Malaya Journal of Matematik, Vol. 8, No. 2, 702-708, 2020

https://doi.org/10.26637/MJM0802/0064

Fuzzy resolvable sets and fuzzy hyperconnected spaces

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Abstract

In this paper, several characterizations of fuzzy resolvable sets are established. The conditions for the existence of fuzzy resolvable sets in fuzzy hyperconnected spaces are obtained. It is established that the pre-images of fuzzy open sets in fuzzy topological spaces under fuzzy continuous, fuzzy semi-continuous and fuzzy contra -continuous functions are fuzzy resolvable sets in fuzzy hyperconnected spaces.

Keywords

Fuzzy somewhere dense set, fuzzy nowhere dense set, fuzzy simply $\star$ open set, fuzzy residual set, fuzzy β-open set, fuzzy P-spaces, somewhat fuzzy continuous function.

AMS Subject Classification

54A40.

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Article History: Received **12** December **2019**; Accepted **26** March **2020**

Contents

1. Introduction

In 1965, L . A. Zadeh [22] introduced the notion of fuzzy sets as a new approach for modeling uncertainties. Many researchers realized the potential of fuzzy idea and have successfully applied it in every branch of Mathematics. In 1968, C.L. Chang [3] introduced the concept of fuzzy topological spaces . In the recent years, there has been a growing trend among many fuzzy topologists to introduce and study different forms of fuzzy sets in fuzzy topology.

In 2002, the notion of fuzzy hyperconnectedness in fuzzy topological spaces was introduced by M. Caldas et al [7]. The concept of resolvable sets in topological spaces were studied in [6]. The notion of fuzzy resolvable sets in topological spaces was introduced and studied in [18]. In continuation of this work, several characterizations of fuzzy resolvable sets are obtained. The mathematical directions for the existence of fuzzy resolvable sets in fuzzy hyperconnected spaces are obtained. It is found that the pre-images of fuzzy open sets in fuzzy topological spaces under fuzzy continuous, fuzzy semi-continuous and fuzzy contra-continuous functions are fuzzy resolvable sets in the fuzzy hyperconnected spaces.

2. Preliminaries

For the purpose of having the exposition self-contained, some basic concepts and results used in the sequel, are presented. In this paper, (X,T) or simply X, means a fuzzy topological space due to Chang (1968). A fuzzy set λ defined in X, is a mapping from the set X into the unit interval $I=[0,1]$. The fuzzy set 0_X will be defined as $0_X(x)=0$, for all $x\in X$ and the fuzzy set 1_X will be defined as $1_X(x)=1$, for all $x\in X$ and $\lambda'(x)=1-\lambda(x)$, for all $x\in X$.

Definition 2.1. *[3] Let λ be any fuzzy set in the fuzzy topological space (X,T). The fuzzy interior and the fuzzy closure of λ are defined respectively as follows:*

(i) $Int(\lambda)=\vee\{\mu/\mu\leq\lambda,\mu\in T\}$;

(ii) $Cl(\lambda)=\wedge\{\mu/\lambda\leq\mu,1-\mu\in T\}$.

Lemma 2.2. *[1] For a fuzzy set λ of a fuzzy space X,*

(i) $1-int(\lambda)=cl(1-\lambda)$ *and*

(ii) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.3. *A fuzzy set λ in a fuzzy topological space (X,T) is called*

(1) Fuzzy semi-open if $\lambda \leq cl\ int\ (\lambda)$ and fuzzy semi-closed if $int\ cl\ (\lambda) \leq \lambda$ [1].

(2) Fuzzy β-open if $\lambda \leq cl\ int\ cl(\lambda)$ and fuzzy β-closed if $int\ cl\ int\ (\lambda) \leq \lambda$ [2].

(2) Fuzzy regular-open if $\lambda = int cl(\lambda)$ and fuzzy regular-closed if $\lambda = cl int(\lambda)$ [1].

Definition 2.4. *Let λ be the fuzzy set in the fuzzy space (X,T). Then λ is called a*

(i) Fuzzy dense set if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $cl\ (\lambda) = 1$, in the fuzzy space (X,T)[10].

(ii) Fuzzy nowhere dense set if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\ [cl(\lambda)] = 0$, in (X,T)[10].

(iii) Fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category [10].

(iv) fuzzy σ-nowhere dense set if λ is an fuzzy F_σ-set in (X,T) such that $Int\ (\lambda) = 0$ [14].

(v) fuzzy simply open set if $Bd\ (\lambda)$ is an fuzzy nowhere dense set in (X,T). That is, λ is a fuzzy simply open set in (X,T) if $[Cl\ (\lambda) \wedge Cl(1-\lambda)]$ is an fuzzy nowhere dense set in (X,T) [15] .

(vi) Fuzzy simply $\star$ open set if $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T) [16].

(vii) Fuzzy somewhere dense set if $int\ cl\ (\lambda) \neq 0$ in (X,T) [11].

(viii) Fuzzy σ-first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy σ-nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy σ-second category [14].

(ix) Fuzzy pseudo-open set if $\lambda = \mu \vee \delta$ where μ is the non-zero fuzzy open set and δ is the fuzzy first-category set in (X,T)[17].

(x) Fuzzy resolvable set in (X,T) if for each fuzzy closed set μ in (X,T), $\{cl\ (\mu \wedge \lambda) \wedge cl(\mu \wedge [1-\lambda])\}$ is a fuzzy nowhere dense set in (X,T) [18].

Definition 2.5. *Let (X,T) be the fuzzy topological space and (X,T) is called*

(i) Fuzzy hyper-connected space if every non-null fuzzy open subset of (X,T) is fuzzy dense in (X,T) [7].

(ii) Fuzzy P-space if each fuzzy G_δ-set in (X,T) is fuzzy open in (X,T) [9].

(iii) Fuzzy resolvable space if there exists fuzzy dense set λ in (X,T) such that $cl\ (1-\lambda) = 1$. Otherwise (X,T) is called the fuzzy irresolvable space [12].

Definition 2.6. *[13] Let λ be an fuzzy first category set in the fuzzy topological space in (X,T). Then, $1-\lambda$ is called an fuzzy residual set in (X,T).*

Definition 2.7. *Let (X,T) and (Y,S) be any two fuzzy topological spaces. The function $f:(X,T) \to (Y,S)$ is called a*

(i) Fuzzy continuous function if for the fuzzy open set λ in $(Y,S) f^{-1}(\lambda)$ is fuzzy open in (X,T) [3].

(ii) Fuzzy contra-continuous function if $f^{-1}(\lambda)$ is fuzzy closed in (X,T), for the fuzzy open set λ in (Y,S) [4].

(iii) Somewhat fuzzy continuous function if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$, there exists a non-zero fuzzy open set μ in (X,T) such that $\mu \leq f^{-1}(\lambda)$. That is, $Int\ [f^{-1}(\lambda)] \neq 0$, in (X,T) [10].

(iv) Fuzzy semi-continuous function if for the fuzzy open set λ in $(Y,S) f^{-1}(\lambda)$ is fuzzy semi-open in (X,T) [1].

Theorem 2.8. *[17] If λ is the fuzzy pseudo -open set in the fuzzy hyperconnected space (X,T), then λ is the fuzzy resolvable set in (X,T).*

Theorem 2.9. *[17] If $\lambda = \mu \vee \delta$, where μ is an fuzzy open and δ is an fuzzy σ-first category set in the fuzzy hyperconnected and fuzzy P-space (X,T), then λ is an fuzzy pseudo-open set in (X,T).*

Theorem 2.10. *[5] In an fuzzy hyperconnected space (X,T), any fuzzy subset λ of X is an fuzzy semi-open set if $Int\ (\lambda) \neq 0_X$.*

Theorem 2.11. *[16] If λ is an fuzzy simply $\star$ open set in the fuzzy hyperconnected space (X,T), then λ is an fuzzy simply open set in (X,T).*

Theorem 2.12. *[18] If λ is an fuzzy simply open set in the fuzzy topological space (X,T), then λ is an fuzzy resolvable set in (X,T).*

Theorem 2.13. *[20] If λ is the fuzzy somewhere dense set in the fuzzy topological space (X,T) in which fuzzy open sets are fuzzy resolvable sets, $Int\ [\lambda \wedge (1-\lambda)] = 0$, in (X,T).*

Theorem 2.14. *[20] If λ is the fuzzy somewhere dense set in the fuzzy topological space (X,T) in which fuzzy open sets are fuzzy resolvable sets, $cl\ [\lambda \vee (1-\lambda)] = 1$, in (X,T).*

Theorem 2.15. *[18] If λ is the fuzzy resolvable set in the fuzzy topological space (X,T), then there exists an fuzzy regular open set δ in (X,T) such that $\delta \leq cl[\lambda \vee (1-\lambda)]$.*

Theorem 2.16. *[5] Let (X,T) be the fuzzy topological space. Then the following properties are equivalent :*

(i) (X,T) *is fuzzy hyperconnected.*

(ii) 1_X *and* 0_X *are the only fuzzy regular open sets in* X.

Theorem 2.17. *[19] If λ is an fuzzy simply $\star$ open set in the fuzzy topological space (X,T), then λ is an fuzzy somewhere dense set in (X,T).*

Theorem 2.18. *[19] If λ is an fuzzy residual set in the fuzzy topological space (X,T) in which fuzzy first category sets are not fuzzy dense sets, then λ is an fuzzy somewhere dense set in (X,T).*

Theorem 2.19. *[19] If λ is an fuzzy residual set in the fuzzy P-space (X,T), then λ is an fuzzy somewhere dense set in (X,T).*

Theorem 2.20. *[19] If λ is the non-zero fuzzy β-open set in the fuzzy topological space (X,T), then λ is the fuzzy somewhere dense set in (X,T).*

Theorem 2.21. *[18] If λ is an fuzzy resolvable set in the fuzzy topological space (X,T), then $1-\lambda$ is also an fuzzy resolvable set in (X,T).*

Theorem 2.22. *[8] Let (X,T) be the fuzzy second category but not fuzzy Baire, and fuzzy hyper connected space. If λ is an fuzzy residual set in (X,T), then λ is an fuzzy simply open set in (X,T).*

Theorem 2.23. *[21] If the fuzzy topological space (X,T) is the fuzzy hyperconnected space, then (X,T) is the fuzzy irresolvable space.*

Theorem 2.24. *[15] If λ is an fuzzy semi-open set in the fuzzy topological space (X,T), then λ is an fuzzy simply open set in (X,T).*

3. Fuzzy resolvable sets

Proposition 3.1. *If the fuzzy set λ is fuzzy open in the fuzzy hyperconnected space (X,T), then λ is the fuzzy resolvable set in (X,T).*

Proof. Let λ be the fuzzy open set in (X,T). Since (X,T) is the fuzzy hyperconnected space, the fuzzy open set λ is fuzzy dense in (X,T) and then Cl $(\lambda)=1$, in (X,T). Now, for the fuzzy closed set μ in (X,T),

$$\begin{aligned}
&[Cl\{\mu\wedge\lambda\}]\wedge[Cl\{\mu\wedge(1-\lambda)\}]\\
&\leq[Cl\{\mu\}\wedge Cl\{\lambda\}]\wedge[Cl\{\mu\}\wedge Cl\{(1-\lambda)\}]\\
&=[Cl\{\mu\}\wedge 1]\wedge[Cl\{\mu\}\wedge Cl\{(1-\lambda)\}]\\
&=Cl\mu\wedge[Cl\{\mu\}\wedge Cl\{(1-\lambda)\}]\\
&=Cl\{\mu\}\wedge Cl\{\mu\}\wedge Cl\{(1-\lambda)\}\\
&=Cl\{\mu\}\wedge Cl\{(1-\lambda)\}\\
&=\mu\wedge(1-\lambda)\,(\text{since}\,\mu\,\text{and}\,1-\lambda\text{ are fuzzy closed})\\
&\leq(1-\lambda)
\end{aligned}$$

This gives that Int Cl $([Cl\{\mu\wedge\lambda\}]\wedge[Cl\{\mu\wedge(1-\lambda)\}])\leq IntCl\{(1-\lambda)\}$. But Int Cl $\{(1-\lambda)\}=1-ClInt(\lambda)=1-Cl(\lambda)=1-1=0$, in (X,T) and thus Int Cl $([Cl\{\mu\wedge\lambda\}]\wedge[Cl\{\mu\wedge(1-\lambda)\}])=0$, in (X,T). Thus the fuzzy open set λ is the fuzzy resolvable set in (X,T). Hence each fuzzy open set in the fuzzy hyper connected space (X,T) is the fuzzy resolvable set in (X,T). ☐

Remark 3.2. *From the proposition 3.1, one will obtain the following:*

The fuzzy open sets in the fuzzy hyper connected spaces are fuzzy resolvable sets. But the converse need not hold in general. That is, the fuzzy resolvable sets may fail to be fuzzy open sets in the fuzzy hyper connected spaces. The following example shows the existence of the fuzzy resolvable but not fuzzy open set in the topological space (X,T).

Example 3.1. *Consider the set $X=\{a,b,c\}$. Let $I=[0,1]$. The fuzzy sets λ,μ and α are defined on X as follows:*

$$\begin{aligned}
\lambda:X\to I &\quad \text{is defined by}\quad \lambda(a)=0.5;\lambda(b)=0.6;\lambda(c)=0.6,\\
\mu:X\to I &\quad \text{is defined by}\quad \mu(a)=0.5;\mu(b)=0.7;\mu(c)=0.6.\\
\alpha:X\to I &\quad \text{is defined by}\quad \alpha(a)=0.4;\alpha(b)=0.5;\alpha(c)=0.3.
\end{aligned}$$

Now $T=\{0,\lambda,\mu,1\}$ is the fuzzy topology for X. By computation , one can find that Cl $(\lambda)=1$ and Cl $(\mu)=1$. Thus (X,T) is the fuzzy hyperconnected space. Also by computation,

$$\begin{aligned}
&Int\,Cl([Cl\{(1-\lambda)\wedge\alpha\}]\wedge[Cl\{(1-\lambda)\wedge(1-\alpha)\}])=0,\\
&Int\,Cl([Cl\{(1-\mu)\wedge\alpha\}]\wedge[Cl\{(1-\mu)\wedge(1-\alpha)\}])=0.
\end{aligned}$$

Thus α is the fuzzy resolvable set in (X,T), but α is not the fuzzy open set in (X,T).

Proposition 3.3. *If λ is the fuzzy set defined on X with Int $(\lambda)\neq 0$, in the fuzzy hyperconnected space (X,T), then*

(i) Int (λ) is the fuzzy resolvable set in (X,T).

(ii) The fuzzy set λ is fuzzy semi-open in (X,T) and there exists an fuzzy resolvable set μ in (X,T) such that $\lambda\geq\mu$.

Proof. Let λ be the non-zero fuzzy set defined on X with int $(\lambda)\neq 0$ in (X,T).
(i). Because (X,T) is the fuzzy hyper connected space, by the proposition 3.1, the fuzzy open set Int (λ) is the fuzzy resolvable set in (X,T).
(ii). Now Int $(\lambda)\neq 0$ in (X,T) implies, by the theorem 2.10, that λ is the fuzzy semi-open set in (X,T). Let $\mu=Int(\lambda)$. Because Int $(\lambda)\leq\lambda$ in (X,T), there exists an fuzzy resolvable set μ in (X,T) such that $\lambda\geq\mu$. ☐

Proposition 3.4. *If $\lambda=\mu\vee\delta$, where μ is the fuzzy open and δ is the fuzzy σ-first category set in the fuzzy hyperconnected and fuzzy P-space (X,T), then λ is the fuzzy resolvable set in (X,T).*

Proof. Let λ be the fuzzy set defined on X such that $\lambda=\mu\vee\delta$, where μ is the fuzzy open set and δ is the fuzzy σ-first category set in (X,T). Because (X,T) is the fuzzy hyperconnected and fuzzy P-space, by the theorem 2.9, λ is the fuzzy pseudo-open set in (X,T). Since fuzzy pseudo-open sets are fuzzy resolvable sets in the fuzzy hyperconnected spaces, by the theorem 2.8, λ is the fuzzy resolvable set in (X,T). □

Proposition 3.5. *If λ is the fuzzy simply $\star$ open set in the fuzzy hyper connected space (X,T), then λ is the fuzzy resolvable set in (X,T).*

Proof. Let λ be the fuzzy simply $\star$ open set in (X,T). Because (X,T) is the fuzzy hyper connected space, by the theorem 2.11 , λ is the fuzzy simply open set in (X,T). Then, by the theorem 2.12, the fuzzy simply open set λ is the fuzzy resolvable set in (X,T). □

Remark 3.6. *The above proposition can also be stated as follows:*

If λ is the fuzzy open set in the fuzzy hyper connected space (X,T), then for the fuzzy nowhere dense set μ in $(X,T), \lambda \vee \mu$ is also the fuzzy resolvable set in (X,T).

Proposition 3.7. *If λ is the fuzzy somewhere dense set in the fuzzy hyperconnected space (X,T), then there exists fuzzy resolvable sets Int $[\lambda]$ and Int $[1-\lambda]$ in (X,T), such that Int $[\lambda \wedge (1-\lambda)] = 0$, in (X,T).*

Proof. Proof : Let λ be the fuzzy somewhere dense set in (X,T). Because (X,T) is the fuzzy hyperconnected space, by the theorem 2.13, Int $[\lambda \wedge (1-\lambda)] = 0$, in (X,T). Since Int $[\lambda \wedge (1-\lambda)] = Int[\lambda] \wedge Int[1-\lambda], Int[\lambda] \wedge Int[1-\lambda] = 0$, in (X,T). Because (X,T) is the fuzzy hyper connected space, by the proposition 3.1, the fuzzy open sets Int $[\lambda]$ and Int $[1-\lambda]$ are fuzzy resolvable sets in (X,T). Thus, int $[\lambda] \wedge int[1-\lambda] = 0$, where Int $[\lambda]$ and Int$[1-\lambda]$ are fuzzy resolvable sets in (X,T). □

Proposition 3.8. *If λ is the fuzzy somewhere dense set in the fuzzy hyper connected space (X,T), then there exists a fuzzy resolvable set μ in (X,T) such that Cl $[\lambda] \geq \mu$.*

Proof. Let λ be the fuzzy somewhere dense set in (X,T). Then, Int Cl $(\lambda) \neq 0$, in (X,T). Then, there will be an fuzzy open set μ in (X,T) such that Cl $(\lambda) \geq \mu$. Since (X,T) is the fuzzy hyperconnected space, by the proposition 3.1 , the fuzzy open set μ is the fuzzy resolvable set in (X,T). Thus, there exists a fuzzy resolvable set μ in (X,T) such that Cl $(\lambda) \geq \mu$. □

Proposition 3.9. *If λ is the fuzzy somewhere dense set in the fuzzy hyperconnected space (X,T), then Cl $[\lambda] \vee Cl[1-\lambda] = 1$, in (X,T).*

Proof. Let λ be the fuzzy somewhere dense set in (X,T). Because (X,T) is the fuzzy hyperconnected space, by the proposition 3.7, for the fuzzy somewhere dense set λ in $(X,T,)Int[\lambda \wedge (1-\lambda)] = 0$, in (X,T). This gives that $1 - Int[\lambda \wedge (1-\lambda)] = 1-0 = 1$ and then Cl $[1-\{\lambda \wedge (1-\lambda)\}] = 1$. This means that Cl $[(1-\lambda) \vee 1-(1-\lambda)] = 1$ and then Cl $[\lambda \vee (1-\lambda)] = 1$, in (X,T). Now Cl $[\lambda \vee (1-\lambda)] = Cl[\lambda] \vee Cl[1-\lambda]$, implies that Cl $[\lambda] \vee Cl[1-\lambda] = 1$, in (X,T). □

Proposition 3.10. *If λ is the fuzzy resolvable set in the fuzzy hyper connected space (X,T), then either there exists no fuzzy regular open set δ in (X,T) such that Cl $[\lambda \vee (1-\lambda)] \geq \delta$ or Cl $[\lambda \vee (1-\lambda)] = 1$, in (X,T).*

Proof. Let λ be the fuzzy resolvable set in (X,T). Because (X,T) is the fuzzy hyper connected space (X,T), by the theorem 2.15, there exists an fuzzy regular open set δ in (X,T) such that $\delta \leq cl[\lambda \vee (1-\lambda)]$. Again since (X,T) is the fuzzy hyper connected space, by the theorem 2.16, 1_X and 0_X are the only fuzzy regular open sets in X. Then, either $\delta = 0$ or $\delta = 1$. If $\delta = 0$, then there exists no fuzzy regular open set δ in (X,T) such that $\delta \leq Cl[\lambda \vee (1-\lambda)]$, in (X,T). If $\delta = 1$, then $1 \leq Cl[\lambda \vee (1-\lambda)]$. That is, Cl $[\lambda \vee (1-\lambda)] = 1$, in (X,T). □

Proposition 3.11. *If λ is the non-zero fuzzy β-open set in the fuzzy hyperconnected space (X,T), then Cl $[\lambda] \vee Cl[1-\lambda] = 1$, in (X,T).*

Proof. Let λ be the non-zero fuzzy β-open set in (X,T). Then, $\lambda \leq ClIntcl(\lambda)$, in (X,T). This means that, Int Cl $\lambda \neq 0$. For, otherwise if Int Cl $\lambda = 0$ and then $\lambda \leq Cl0 = 0$ and this will imply that $\lambda = 0$, a contradiction. Thus, λ is the fuzzy somewhere dense set in (X,T). Because (X,T) is the fuzzy hyper connected space, by the proposition 3.9, for the fuzzy somewhere dense set λ in $(X,T), Cl[\lambda] \vee Cl[1-\lambda] = 1$, in (X,T). □

Proposition 3.12. *If λ is a non-zero fuzzy simply $\star$ open set in a fuzzy hyper connected space $(X,T), cl[\lambda] \vee cl[1-\lambda] = 1$, in (X,T).*

Proof. Let λ be a non-zero fuzzy simply $\star$ open set in (X,T). Then, by theorem 2.17, λ is a fuzzy somewhere dense set in (X,T). Since (X,T) is a fuzzy hyper connected space (X,T), by proposition 3.9, for the fuzzy somewhere dense set λ in $(X,T), cl[\lambda] \vee cl[1-\lambda] = 1$, in (X,T). □

Proposition 3.13. *If λ is the fuzzy residual set in the fuzzy hyper connected space (X,T), in which fuzzy first category sets are not fuzzy dense sets, then Cl $[\lambda] \vee Cl[1-\lambda] = 1$, in (X,T).*

Proof. Let λ be the non-zero fuzzy residual set in (X,T) in which fuzzy first category sets are not fuzzy dense sets. Then, by the theorem 2.18, λ is the fuzzy somewhere dense set in (X,T). Because (X,T) is the fuzzy hyper connected space (X,T), by the proposition 3.9, for the fuzzy somewhere dense set λ in $(X,T), Cl[\lambda] \vee Cl[1-\lambda] = 1$, in (X,T). □

Proposition 3.14. *If λ is the fuzzy residual set in the fuzzy hyper connected and fuzzy P-space (X,T), then Cl $[\lambda] \vee Cl[1-\lambda] = 1$, in (X,T).*

Proof. Let λ be the non-zero fuzzy residual set in (X,T). Because (X,T) is the fuzzy P-space, by the theorem 2.19, λ is the fuzzy somewhere dense set in (X,T). Because (X,T) is the fuzzy hyperconnected space, by the proposition 3.9, for the fuzzy somewhere dense set λ in $(X,T), Cl[\lambda] \vee Cl[1-\lambda] = 1$, in (X,T). □

The following propositions establish the existence of the fuzzy resolvable sets in fuzzy hyperconnected spaces.

Proposition 3.15. *If λ is the non-zero fuzzy β-open set in the fuzzy hyperconnected space (X,T), then there exists an fuzzy resolvable set μ in (X,T) such that $\mu \leq Cl[\lambda]$.*

Proof. Let λ be the non-zero fuzzy β-open set in (X,T). Then, by the theorem 2.20, λ is the fuzzy somewhere dense set in (X,T). Because (X,T) is the fuzzy hyperconnected space, by the proposition 3.8, there exists an fuzzy resolvable set μ in (X,T) such that $\mu \leq Cl[\lambda]$. □

Proposition 3.16. *If λ is the non-zero fuzzy simply $\star$ open set in the fuzzy hyperconnected space (X,T), then there exists an fuzzy resolvable set μ in (X,T) such that $\mu \leq Cl[\lambda]$.*

Proof. Let λ be the non-zero fuzzy simply $\star$ open set in (X,T). Then, by the theorem 2.17, λ is the fuzzy somewhere dense set in (X,T). Since (X,T) is the fuzzy hyper connected space, by the proposition 3.8, there exists an fuzzy resolvable set μ in (X,T) such that $\mu \leq Cl[\lambda]$. □

Proposition 3.17. *If λ is the non-zero fuzzy residual set in the fuzzy hyper connected space (X,T), in which fuzzy first category sets are not fuzzy dense sets, then there exists an fuzzy resolvable set μ in (X,T) such that $\mu \leq Cl[\lambda]$.*

Proof. Let λ be the non-zero fuzzy residual set in (X,T) in which fuzzy first category sets are not fuzzy dense sets. Then, by the theorem 2.18, λ is the fuzzy somewhere dense set in (X,T). Since (X,T) is the fuzzy hyper connected space, by the proposition 3.8, there exists a fuzzy resolvable set μ in (X,T) such that $\mu \leq Cl[\lambda]$. □

Proposition 3.18. *If λ is the fuzzy residual set in the fuzzy hyper connected and fuzzy P-space (X,T), then there exists an fuzzy resolvable set μ in (X,T) such that $\mu \leq Cl[\lambda]$.*

Proof. Let λ be the non-zero fuzzy residual set in (X,T). Because (X,T) is the fuzzy P-space, by the theorem 2.19, λ is the fuzzy somewhere dense set in (X,T). Since (X,T) is the fuzzy hyperconnected space, by the proposition 3.8, there exists an fuzzy resolvable set μ in (X,T) such that $\mu \leq Cl[\lambda]$. □

Proposition 3.19. *If λ is the fuzzy set defined on X with Int $(\lambda) \neq 0$, in the fuzzy hyperconnected space (X,T), then*

(i) the fuzzy resolvable set Int (λ) is the fuzzy dense set in (X,T).

(ii) the fuzzy set λ is the fuzzy dense set in (X,T).

Proof. (i). Let λ be the fuzzy set defined on X with Int $(\lambda) \neq 0$, in (X,T). Because (X,T) is the fuzzy hyper connected space, by the proposition 3.3, Int (λ) is the fuzzy resolvable set in (X,T). Now Int (λ) is the fuzzy open set in (X,T) implies that Cl $[Int(\lambda)] = 1$. Hence Int (λ) is the fuzzy dense set in (X,T).

(ii). From (i), Cl $[Int(\lambda)] = 1$, for the fuzzy set defined on X with Int $(\lambda) \neq 0$, in (X,T). But Cl $[Int(\lambda)] \leq Cl(\lambda)$, implies that $1 \leq cl(\lambda)$. That is, Cl $(\lambda) = 1$ and hence the fuzzy set λ is the fuzzy dense set in (X,T). □

Proposition 3.20. *If λ is the fuzzy set defined on X with Int $(\lambda) \neq 0$, in the fuzzy hyper connected space (X,T), then Int (λ) and Cl $(1-\lambda)$ are fuzzy resolvable sets in (X,T).*

Proof. Let λ be the fuzzy set defined on X with Int $(\lambda) \neq 0$, in (X,T). Because (X,T) is the fuzzy hyper connected space, by the proposition 3.3, Int (λ) is the fuzzy resolvable set in (X,T). Then, by the theorem 2.21, $1-Int(\lambda)$ is also the fuzzy resolvable set in (X,T). Now Cl $(1-\lambda) = 1-Int(\lambda)$, shows that Cl $(1-\lambda)$ is the fuzzy resolvable set in (X,T). □

Proposition 3.21. *If λ is the fuzzy set defined on X with Int $(\lambda) \neq 0$, in the fuzzy hyperconnected space (X,T), Int $[Cl(\lambda)]$ and $1-IntCl(\lambda)$ are fuzzy resolvable sets in (X,T).*

Proof. Let λ be the fuzzy set defined on X with Int $(\lambda) \neq 0$, in (X,T). Now Int $(\lambda) \leq IntCl(\lambda)$, gives that Int Cl $(\lambda) \neq 0$, in (X,T). Thus Cl (λ) is the fuzzy set defined on X with Int $[Cl(\lambda)] \neq 0$, in (X,T). Then, by the proposition 3.20, Int [Cl (λ)] and Cl $(1-Cl(\lambda))$ are fuzzy resolvable sets in (X,T). Thus Int $[Cl(\lambda)]$ and $1-IntCl(\lambda)$ are fuzzy resolvable sets in (X,T). □

Proposition 3.22. *If λ is the fuzzy open set in the fuzzy hyperconnected space (X,T), then λ is the fuzzy resolvable set in (X,T) such that Cl $(\lambda) = 1$ and Cl $(1-\lambda) \neq 1$.*

Proof. Let λ be the non-zero fuzzy open set in (X,T). Because (X,T) is the fuzzy hyperconnected space, λ is the fuzzy dense set and then Cl $(\lambda) = 1$ in (X,T). By the theorem 2.23, (X,T) is the fuzzy irresolvable space and hence for the fuzzy dense set λ in $(X,T), Cl(1-\lambda) \neq 1$. Also by the proposition 3.1, λ is the fuzzy resolvable set in (X,T). Hence λ is the fuzzy resolvable set in (X,T) such that Cl $(\lambda) = 1$ and Cl $(1-\lambda) \neq 1$. □

Proposition 3.23. *Let (X,T) be the fuzzy second category [but not fuzzy Baire] and fuzzy hyper connected space. If λ is the fuzzy residual set in (X,T), then λ is the fuzzy resolvable set in (X,T).*

Proof. Let λ be the fuzzy residual set in (X,T). Since (X,T) is fuzzy second category [but not fuzzy Baire] and fuzzy hyper connected, by the theorem 2.22, the fuzzy residual set λ in (X,T), is the fuzzy simply open set in (X,T). Then, by the theorem 2.12 , λ is the fuzzy resolvable set in (X,T). □

4. Fuzzy resolvable sets and fuzzy functions

Proposition 4.1. *If $f : (X,T) \to (Y,S)$ is the somewhat fuzzy continuous function from the fuzzy hyperconnected space (X,T) into the fuzzy topological space (Y,S), then for a non-zero fuzzy open set λ in (Y,S),*

(i) Int $[f^{-1}(\lambda)]$ is the fuzzy resolvable set in (X,T).

(ii) $f^{-1}(\lambda)]$ is the fuzzy dense set in (X,T).

(iii) Cl $(f^{-1}(1-\lambda)$ is the fuzzy resolvable set in (X,T).

Proof. (i). Let $f : (X,T) \to (Y,S)$ be the somewhat fuzzy continuous function from (X,T) into (Y,S). Then, for the non-zero fuzzy open set λ in $(Y,S), int[f^{-1}(\lambda)] \neq 0$, in (X,T). Because (X,T) is the fuzzy hyperconnected space, by the proposition 3.3, int $[f^{-1}(\lambda)]$ is the fuzzy resolvable set in (X,T).

(ii). Now int $[f^{-1}(\lambda)]$ is the fuzzy open set in the fuzzy hyper-connected space (X,T) shows that Cl $\{Int[f^{-1}(\lambda)]\} = 1$ and Cl $\{Int[f^{-1}(\lambda)]\} \leq Cl[f^{-1}(\lambda)]$ implies that $1 \leq Cl[f^{-1}(\lambda)]$. That is, Cl $[f^{-1}(\lambda)] = 1$, in (X,T). Hence $f^{-1}(\lambda)$ is the fuzzy dense set in (X,T).

(iii). Now by (i), int $[f^{-1}(\lambda)]$ is the fuzzy resolvable set in (X,T). Then, by the theorem 2.21, $1 - Int[f^{-1}(\lambda)]$ is also the fuzzy resolvable set in (X,T). Now Cl $(f^{-1}(1-\lambda)) = Cl(1 - f^{-1}(\lambda)) = 1 - int[f^{-1}(\lambda)]$, implies that Cl $(f^{-1}(1-\lambda))$ is the fuzzy resolvable set in (X,T). □

Proposition 4.2. *If $f : (X,T) \to (Y,S)$ is the fuzzy continuous function from the fuzzy hyperconnected space (X,T) into the fuzzy topological space (Y,S), then for the non-zero fuzzy open set λ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in (X,T) such that Cl $(f^{-1}(\lambda)) = 1$ and Cl $(f^{-1}(1-\lambda)) \neq 1$.*

Proof. Let $f : (X,T) \to (Y,S)$ be the fuzzy continuous function from (X,T) into (Y,S). Then, for the non-zero fuzzy open set λ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy open set λ in (X,T). Because (X,T) is the fuzzy hyperconnected space, by the proposition 3.1, $f^{-1}(\lambda)$ is the fuzzy resolvable set in (X,T).

Since (X,T) is the fuzzy hyperconnected space, the fuzzy open set $f^{-1}(\lambda)$ is the fuzzy dense set and then Cl $(f^{-1}(\lambda)) = 1$ in (X,T). By the theorem 2.23, (X,T) is the fuzzy irresolvable space and hence for the fuzzy dense set $f^{-1}(\lambda)$ in $(X,T), Cl(1 - f^{-1}(\lambda)) \neq 1$. This shows that cl $(f^{-1}(1-\lambda)) \neq 1$, in (X,T). □

Proposition 4.3. *If $f : (X,T) \to (Y,S)$ is the fuzzy contra-continuous function from the fuzzy hyperconnected space (X,T) into the fuzzy topological space (Y,S), then for the non-zero fuzzy open set λ in $(Y,S), f^{-1}(\lambda)$ and $f^{-1}(1-\lambda)$ are fuzzy resolvable sets in (X,T).*

Proof. Let $f : (X,T) \to (Y,S)$ be the fuzzy contra-continuous function from (X,T) into (Y,S). Then, for the non-zero fuzzy open set λ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy closed set in (X,T). Then $1 - f^{-1}(\lambda)$ is the fuzzy open set in (X,T). Because (X,T) is the fuzzy hyperconnected space, by the proposition 3.1, $1 - f^{-1}(\lambda)$ is the fuzzy resolvable set in (X,T). This gives that $f^{-1}(1-\lambda)$ is the fuzzy resolvable set in (X,T). By the theorem 2.21, $1 - [1 - f^{-1}(\lambda)]$ is the fuzzy resolvable set in (X,T) and thus $f^{-1}(\lambda)$ is the fuzzy resolvable set in (X,T). □

Proposition 4.4. *If $f : (X,T) \to (Y,S)$ is the fuzzy semi- continuous function from the fuzzy topological space (X,T) into the fuzzy topological space (Y,S), then for the non-zero fuzzy open set λ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in (X,T).*

Proof. Let $f : (X,T) \to (Y,S)$ be the fuzzy semi-continuous function from (X,T) into(Y,S). Then, for the non-zero fuzzy open set λ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy semi-open set λ in (X,T). Then, by the theorem 2.17, λ is the fuzzy simply open set in (X,T) and by the theorem 2.5, 3.1, $f^{-1}(\lambda)$ is the fuzzy resolvable set in (X,T). □

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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