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Dominator and total dominator chromatic number of Mongolian tent and fire cracker graphs

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Abstract
A dominator coloring of a graph G is a proper coloring in which every vertex of G dominates at least one color class. The dominator chromatic number of $G, \chi_d(G)$,is defined by the minimum number of colors needed in a dominator coloring of G. A total dominator coloring of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number of $G, \chi_{td}(G)$, is the minimum number of colors needed in a total dominator coloring of G. In this paper, we obtain dominator and total dominator chromatic number of Mongolian tent graphs and fire cracker graphs.

Keywords
Dominator chromatic number, total dominator chromatic number, Mongolian tent graph, fire cracker graph.

AMS Subject Classification
05C15, 05C69.

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Contents

1. Introduction

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in [1]. The open neighborhood $N(v)$ of a vertex $v \in V(G)$ consists of the set of all vertices adjacent to v. The closed neighborhood of v is $N[v] = N(v) \cup v$. The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H, we define the cartesian product, denoted by $G \times H$, to be the graph with vertex set $V(G) \times V(H)$ and edges between two vertices (u_1, v_1) and (u_2, v_2) iff either $u_1 = u_2$ and $v_1v_2 \in E(H)$ or $u_1u_2 \in E(G)$ and $v_1 = v_2$. A star graph S_n is the complete bipartite graph $K_{1,n-1}$ (A tree with one internal node and $n-1$ leaves). A grid graphs can be defined as $P_m \times P_n$ where $m, n \geq 2$ and denoted by $P_{m \times n}$.

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$. A dominator coloring of G is a proper coloring of G in which every vertex of G dominates at least one color class. The dominator chromatic number, $\chi_d(G)$, is defined by the minimum number of colors needed in a dominator coloring of G. A total dominator coloring (td-coloring) of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number is denoted by $\chi_{td}(G)$ and is defined by the minimum number of colors needed in a total dominator coloring of G. This concept was studied by Vijiyalekshmi in [2, 3]. This notion is also referred as a Smarandachely k-dominator coloring of $G(k \geq 1)$ and was discussed by Vijiyalekshmi in [2, 3]. For an integer $k \geq 1$, a Smarandachely k-dominator coloring of G is a proper coloring of G such that every vertex in G properly dominates a k color class. The smallest number of colors for which there exist a Smarandachely k-dominator coloring of G is called the Smarandachely k-dominator chromatic number of G, and is denoted by $\chi^s_{td}(G)$.

In a proper coloring $\mathscr{C}$ of G, a color class of $\mathscr{C}$ is a set consisting of all those vertices assigned the same color. Let $\mathscr{C}^1$ be a minimal dominator coloring of G. We say that a color class $c_i \in \mathscr{C}^1$ is called a non-dominated color class ($n-d$ color class) if it is not dominated by any vertex of G. These color classes are also called repeated color classes. A ladder graph can be defined as $P_2 \times P_n$ where $n \geq 2$ and is denoted by L_n. The Mongolian tent graph $M_{m,n}$ is a graph obtained from $P_m \times P_n$ by adding a new vertex u to the vertices $v_1j/j = 1,3,5,\ldots..,n$. An (n,k)- firecracker graph $F_{n,k}$ is a graph obtained by the concatenation of n,k-stars by linking one leaf from each.

2. Preliminaries

In this section, we recall the crucial theorems [3, 4, 7] which are very useful in our work.

Theorem 2.1. *[4] The path P_n of order $n \geq 2$ has*

$$\chi_d(p_n) = \begin{cases} \lceil \frac{n}{3} \rceil + 1 & if\, n = 2,3,4,5,7 \\ \lceil \frac{n}{3} \rceil + 2 & otherwise. \end{cases}$$

Theorem 2.2. *[3] Let G be P_n or C_n. Then*

$$\chi_{td}(p_n) = \chi_{td}(C_n) = \begin{cases} 2\lfloor \frac{n}{4} \rfloor + 2 & if\, n \equiv 0(mod\,4) \\ 2\lfloor \frac{n}{4} \rfloor + 3 & if\, n \equiv 1(mod\,4) \\ 2\lfloor \frac{n+2}{4} \rfloor + 2, & otherwise. \end{cases}$$

Theorem 2.3. *[3] For every $n \geq 2$, the total dominator chromatic number of a ladder graph L_n is*

$$\chi_{td}\ (L_n) = \begin{cases} 2\lfloor \frac{p}{6} \rfloor + 2, & if\, p \equiv 0(mod6) \\ \begin{cases} 2\lfloor \frac{p-2}{6} \rfloor + 4; \\ 2\lfloor \frac{p-4}{6} \rfloor + 4, \end{cases} & Otherwise. \end{cases}$$

Theorem 2.4. *[7] If m is even and n is odd, then*

$$\chi_d(p_{m\times n}) = \begin{cases} \frac{m(n-1)}{4} + \lceil \frac{n}{3} \rceil + 2 & if\, m, n-1 \equiv 0(mod\,4) \\ \frac{m(n-1)}{4} + \lceil \frac{n}{3} \rceil + 3 & otherwise. \end{cases}$$

Theorem 2.5. *[7] If m is odd and n is odd, then*

$$\chi_d(p_{m\times n}) = \begin{cases} \frac{(m-1)(n-1)}{4} + \lceil \frac{m+n-1}{3} \rceil + 2 & if\, m-1, n-1 \equiv 0(mod\,4) \\ \frac{(m-1)(n-1)}{4} + \lceil \frac{m+n-1}{3} \rceil + 3 & otherwise. \end{cases}$$

In this paper, we obtain the least value for dominator and total dominator chromatic number of Mongolian tent graphs and fire cracker graphs.

3. Main Result

Dominator Chromatic Number of Mongolian Tent graphs

Theorem 3.1. *If m is odd, then*

$$\chi_d(M_{m,n}) = \begin{cases} \frac{(m-1)(n-1)}{4} + \lceil \frac{m+n-1}{3} \rceil + 3 & if\, m,n \equiv 1(mod\,4) \\ \frac{(m-1)(n-1)}{4} + \lceil \frac{m+n-1}{3} \rceil + 4 & otherwise. \end{cases}$$

Proof. Let $M_{m,n}$ be a Mongolian graph and $V(G) = \{u\} \cup \{v_{ij}/1 \leq i \leq m \text{ and } 1 \leq j \leq n\}$.

We consider two cases

Case (1): When $m \equiv 1(mod\,4)$ & $n \equiv 1(mod\,4)$.

Since the vertex u is adjacent to the vertices $\{v_{1j}/j = 1,3,5,\ldots..,n\}$ and these vertices received either the repeated color 2 or 3 or unique non repeated color. So we assign one unique non repeated color say 1 to the vertex u and we know $\chi_d(M_{m,n}) = 1 + \chi_d(P_{m\times n})$.

By Theorem 2.5, we get

$$\chi_d(M_{m,n}) = \frac{(m-1)(n-1)}{4} + \left\lceil \frac{m+n-1}{4} \right\rceil + 3.$$

Case (2): We have three sub cases.

Sub case (2.1) : When $m \equiv 1(mod\,4)$ & $n \equiv 3(mod\,4)$.

By case (1) & Theorem 2.5, we get,

$$\chi_d(M_{m,n}) = \frac{(m-1)(n-1)}{4} + \lceil \frac{m+n-1}{3} \rceil + 4.$$

Sub case (2.2) : When $m \equiv 3(mod\,4)$ & $n \equiv 1(mod\,4)$.

By case (2.1) & Theorem 2.5, we get

$$\chi_d(M_{m,n}) = \frac{(m-1)(n-1)}{4} + \lceil \frac{m+n-1}{3} \rceil + 4.$$

Sub case (2.3) : When $m \equiv 3(mod\,4)$ & $n \equiv 3(mod\,4)$.

By case (2.1) & Theorem 2.5, we get

$$\chi_d(M_{m,n}) = \frac{(m-1)(n-1)}{4} + \lceil \frac{m+n-1}{3} \rceil + 4.$$

□

Illustration: Consider $M_{5,7}$

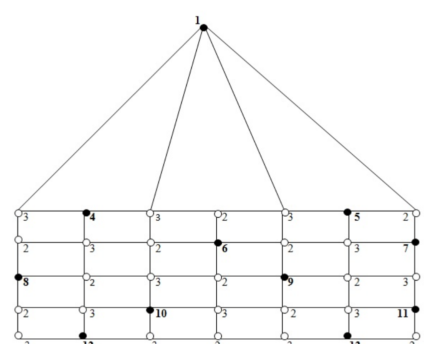

Figure 1

$$\chi_{td}(M_{5,7}) = 13.$$

Theorem 3.2. *If m is even, then*

$$\chi_d(M_{m,n}) = \begin{cases} \frac{m(n-1)}{4} + \lceil \frac{m}{3} \rceil + 3 & if\, m \equiv 0(mod\,4), n \equiv 1(mod\,4) \\ \frac{m(n-1)}{4} + \lceil \frac{m}{3} \rceil + 4 & otherwise. \end{cases}$$

Proof. Let $M_{m,n}$ be a Mongolian graph and $V(G) = \{u\} \cup \{v_{ij}/1 \leq i \leq m \quad \text{and} \quad 1 \leq j \leq n\}$.

We consider two cases

Case (1): When $m \equiv 0(mod\,4)$ & $n \equiv 1(mod\,4)$.

So we assign one unique non repeated color say 1 to the vertex u and the remaining vertices of $M_{m,n}$ received color. So $\chi_d(M_{m,n}) = 1 + \chi_d(P_{m\times n})$.

By Theorem 2.4, we get

$$\chi_d(M_{m,n}) = \frac{m(n-1)}{4} + \left\lceil \frac{m}{3} \right\rceil + 3.$$

Case (2): We have three sub cases.

Sub case (2.1) : When $m \equiv 0(mod\,4)$ & $n \equiv 3(mod\,4)$.

By case (1) & Theorem 2.4, we get,

$$\chi_d(M_{m,n}) = \frac{m(n-1)}{4} + \left\lceil \frac{m}{3} \right\rceil + 4.$$

Sub case (2.2) : When $m \equiv 2(mod\,4)$ & $n \equiv 1(mod\,4)$.

By case (2.1) & Theorem 2.4, we get

$$\chi_d(M_{m,n}) = \frac{m(n-1)}{4} + \left\lceil \frac{m}{3} \right\rceil + 3.$$

Sub case (2.3) : When $m \equiv 2(mod\,4)$ & $n \equiv 3(mod\,4)$.

By case (2.1) & Theorem 2.4, we get

$$\chi_d(M_{m,n}) = \frac{m(n-1)}{4} + \left\lceil \frac{m}{3} \right\rceil + 3.$$

□

Illustration: Consider $M_{4,7}$

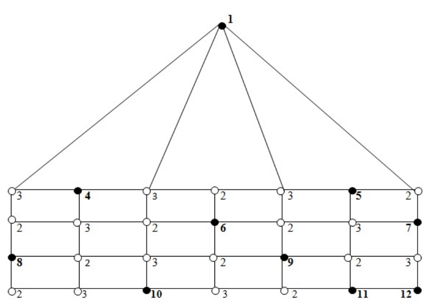

Figure 2

$$\chi_{td}(M_{4,7}) = 12.$$

Total Dominator Chromatic Number of Mongolian Tent graphs

Theorem 3.3.

$$\chi_{td}(M_{m,b}) = \begin{cases} \frac{mn}{3} + 2 & n \equiv 0(mod\,3) \\ \frac{mn}{3} + \chi_{td}(p_m) - 2 & n \equiv 1(mod\,3) \\ \frac{mn}{3} + 2\lceil \frac{m}{3} \rceil + 2 & n \equiv 3(mod\,3). \end{cases}$$

Proof. Let $G = M_{m,n}$ be a Mongolian graph and $V(G) = \{u\} \cup \{v_{ij}/1 \leq i \leq m \quad \text{and} \quad 1 \leq j \leq n\}$.

We consider three cases

Case (i): When $n \equiv 0(mod\,3)$.

Let $D = \{v_{ij}/1 \leq i \leq m \text{ and } j = 2,5,\ldots,(n-1)\}$ be a γ_t set of G. we assign one distinct color say $3,4,5,\ldots,\frac{mn}{3}+2$ to each vertex in D and assign two repeated color $1,2$ to the remaining vertices of G such that adjacent vertices receives different colors. So

$$\chi_{td}(G) = \frac{mn}{3} + 2.$$

Case (ii): When $n \equiv 1(mod\,3)$.

Let $G_1 = P_{m\times(n-1)}$. Since $n-1 \equiv 0(mod\,3), G$ is obtained by G_1 followed by P_m. By case (i), G_1 can be colored with $\frac{m(n-1)}{3} + 2$ colors. So

$$\chi_{td}(G_1) = \frac{m(n-1)}{3} + 2.$$

Thus

$$\chi_{td}(G) = \chi_{td}(G_1) + \chi_{td}(P_m) - 2.$$

From Theorem 2.2, we get

$$\chi_{td}(p_n) = \chi_{td}(C_n) = \begin{cases} 2\lfloor \frac{m}{4} \rfloor + 2 & if\, m \equiv 0(mod\,4) \\ 2\lfloor \frac{m}{4} \rfloor + 3 & if\, m \equiv 1(mod\,4) \\ 2\lfloor \frac{m+2}{4} \rfloor + 2, & otherwise. \end{cases}$$

So

$$\chi_{td}(G) = \begin{cases} \frac{m(n-1)}{3} + 2\lfloor \frac{m}{4} \rfloor + 2 & if\, m \equiv 0(mod\,4) \\ \frac{m(n-1)}{3} + 2\lfloor \frac{m}{4} \rfloor + 3 & if\, m \equiv 1(mod\,4) \\ \frac{m(n-1)}{3} + 2\lfloor \frac{m+2}{4} \rfloor + 2, & otherwise. \end{cases}$$

Case (iii): When $n \equiv 2(mod\,3)$. Let $G_2 = P_{m\times(n-2)}$. Since $n-2 \equiv 0(mod\,3), G$ is obtained by G_2 followed by L_m. By case (i), G_2 can be colored with $\frac{m(n-2)}{3} + 2$ colors. So $\chi_{td}(G_2) = \frac{m(n-2)}{3} + 2$ and $\chi_{td}(G) = \chi_{td}(G_2) + \chi_{td}(L_m) - 2$. Thus $\chi_{td}(G) = \frac{mn}{3} + 2\lceil \frac{m}{3} \rceil + 2$. □

Illustration: Consider $M_{6,9}$

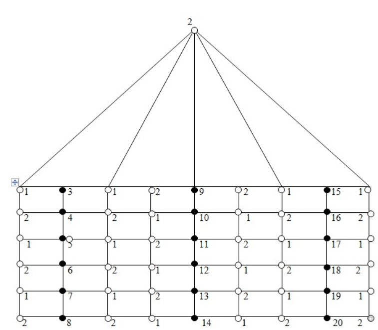

Figure 3

$$\chi_{td}(M_{6,9}) = 12.$$

Dominator Chromatic Number and Total Dominator Chromatic Number of Firecracker graphs

Theorem 3.4. *For a firecracker graphs*

$$(F_{M,n}) = \begin{cases} m+1, & m=2 \\ m+2, & m \geq 3. \end{cases}$$

Proof. Let $V(F_{m,n}) = \{u_{ij}/1 \leq i \leq m \text{ and } 1 \leq j \leq n\}$ with degree of $u_{i1} = n-1, 1 \leq i \leq m$ and u_{i1} is adjacent to $u_{1j}, 2 \leq j \leq n$.

If $m = 2$, the proof is obvious.

Let $m \geq 3$, we consider two cases:

Case 1: If m is odd, we assign the repeated color say 1 to the vertices to $u_(ij), j \neq 1$ and $i = 1,3,5,\dots,m$.

Also we assign the repeated color say 2 to the vertices to $u_(ij), j \neq 1$ and $i = 2,4,6,\dots,m-1$.

Case 2: If m is even, we assign the repeated color say 1 to the vertices to $u_(ij), j \neq 1$ and $i = 1,3,5,\dots,m-1$.

Also we assign the repeated color say 2 to the vertices to $u_(ij), j \neq 1$ and $i = 2,4,6,\dots,m$.

Assign distinct m colors say $3,4,\dots,m+2$ to the vertices $u_{i1}, 1 \leq i \leq m$, we get a dominator coloring of $F_{m,n}$. So $\chi_d(F_{m,n}) = m+2$.

□

Illustration: Consider $F_{4,5}$

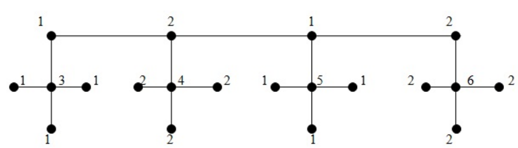

Figure 4

$$\chi_{td}(F_{4,5}) = 6.$$

Theorem 3.5. *For a firecracker graphs*

$$\chi_{td}(F_{M,n}) = \begin{cases} m+1, & m=2 \\ 2m, & m \geq 3. \end{cases}$$

Proof. If $m = 2$, the proof is obvious.

Let $m \geq 3$, we assign the distinct color say $1,2,3,4,\dots,m$ to the vertices to $u_{(i1)}, 1 \leq i \leq m$ and a distinct color say $m+i, 1 \leq i \leq m$ to the vertices$u_{(ij)}, 1 \leq i \leq m \,\&\, j \neq 1$ and we get a total dominator coloring of $F_{m,n}$. So $\chi_{td}(F_{m,n}) = m+2$. □

Illustration: Consider $F_{4,5}$

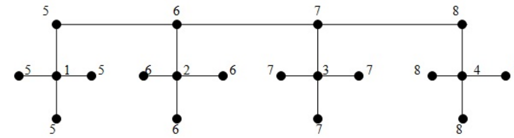

Figure 5

$$\chi_{td}(F_{4,5}) = 8.$$

4. Conclusion

In this paper, we obtain dominator and total dominator chromatic number of Mongolian tent graphs and fire cracker graphs.

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