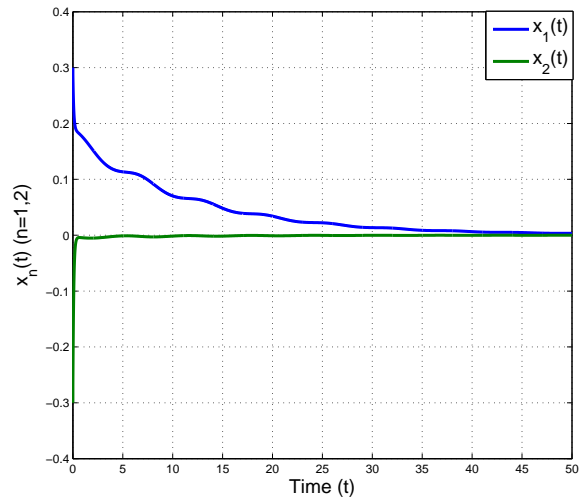


Figure 2. State responses of the closed loop system (2.5)

Based on the same initial state criteria, the state trajectories of the considered system are provided in Fig.2. Under the figure we see that the modeled sampled-data controllers make the closed-loop states converge to zero. Thus, the control method gives that the constructed sampled-data technique provided in Theorem 2 can make the system stable in the presence of time delays effectively.

Example 4.2. This simulation example is developed via a simple nonlinear mass spring damper physical system which is given in Fig. 3. More details about the proposed model and its values are available in [9].

Mention $x(t) = [y(t) \ \dot{y}(t)]^T$. Further, the state space representation of the mass spring damper model can be expressed by the proposed fuzzy system (2.5) with two rules in the up-



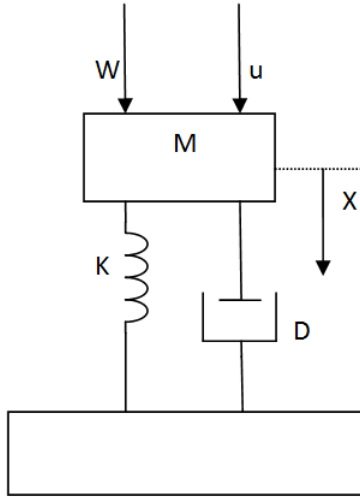


Figure 3. Mass spring damper system.

coming path:

Plant rule 1: **IF** $\dot{y}(t)$ is about \mathcal{F}_1^1 , **THEN**

$$\dot{x}(t) = (A_1)x(t) + B_1u(t) + B_1w(t),$$

$$z(t) = C_1x(t) + D_1u^f(t),$$

Plant rule 2: **IF** $\dot{y}(t)$ is about \mathcal{F}_1^2 , **THEN**

$$\dot{x}(t) = (A_2)x(t) + B_2u(t) + B_2w(t),$$

$$z(t) = C_2x(t) + D_2u(t),$$

where

$$A_1 = \begin{bmatrix} 0.0 & -0.02 \\ 1 & 0 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$B_{w1} = B_{w2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, D_1 = D_2 = 1,$$

$$A_2 = \begin{bmatrix} -0.225 & -0.02 \\ 1 & 0 \end{bmatrix}.$$

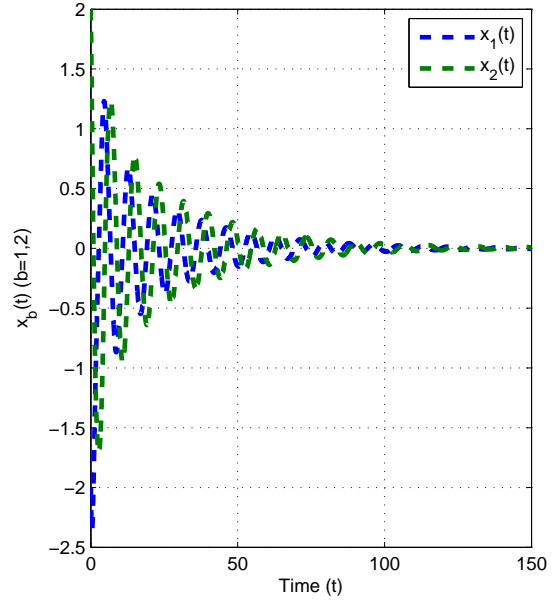
Moreover, the two membership functions are given as $\mathcal{F}_1^1 = \mu_1(x(t)) = 1 - \frac{x_1^2(t)}{2.25}$ and $\mathcal{F}_1^2 = \mu_2(x(t)) = \frac{x_1^2(t)}{2.25}$.

The purpose of this example is to design the fuzzy feedback controller (2.4) such that the closed-loop uncertain fuzzy system (2.5) is asymptotically stable. Take the remaining simulation parameters as $\mu = 0.2$. By determining the LMI based criteria provided in Theorem 3.2 with $\tilde{\gamma} = 1.9$, we get a set of feasible solutions that are not shown here. Under these, the sampled-data gain values can be estimated as

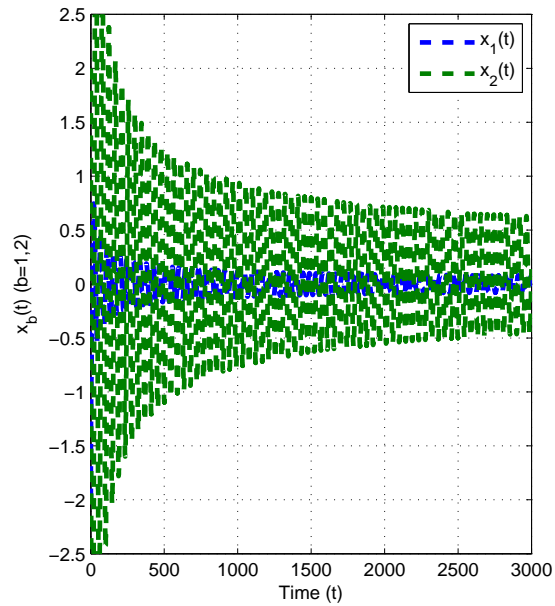
$$K_1 = [-0.0402 \quad -0.2725] \quad \text{and}$$

$$K_2 = [-0.0393 \quad -0.2637].$$

With the aforementioned gain values and the initial state $x(0) = [-2, 2]^T$, the state responses of the sampled-data control system and unforced fuzzy system (2.5) are shown in Figs.



(a) State responses of the closed loop system
Figure 4. Simulation results of the T-S fuzzy system (2.5)



(b) State responses of the unforced system
Figure 5. Simulation results of the T-S fuzzy system (2.5)

5(a) and 5(b). From the simulation analysis, the important and the necessity of the modeled sampled-data scheme (2.4) can be observed.



5. Conclusion

In this work, a novel condition has been constructed for the fuzzy model with the sampled-data scheme. By developing a new LKF, a new stability criteria has been obtained in the form of LMIs. The primary objective has been proposed on designing a sampled-data scheme such that for all admissible uncertainties, the considered system is asymptotically stable. Eventually, numerical simulations have been provided to illustrate the less conservativeness of the considered method.

Acknowledgment

The authors gratefully acknowledge the Dr.T. Saravanakumar, Department of Mathematics, Anna University Regional Campus, Coimbatore — 641046, for his kind help and suggestions.

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ISSN(P):2319 – 3786

Malaya Journal of Matematik

ISSN(O):2321 – 5666

